Handling haptics as a stand-alone medium

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ABSTRACT

This paper provides an information theoretic perspective of haptic rendering that aims to formulate the necessary theoretical foundations to lead to a compact and holistic representation of haptic media. Initially, an information theoretic view on haptic rendering is presented that enables the development of metrics enabling measuring of haptic information. Following the principles of information theory, two novel haptic information rendering pipelines are provided. Moreover, several quantities like entropy, mutual information and filters are defined and instantiated for the case of haptic rendering. Explicit examples of their potential use are analyzed. The paper concludes with examples and a discussion on how information theory can be used to provide a holistic haptic media representation scheme that can be used in a variety of application including coding, indexing and rendering of stand-alone haptic media.

Keywords

Haptic rendering, information theory, haptic coding, indexing.

1 INTRODUCTION

As information and communication technologies become more mature the amount of information generated and exchanged exceeds every previously imaginable limit. While this amount of information is expected to double each year, it is mainly comprised of media information [Chinchor10] including images, videos, sounds, etc. From a holistic media perspective, their historical evolution clearly demonstrates an increment in the dimensionality of the underlying media sources. Starting from two-dimensional still images, moving to the three-dimensional moving picture, further adding one and two dimensions of mono- and stereo-sound respectively and recently also including the sense of depth in stereoscopic visual representations, currently available and widespread media can be considered as six dimensional. Even if more dimensions can be assumed for different features like color, etc. they are considered as features of the 6D space.

The aforementioned increment in the dimensionality of media has been quite remarkable leading to impressive and immersive media. However, new dimensions will be inevitably added in the media of the future, possibly including digital representations of signals triggering the human senses, that have not been addressed so far in a systematic way, like touch, olfaction and taste. While all three senses could provide a significant added value in the realism and quality of the future media, research in the haptics domain has progressed significantly to allow for an attempt of systematic and formal definition, representation, processing and rendering of the underlying haptic media.

Figure 1, illustrates the complex issue of media source and interrelation, from an information source perspective. On the left part the physical environment is the source of all “direct” media. In particular, light interactions, pressure oscillations and collisions/interaction are considered as the source of “direct visual”, “direct audio” and “direct haptic” information respectively. It is evident that all direct media are correlated, since they are due to the same source even if they are caused by different interactions. Moreover, the “media space” is augmented by auxiliary information that is characterized as “symbolic”. For example “symbolic visual” information includes some forms of visual effects, overlays or even augmented reality renderings; “symbolic audio” is used to describe audio effects or music, while term “symbolic haptic” information is used to describe the synthetic haptic information, including haptic icons [MacLean08]. It should be emphasized that even if the symbolic media are not correlated per se, they could exhibit some correlation depending on the particular case.

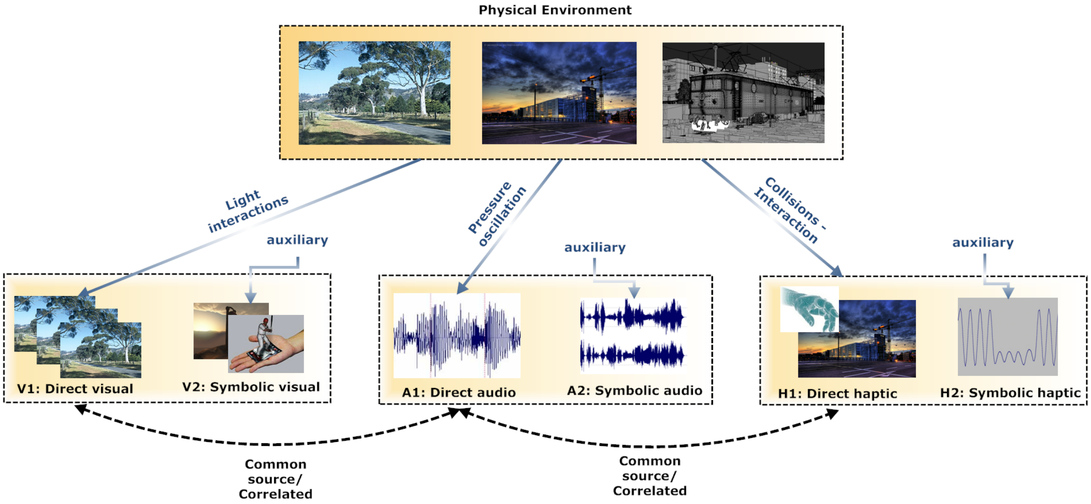

Figure 1: View of haptics as a medium. Left: Physical environment is the source of all "direct" media. Light interactions, pressure oscillations and collisions/interaction are considered as the source of "direct visual", "direct audio" and "direct haptic" information respectively. Auxiliary symbolic media can be added for all communication channels. "Symbolic visual" information includes some forms of visual effects, "symbolic audio" is used to describe audio effects or music, while term "symbolic haptic" information is used to describe the synthetic haptic information, including haptic icons.

The proposed paper makes a first attempt to define a theoretical framework of haptic media from an information theoretic perspective that could be in turn used as a mathematical background for several processes including representation, processing, rendering and streaming transmission of haptic media.

1.1 Related work

Human perception combines information of various sensors, including visual, aural, haptic, olfactory, etc., in order to perceive the environment. A very descriptive analysis on the importance of the sense of touch is given in [Torre06] through the effects of its loss that include inability to eat and walk. On contrary to the audio and vision channels, the haptic media have not yet been addressed systematically from an information theoretic perspective, leading to non-holistic, fragmented and problem-targeted research.

Haptics research can be divided into three main categories [Lin08]: Machine Haptics, Human Haptics and Computer Haptics [Srinivasan97]. Machine Haptics is related to the design of haptic devices and interfaces, while Human Haptics is devoted to the study of the human perceptual abilities related to the sense of touch. Computer Haptics, or alternatively haptic rendering, studies the artificial generation and rendering of haptic stimuli for the human user. It should be mentioned that the proposed framework takes into account recent research on human haptics, while it provides mathematical tools targeting mainly the area of computer haptics.

The simplest haptic rendering approaches focus on the interaction with the virtual environment using a single point [Moustakas06], [Moustakas07b], [Kaklanis09], [Kaklanis10]. Many approaches have been proposed so far both for polygonal, non-polygonal models, or even for the artificial generation of surface effects like stiffness, texture or friction [Moustakas07], [Moustakas07b], [Laycock07], [Kostopoulos07], [Nikolakis06]. The assumption, however, of a single interaction point limits the realism of haptic interaction since it is contradictory to the rendering of more complex effects like torque. On contrary, multipoint, or object based haptic rendering approaches use a particular virtual object to interact with the environment and therefore, besides the position of the object, its orientation becomes critical for the rendering of torques [Laycock07], [Nikolakis06]. Apart from techniques for polygonal and non-polygonal models [Laycock07], voxel based approaches for haptic rendering [Petersik01] including volumetric haptic rendering schemes [Palmerius08] have lately emerged. Additionally, research has also tackled with partial success the problem of haptic rendering of dynamic systems like deformable models and fluids [Barbic09], [Cirio11].

From information theoretic perspective, it is worth mentioning that surprisingly coding and compression of haptic data has not been researched extensively so far. Most of the approaches deal with aspects of haptic data transmission in the context of telepresence and teleaction systems, focusing mainly on stability and latency issues [Hirche05], [Ou02], [Souayed03], [Kron04]. Differential and entropy coding has been successfully applied in [Kron04], while other traditional approaches including, DPCM ADPCM, Huffman coding have been applied to haptic signals in [Kron04], [Ortega02] and [Hikichi01] respectively. In [Borst05] predictive coding has been applied for haptic signals in order to optimize the communication in teleaction systems with respect to sampling rate, while in [Hinterseer08], [Kuschel09], [Zadeh08], [Vittorias09] perceptual coding mechanisms are developed for haptic data streams. The major limitation of the current approaches is that, since they are targeting mainly telepresence and teleaction systems, they refer mainly to representation and coding of single haptic timeseries that correspond to the force that should exert a remotely manipulated device.

1.2 Motivation and contributions

In general, with the exception of some approaches related to haptic rendering of distance or force fields [Moustakas07], [Barlit07], one of the biggest limitations of current schemes is that haptic rendering is considered only as a result of the interaction of a human user with an underlying (usually 3D) environment. Even if this approach is inspired by the real world, where the sense of touch is triggered by collisions of the human body with the environment, it has three significant drawbacks: i) haptic media cannot be defined as a signal related to a specific physical environment, ii) Off-line processing of haptic media for optimization, compression, indexing, etc. becomes impossible, iii) The requirement for 1kHz update rate, that is considered as the most significant constraining factor imposed on haptic rendering algorithms [Laycock07], can rarely be satisfied due to the need for on-line processing of the entire haptic rendering pipeline as also mentioned in a high-level theoretical attempt on haptic media using the existing concepts on haptic interaction described in [Cha09].

The proposed framework aims to provide a new view on haptic rendering from an information theoretic perspective that will in turn provide the necessary theoretical foundations to lead to a compact and holistic representation of haptic media. As a result all three aforementioned limitations will be theoretically overcome providing a new potential for the field of haptic rendering. In particular, the proposed approach does not restrict its framework on interaction-based rendering, but provides a holistic approach, that can also accommodate synthetic authored haptic media, where interaction-based rendering is a special case.

The rest of the paper is organized as follows. Section 2 outlines the links between information theory and haptic rendering and presents three haptic rendering pipelines. In Section 3 well known information theoretic quantities are introduced and instantiated from a haptic rendering perspective, while Section 4 presents how device and perceptual haptic filters could be designed and applied in the context of the proposed framework. Finally, Section 5 discusses potential applications and open problems for future work, while conclusions are drawn in Section 6

2 INFORMATION THEORY AND HAPTIC RENDERING

The typical models of a communication system has been used as a basis to develop information theory [Shannon48]. Following a similar methodology, this section describes how haptic rendering can be seen as a communication channel and how typical quantities of information theory can be adopted to more efficiently describe haptics as a medium.

Figure 2a illustrates a typical communication system; The source is generating data that are compressed in the “encoder” and transmitted as a signal through the communication channel. At the receiver the signal is decoded and the resulting data is provided to the destination for rendering or processing. It should be emphasized that the diagrams of Figure 2 refer only to source coding. Even if it is possible to derive correspondences between channel coding and haptic rendering as well, this issue remains out of the scope of the present paper.

Figure 2c illustrates a general block diagram that is used in many recently proposed systems for coding and transmission of haptic information. In particular based on traditional collision and reaction models, the force feedback of the interaction of a specific haptic-probe with the environment is initially estimated. Then the force is filtered, usually to avoid force dicontinuities, force effects are added, including texture or friction, while the final resulting force is mapped onto the specific display capabilities, e.g. resolution, workspace. Then the resulting time-series is transmitted using traditional coding-decoding approaches and rendered at the receiver. The major characteristic of this approach, called so forth “soft HR system”, is the fact that the force at the transmitter side at each time-step is calculated based on the actions of the end-user at the receiver side. This issue reduces from the one side the amount of haptic information to be transmitted but on the other side, introduces latency issues and places severe limitations when targeting at explicit and stand-alone haptic media that can be processed and rendered as a single entity, without the need to know the user actions beforehand.

Towards this target, Figure 2b depicts a general potential “strong HR system” that implies that haptic information is available as a multivariate time-series that

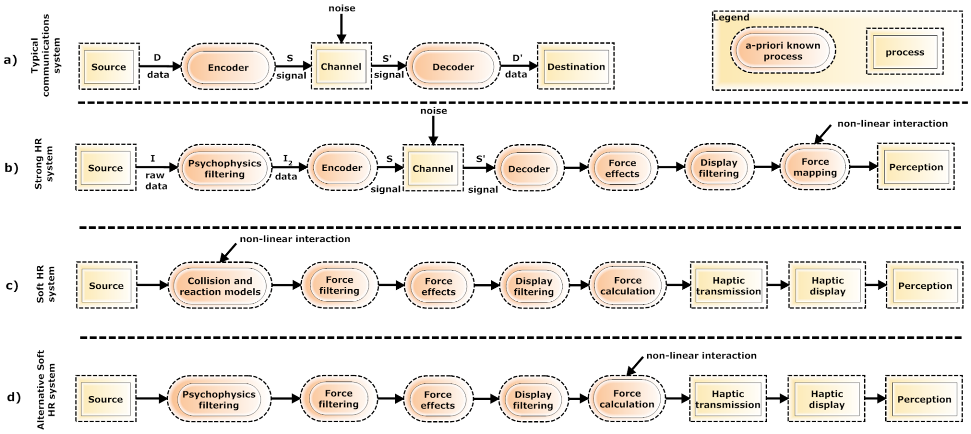

Figure 2: Haptic rendering from an information theoretic perspective. a) Typical communication system, b) Strong haptic rendering (HR) system: Haptic media are encoded in a similar manner to Figure 2a. Since force mapping is performed at the decoder it is device agnostic, c) Soft HR system: Is based on interaction-based haptic rendering; block “Collision and reaction models” is influenced by the end user actions and haptic information is transmitted only for the necessary degrees of freedom that are rendered, d) Alternative Soft HR System: Follows the approach of Figure 2b placing however all haptic rendering processes at the encoder side, thus being device specific.

needs to be compressed and transmitted, after applying psychophysics filtering, through a communication channel. At the decoder the raw data are recovered and enriched with force effects and mapped for rendering on a specific display. This scheme has the advantage that haptic media is actually encoded as a time-varying, device agnostic vector field that can be used by any kind of haptic rendering approach. Special effects and force mapping are added at the receiver side according to potential metadata sent by the transmitter or even based on the preferences of the receiver.

Similarly, Figure 2d introduces an alternative to Figure 2b and Figure 2c communication system that tries to capture some interesting properties of both sides. Actually the main difference between the “Alternative Soft HR system” and the “Strong HR system” is that it places the force effects and display filtering and mapping at the transmitter side. This approach still treats haptic information as a stand-alone entity, while it also reduces the amount of information to be transmitted with respect to the “strong HR system”. On the other side, it is less general since different receivers would require different streams to be transmitted, which is not ideal if haptic media would need to be transmitted in a broadcasting sense.

In the following, the potential use of several information theoretic quantities for the case of haptic media is described, that can be applied for both the “Strong HR system” and the “Alternative Soft HR system”.

3 MEASURING HAPTIC INFORMATION

In the following the major information theoretic quantities, namely entropy and mutual information, that will be used in the subsequent analysis will be described and adapted for the case of haptic media.

3.1 Entropy

Entropy is maybe the fundamental measure in information theory. It can be seen either as the uncertainty of a given random variable or as the theoretical minimum number of bits that are required to represent the variable.

The entropy of a discrete random variable **F** that takes values in an alphabet **A** and has a probability mass function $p_{\mathbf{F}}(f)$ is given by

$$H(F) = -\sum_{f \in A} p_F(f) \log_2 p_F(f) \qquad (1)$$

Let us know consider, without loss of generality, a haptic workspace of $5 \times 5 \times 5$ voxels. For each point in space in between the voxel centres, tri-linear interpolation is used to render the force if necessary. Moreover let us also assume that the force that is attributed to each voxel is quantized and takes values from an alphabet of size 256. The probability mass function of each voxel is independent and identically distributed, so there is $p = 1/2^8$. Now letting V denote the random variable for a single voxel and S denote the full workspace, the total entropy is calculated as follows:

$$H(S) = \sum_{i=1}^{125} H(V_i) = -\sum_{i=1}^{125}\sum_{j=1}^{256} \frac{1}{2^8} \log_2 \frac{1}{2^8} = 1000 \qquad (2)$$

The above result is theoretically expected since for each voxel 1 byte is needed to represent the force value and there are 125 voxels in the workspace.

Haptic workspace capacity:

It is reasonable to assume that in most applications the reference space or virtual space is of different size compared to the haptic workspace. Let us now assume a mapping M of the haptic information of the actual space into haptic information of the haptic workspace. The latter maybe restricted by several parameters including haptic display limitation on the workspace size, resolution, force exertion amplitude limitations, arithmetics, etc. Let us know define as Workspace Capacity $C(M)$ the average amount of information that the specific haptic workspace setting can render. Since the above parameters are usually constant $C(M)$ is the entropy of a random variable of the specific mapping M. For a specific input haptic space W, following the principle of the data processing inequality the following equation holds:

$$C(M) = \min\left(C(M(W)), H(W)\right) \tag{3}$$

where the *min* function encodes the fact the mapping is passive, i.e. no energy/information can be generated through the mapping procedure, e.g. in the typically not usual case where the haptic information space is less detailed than the haptic workspace itself.

Since the quantity C has the same unit as entropy H, we can define the following measures:

$$Workspace\ Mapping\ Ratio\ (WMR) = \frac{C(M)}{H(W)} \tag{4}$$

$$Haptic\ Inf.\ Loss\ (HIL) = \frac{\max\left(H(W) - C(M), 0\right)}{H(W)} \tag{5}$$

The above measures are meaningful under the reasonable constraints $H(W) > 0$ and $C(M) > 0$. The WMR reflects the amount of information that is transferred for haptic rendering through the haptic workspace, while the HIL encodes the amount of information that is lost in the above procedure.

3.2 Joint Entropy and Mutual Information

The fact that haptic interaction involves in general several computationally intensive procedures like 3D mesh manipulation leads researchers in the adoption of a level-of-detail (LoD) haptic rendering scheme. This means that each object or space partitioning element can be rendered in different scales. For example a user might need initially to interact with the complete environment and then zoom in a specific area so as to analyze it in more detail and in higher resolution.

In information theoretic terms this case can be described through different mappings M. In particular there is one mapping related to the whole scene M_{all} and several LoD mappings $M_{LOD(i)}$. Obviously M_{all} and $M_{LOD(i)}$ are related.

Let us know assume two random variables X and Y that represent M_{all} and $M_{LOD(i)}$ respectively, assuming that the latter act as mappings on the input space Z shared by both full and LoD representations. Let also $H(X,Y)$ represents their joint entropy that is defined through their joint pmf $p_{X,Y}(x,y)$, $x \in A_x$, $y \in A_y$:

$$H(X,Y) = -\sum_{x \in A_x} \sum_{y \in A_y} p_{X,Y}(x,y) \log_2 p_{X,Y}(x,y) \tag{6}$$

It is interesting to mention that, since entropy is a measure of uncertainty, the triangle inequality $H(X,Y) \leq H(X) + H(Y)$ "points out" that having two correlated views reduces uncertainty with direct consequences in haptic data coding. Similarly the inequality of the conditional entropy between two random variables $H(X|Y) \leq H(Y)$ indicates that the LoD rendering is heavily influenced by the by the overall rendering information.

Similarly mutual information between the two random variables X and Y is defined as follows:

$$I(X,Y) = \sum_{x \in A_x} \sum_{y \in A_y} p_{X,Y}(x,y) \log_2 \frac{p_{X,Y}(x,y)}{p_X(x)\, p_Y(y)} \tag{7}$$

or

$$I(X,Y) = H(X) + H(Y) - H(X,Y) \tag{8}$$

The mutual information $I(X;Y)$, that specifies the amount of information provided by Y about X, is symmetric and non negative. Mutual information can be thus very easily used as a high level information theoretic similarity measure between haptic media.

In particular similarity can be estimated on a region basis e.g. LoD representation, so as to be able to estimate entropies. There is, however, one important drawback to mutual information as a way of comparing vector fields; it fails to take the topology into account since it is calculated only over vector/force values, and not voxel positions. This limitation can be however overcome by increasing the dimensionality of the problem and including geometry information.

4 DEVICE AND PERCEPTUAL HAPTIC FILTERS

Haptic media are assumed to be defined within the input haptic space W that refers to a virtual environment augmented with haptic information. W actually defines

Property	Phantom Omni	Phantom Desktop
Workspace	160Wx120Hx70D mm	160Wx120Hx120D mm
Position resolution	0.055 mm	0.023 mm
Continuous Force	0.88 N	1.75 N

Table 1: Haptic devices properties: "Workspace corresponds to the size of the volume where the haptic probe can move, "positional resolution" to the sensing resolution in mms and continuous force to the maximum force that can be continuously applied on the device.

a mapping of the $\Re^3$ Euclidean space into the $\Re^3$ force field that refers to the force exerted to a point object lying on the specific point in space. It should be emphasized that in the general case the haptic space evolves over time and therefore the static W representation becomes a timeseries W_t.

Device Haptic Filters: It is apparent that since haptic media are defined in $\Re^3$ compared to $\Re^2$of the visual media, their storage complexity becomes in the general case $O(n^3)$. However, unfortunately haptic displays have not evolved similarly to visual displays and therefore the spatial resolution they can provide to the user is extremely low with respect to the sensing potential of humans. This is reflected to a very low workspace mapping ratio (equation 4) that can be rendered using typical devices. In information theoretic terms we call this effect as device filter that limits the amount of information that can be perceived. The device filter actually performs a quantization of both the input space $\Re^3$ in terms of the device spatial resolution and the $\Re^3$ output space in terms of the force magnitude that can be exerted. Considering now without loss of generality, the case of input space quantization, the device filter is a workspace mapping C_D as defined in Section 3.1, where the workspace refers to the specific haptic device workspace characteristics.

Now let us consider the case of two specific popular haptic devices, namely the Sensable Phantom Omni and Phantom Desktop. Table 1 presents some of their basic properties of interest in our analysis.

Now let us emphasize on the position resolution property, where the Phantom Desktop exhibits almost double the resolution of the Omni. Consider now a static haptic signal S_W defined in a space $Q \in R^3$ and sampled with a resolution of $0.01mm$. Assuming now that the size of Q is equal to the workspaces of the haptic devices, it becomes evident that the haptic information loss for the case of the Omni HIL_O will be always higher or equal to the one of the Desktop HIL_D. In other words, when using the Phantom Omni a lower amount of information is perceived by the end user, since the device filter of the Omni C_O filters out more information with respect to the Desktop filter C_D. Therefore, following the concept of the "alternative soft HR system", illustrated in Figure 2d, a device dependent encoding scheme would transform (e.g. resampling, lowpass filtering, etc.) the initial signal S_W into a new one S_T that produces minimal HIL. This transformation can be formulated as an optimization problem as follows:

$$\mathbf{t} = \arg\min_{\mathbf{t}} \left(HIL\left(S_T, C_D\right)\right)$$

where $\mathbf{t}$ is the state vector of the transformation of S_W into S_T.

Now, for the case of the "hard HR system", illustrated in Figure 2b, the transmitter should encode the signal as is, unless information is available about the supported haptic devices. In the latter case the transformation should be applied only for the device of higher resolution and therefore the zero HIL would be valid only for this specific device

Perceptual Haptic Filters:

Besides device display capacity, the perceptual capabilities of the human with respect to temporal resolution have specific limitations [Hirche05], [Hinterseer08], [Kuschel09] that can be used in order to compress haptic media.

Some approaches presented in the literature focusing on telepresence and teleaction [Hinterseer08] tackle the problem of perceptual coding, for the specific case of point-based interaction, trying to define masking thresholds of force differences between consecutive frames beyond which a difference in the force fed back to the user cannot be perceived.

The complexity of this problem for the general case of haptic media and general multi DoF haptic devices is very high, since it does not only depend on the temporal relation between two successive force stimuli but also on the body part that they are applied, the spatial proximity of concurrent stimuli etc. Since, a detailed analysis is out of the scope of present paper, without loss of generality, let us consider the "deadband" approach described in [Hinterseer08].

In particular, in [Hinterseer08] a perceptual mechanism for haptic data compression is proposed based on Weber's Law and the "Just Noticeable Differences" (JND) principle. At a glance, for specific timeseries of multidimensional in the general case haptic data, at a specific time instance the respective haptic sample is trasmitted only if it will be perceived by the user, i.e. if it lies outside the perceptual mask defined by the previously transmitted samples.

This procedure can be seen as a *perceptual filter* that is applied on the input data stream modifying its entropy. Then all quantities described in Section 3 are applicable on the perceptually filtered input. Now the design problem lies on the definition of a perceptual, multivariate in the general case, filter that takes into account the potential haptic rendering schemes, in terms of number of degrees of freedom, perceptual correlation between them, etc.

5 APPLICATIONS AND DISCUSSION

The potential applications of an information theoretic framework of haptic rendering are numerous. First of all, the proposed representation is general, holistic and can accommodate interaction-based rendering as a special case. Moreover, typical multimedia operations including off-line processing, like haptic editing, optimization and indexing become possible. Additionally, in the previous sections more potential applications were outlined including LoD haptic rendering and compression, similarity estimation and design of device and perceptual haptic filters necessary for haptic media coding. Other interesting applications include optimal adaptive workspace partitioning in disjoint subspaces so as to minimize information loss taking into account the rendering device or even correlation analysis between haptic media and visual media, etc.

However, concerning limitations, it should be emphasized that the dimensionality of haptic perception is in the best case much higher with respect to visual perception as very well described in [Hayward11] and in the worst case infinite. Therefore, the transition of interaction-based rendering to open-loop stand-alone streaming haptic media, may introduce a significant increment in dimensionality resulting also in vast amount of haptic information to be transmitted and rendered if not dealt with explicitly. Typical solutions of the computer graphics research community could be employed, like space partitioning and culling. However, to the author's view and as a future research direction, a more fundamental theoretical treatment of dimensionality reduction is necessary so as to result in a complete and tractable information theoretic framework of haptic media. Finally, the proposed framework has to prove its applicability in several applications scenarios, which is a challenging direction for future work.

6 CONCLUSIONS

In this paper an information theoretic view of haptic rendering is presented. Following the major principles of audiovisual communications, several information theoretic quantities are instantiated for the case of haptic rendering and their potential use in the design of haptic media filters and challenging applications is outlined. It should be emphasized that the proposed framework aims to highlight a different way on how haptic rendering could be potentially dealt with, targeting at haptic media that can be processed, edited, indexed as a stand alone entity and not only as a result of the interaction between a user and the media space.

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