Interactive Exploration and Analysis of Pathlines in Flow Data

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ABSTRACT

The rapid development of large-scale scientific computing nowadays allows to inherently respect the unsteady character of natural phenomena in computational flow simulation. With this new trend to more regularly consider time-dependent flow scenarios, an according new need for advanced exploration and analysis solutions emerges. In this paper, we now present three new concepts in pathline analysis which further improve the abilities of analysis: a multi-step analysis which helps to save time and space needed for computation, direct pathline brushing, and the use of pre-configured view arrangements. We have found that a clever combination of these three concepts with already existing methods creates a very powerful tool for pathline analysis. A solution that follows the concept of coordinated multiple views (CMV) with iterative composite brushing enables a quick information drill-down. We illustrate the usefulness of this approach in the context of an example from the automotive industry.

Keywords: Interactive Visual Analysis, Pathlines, Flow Visualization, Exhaust Manifold

1 INTRODUCTION

Advances in hardware and simulation technology make it possible to simulate and visualize flows in time-dependent vector fields. In contrast to a steady state flow, in this case the unsteady vector field also changes over time itself. The analysis of time-dependent flows is far from trivial and still a largely unsolved problem in visualization [15]. There are many ways to visualize flows, and in the case of unsteady flow some very intuitive methods are based on pathlines. Pathlines describe paths of massless particles over time in the flow and thus the analysis of their behavior is tightly related to the analysis of the dynamic behavior of the underlying flow fields.

Interactive visual analysis (IVA) [19, 9] is useful to analyze large and complex data. As a set of pathlines represents such a case, we illustrate how IVA can help domain experts in engineering to analyze simulated time-dependent scenarios.

Our approach starts with the computation of pathlines and a set of pathline attributes. These attributes can be either scalar values or functions of time (time series). We have focused on attributes describing the (local or global) behavior of the pathlines. We focus on classical and well-established attributes from vector algebra (for the unsteady flow field) or differential geometry (for the pathlines). The result of this step is a multivariate dataset collecting all computed pathlines and features and the attributes stored along them. The pathlines in the dataset are seeded at different points in time as the seeding time plays an important role in pathline characteristics.

Our IVA approach utilizes coordinated multiple views (CMV) with linking and brushing. Besides usual 2D views, used to visualize derived attributes, we added a 3D view which depicts the volume geometry and the pathlines themselves. To save space and time, as required by the computation and analysis, we introduce a multi-step analysis, where in one step a lower resolution (bigger cell size) dataset is investigated. In the next step a higher resolution (smaller cells) dataset is visualized and only spatio-temporal areas which were identified in the previous step as interesting are then analyzed further in more detail. This refinement can be repeated several times, but often two steps are already enough.

We also introduce the use of projections, as a way to do direct pathline brushing (a data visualization technique that identifies and highlights data subsets), and pre-configured view arrangements, to help the user with the screen space organization. All of this, combined into one tool, contributes new opportunities to the analysis of pathlines in flow data.

We demonstrate the usefulness of our approach in the context of an exhaust manifold analysis. An exhaust manifold collects the exhaust gases from multiple cylinders into one pipe from an internal combustion engine. The goal of the exhaust manifold design is to decrease flow resistance (back pressure) and to increase the volumetric efficiency of the engine, resulting in a gain in power output or a reduction of fuel consumption. As a team of visualization researchers and a domain expert, we analyzed one such design together. The main goal was to check if the proposed design meets initial design requirements, and, if not, which directions for improvement could be suggested. We illustrate how the clever selection of pathline attributes can support domain experts in the analysis of time-dependent flows through the interactive visual analysis of pathlines and their attributes.

2 RELATED WORK

The idea to segment a flow domain into areas of certain flow properties has been used for 3D steady flow fields for a variety of features, including topological features [12, 26], or vortex regions [14]. Salzbrunn and Scheuermann [17] provide a general framework of this in the context of topological features. Laramee et al. [10] and Salzbrunn et al. [16] give an overview on flow visualization techniques focusing on feature extraction approaches. These techniques are used on 3D time-dependent fields by observing the feature regions over time, observing either topological features [23, 4], or vortex features, see Theisel et al. [22]. Although these approaches provide insight into the flow behavior at arbitrary time steps, the analysis of the dynamic behavior based on pathlines makes specialized approaches necessary. Wiebel and Scheuermann [28] visualize a number of carefully selected pathlines to get static representations of the dynamic flow. Theisel et al. [24] consider a segmentation of the flow domain based on local properties of the pathlines.

Combining several feature detectors in order to investigate combinations of them using IVA has been suggested by Bürger et al. [2]. Shi et al. [18] have a very similar approach to ours, and we build on that adding more attributes, the multi-step approach, (enabling us to work with much bigger datasets interactively), the use of projections as a method to enable direct pathline brushing, and pre-configured view arrangements for better screen space organization.

Texture-based visualization approaches to capture pathline characteristics are given by Weiskopf et al. [27]. The idea of connecting information visualization and scientific visualization approaches is considered to be one of the ‘hot topics’ in visualization [8].

Due to space limitations, we refer to Pobitzer et al. [15] for a more detailed discussion of related work.

3 EXPLORING PATHLINES

Advances in simulation and hardware technology make it now possible to simulate time-dependent flow fields, i.e., flow fields where the underlying vector field changes over time. The analysis of steady state flows (where the underlying vector field does not change) is a relatively mature research field with many good visualization and feature extraction methods. However, the extension to the unsteady state is not straightforward and is an active area of research [15]. The amount and complexity of data in an unsteady state case often grows to such an extent that the fully automatic methods are not sufficient any more. In order to efficiently cope with such a large and complex datasets, IVA approaches (such as presented by Konyha et al. [8]) balance human advantages (perception, experience, imagination) with those of automatic methods. The interactive visual analysis helps the user to be sure that nothing is missed and gives the user a better understanding of the flow by visualizing computed results and allowing interaction with them. It can also be used as a support tool to train novice engineers.

When analyzing the flow, experts are usually interested in problematic areas in the flow, which result, for example, in vortices and swirling. Such a behavior can be easily identified using pathlines and the attributes computed from them.

The main idea is to compute a set of pathlines, as well as a set of attributes for each pathline, and to use a coordinated multiple views system to analyze the data. The data in this case follows a more complex data model than the usual data model used in interactive visual analysis. As we want a pathline to be an entity, we build records in our dataset around pathlines. This means that one record contains pathline coordinates, various scalar attributes (such as, e.g., length, distance between start and end point, etc.), and various time series attributes (such as, e.g., velocity as a function of position along the pathline, curvature, etc.). Scientific data often follow such a data model (see Konyha et al. [9]), and IVA has been proven to be useful for analyzing such data. An IVA tool that supports such data has to be able to depict a family of curves, i.e., a certain time series attribute given for all pathlines, for example all velocity functions. Konyha et al. [9] introduced the curve view and the accompanying line brush which can be used to quickly drill down by simply selecting (brushing) or deselecting curves that cross a line drawn by the user.

A state of the art CMV system which can show several views simultaneously and that also supports linking and brushing (selecting records in one view and highlighting corresponding items from the same records in all other views) is a prerequisite for such an approach. The pathlines data scenario, due to its complexity and

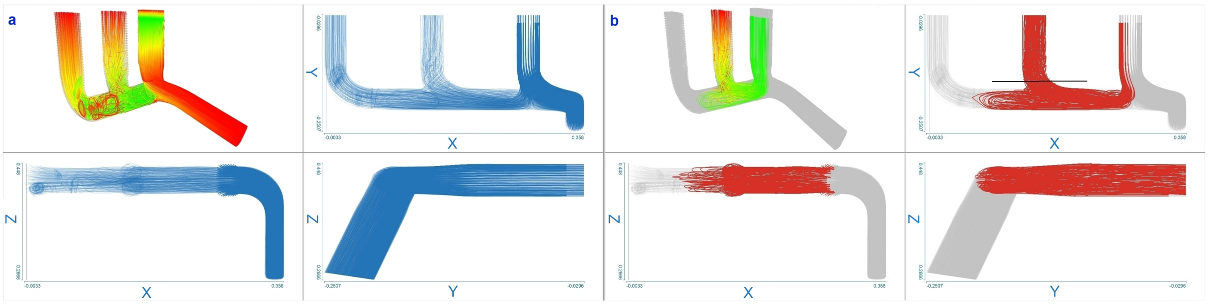

Figure 1: An example of using projection views in the pre-configured view arrangement, which the user can get with one click, to explore the pathlines (about 10000 pathlines in this case). The pre-configured view arrangement consists of a 3D view (top-left) and three projection views (top-right, bottom-left, and bottom-right views). Time is color mapped [13] in the 3D view going linearly from green, for the smallest, to red, for the biggest value, in this case. Other standard color mappings (e.g., rainbow color map, grayscale color map, cyan to mauve color map, etc.) are also available. **a**: 3D view and projection views. **b**: All pathlines passing through the middle tube are brushed in the top-right projection view.

size, requires an especially careful and innovative design to provide an effective exploration tool.

Three key aspects of improvement over the existing methods were identified: multi-step analysis, direct pathline brushing, and the usage of pre-configured view arrangements. The complete set of all pathlines would be really huge in most cases and it would often contain a lot of irrelevant data. There are areas in the flow where a domain expert can the predict flow with a great confidence, and where no interesting features appear, so it is not meaningful to compute a huge number of pathlines in such areas. Our multi-step approach will help us to reduce the data size without losing important information. Furthermore, we need to depict pathlines themselves (in addition to their attributes) and be able to interact with them directly. The curve view shows families of planar curves, but the pathlines are curves in 3D space, so brushing them directly on the 2D screen is difficult. As a solution we present direct pathlines brushing (detailed further below). And finally, in order to allow the user to focus on the actual analysis, we have seen that pre-configured view arrangements are crucial. A clever combination of those three aspects supports an efficient time dependent flow analysis by means of IVA wrt. pathlines and their attributes.

In the next three subsections, each of the identified aspects is described in detail, and in the last subsection available pathline attributes are shortly covered.

3.1 Multi-Step Analysis

An important observation, when thinking about pathlines, is that, depending on the volume size and the resolution (cell size), the pathlines dataset can be very large. This can make IVA very slow. In order to efficiently cope with such large datasets a multi-step analysis approach is proposed. The main idea is to analyze a coarse, lower resolution dataset, and identify possible spatio-temporal (position in the volume and time steps) areas of interest. In case of pathline analysis, the areas of interest will, for example, be the areas were pathlines pass through vortices. Those identified areas of interest are then analyzed in more detail using a higher resolution dataset. This process can be repeated as many times as necessary, but our experience says that often two steps are enough, with the exception of very big or very complex datasets. The identified areas can be further refined at any time and in any of the lower resolution datasets from the previous steps.

The multi-step analysis saves storage space and time needed for the computation and analysis depending on the resolution used and size of the areas of interest. To demonstrate it we tried to compute all pathlines (seeded at the areas of interest as identified by the domain expert) and 16 attributes (5 scalar and 11 time series attributes), on a high resolution (about 2 million cells) dataset for 25 time steps. As assumed, the progress was very slow. It took several hours to compute only about 20% of the pathlines and their attributes, and the produced dataset was over 20GB in size. It is obvious that, from IVA perspective, this is not the right way, so the computation was stopped. Such a large dataset contains many uninteresting pathlines and it unnecessarily uses computational and storage resources. The handling of such a large dataset can also be a problem. Our multi-step approach proved to be much more efficient. Using a lower resolution (36524 cells) dataset to compute pathlines (seeded in the same areas as before) and the same 16 attributes for 25 time steps resulted in a dataset of about 500MB in only a few minutes. After several areas of interest were identified interactively, in the second step a higher resolution (2 million cells) dataset was used to compute pathlines and the 16 attributes seeded only in those areas. This computation again took several minutes, only, and the result was less than 1GB in size. A more detailed IVA of these areas was then possible. The described process resulted in datasets consisting of at most 10000 pathlines in this case.

There are some questions when using this multi-step approach about how to be sure we will not miss anything in the low resolution dataset, where to seed, etc. A careful decision making of the domain expert is required. With the knowledge of the dataset (how complex it is, how big the velocities are, etc.) a decision wrt. the resolution (minimizing the risk of missing important features in the flow) can be made without too much effort and in short time.

3.2 Direct Pathlines Brushing

Using attributes and composite iterative brushing, the user can find pathlines of unwanted (or wanted) behavior very easily, and drill down even from tens of thousands of pathlines to just one in just a few iterations (in many cases it can be done even in one). Still in some cases it would be very useful to be able to brush the pathlines directly. Immediately something like the already mentioned line brush from the curve view comes to mind. But since we are dealing with pathlines in 3D on the 2D screen this would be very hard to use. So the idea of doing orthogonal projections (onto X-Y, X-Z and Y-Z planes) and then using the line brush in the projection views was born. An example of how this looks can be seen in Figure 1. This way, the user can brush pathlines directly using line brushes the same way he/she would do it in the curve view for time series, and to be able to do it precisely, also, zooming is available. Projections also help in getting a better perspective of the position of some point in the 3D volume when looking at it on 2D screen, which in combination with 3D view's translation, rotation and zooming also gives user a better understanding of pathlines' behavior in 3D space.

3.3 Pre-configured View Arrangements

When doing interactive visual analysis very often multiple screen setups are used. The user can have a lot of views open and that can result in an unorganized screen space, where it is hard to focus on the actual analysis. To reduce this problem we propose using pre-configured view arrangements, which help user in working efficiently and focus on what he/she is actually searching for, and not on view arrangement and screen space organization. Identifying standard tasks and providing pre-configured view arrangements for solving those tasks should be used as often as possible. Also the user is given an option to create and edit pre-configured view arrangements. The type of views and their arrangement is chosen by the user and saved. New instance of the pre-configured view arrangements is then available with just one click on the appropriate toolbar button. To enable easier access to projections, pre-configured view arrangement, which can also be seen in Figure 1, was created.

Pathline Attributes		
Geometry Based	Scalar	Length
		Relative start-end distance
		Average particle velocity
		Seeding cell
		Seeding time
	Time Series	Curvature
		Torsion
		Winding angle
		Particle velocity
		Pathlines' coordinates
Vortical Structures	Field Based	Vorticity magnitude [20]
		Swirling strength [25]
		λ_2 [6]
		Hunt's Q [5]
	Core Detectors	Helicity method [11]
		Eigenvector method [21]
		Eigenvector method for unsteady flow [3]

Table 1: Table showing available pathline attributes and vortical structures. All vortical structures are saved as a time series along the pathlines.

3.4 Pathline Attributes

We distinguish between two groups of attributes. Attributes computed from the pathlines' geometry (e.g., their curvature) and attributes computed from flow field characteristics along the pathline. For both groups we can have scalar attributes (e.g., the pathline length) or time series attributes (e.g., torsion along the pathline). For some of the time series attributes additional scalar aggregates, such as maximum or arithmetic mean values, are computed, depending on analysis questions. Table 1 shows attributes that we propose, and which proved to be very useful when used in different combinations. Of course any other attribute can be added depending on the task faced. Most of them are well known and for the description of some more complex ones the reader is referred to Jiang et al. [7].

All of the methods described in the last four subsections complement each other and, when combined intelligently, they form a powerful tool for pathline analysis. In the next section, a few examples are given of how this tool can be utilized and what can be found.

4 APPLICATION EXAMPLE

The dataset we studied in this application case is the result of a flow simulation of an exhaust manifold. The simulation was done using the AVL FIRE tool [1] and the result was a dataset consisting of 36524 cells for the first step (low resolution) and about 2 million cells for the second step of the analysis. This exhaust manifold is used in a six cylinder car, but we have only analyzed one side (exits from three cylinders) since the sides

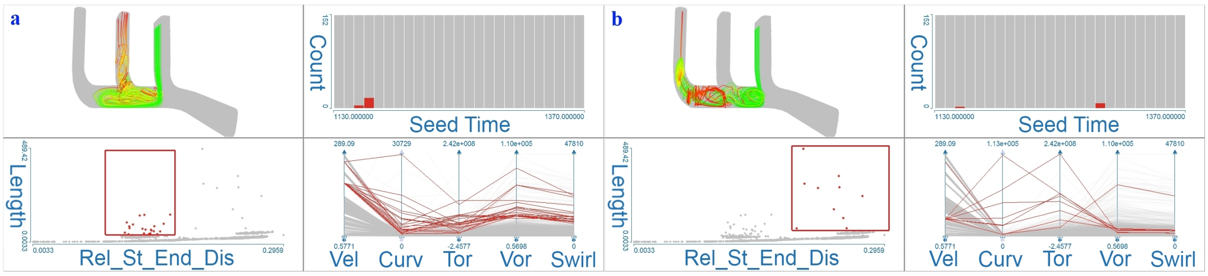

Figure 2: An example of data exploration using scalar attributes in histograms, scatterplots and parallel coordinates. A 3D view with color mapping enabled is used to depict pathlines. **a**: pathlines with medium relative start-end distances and high length values are brushed. In the 3D view we can see that those are the pathlines finishing somewhere in the left pipe. **b**: pathlines with high relative start-end distances and high length values are brushed and in the 3D view we can see that those are the pathlines finishing somewhere in the middle pipe.

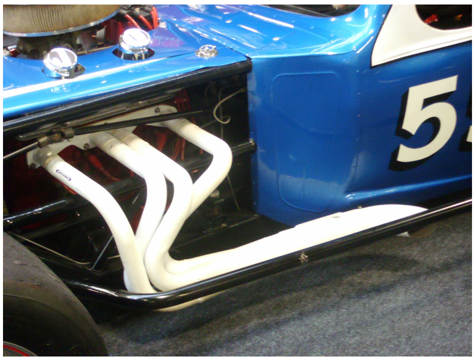

Figure 3: An example of a performance-car manifold. (Image courtesy of http://www.zircotec.com.)

are symmetrical. From the simulation dataset pathlines and their selected attributes are computed. The dataset proved to be very challenging because of its pulsing nature, where only one pipe (on one side) is active at specific time points and for only a short period of time. Also, as velocity values of the flow field are very high we had to pay special attention to how to compute pathlines, as they would otherwise leave the, in our case, relatively small volume very early. To cope with that, we decided to use cellular integration instead the one with a constant time step. Cellular integration iterates cell by cell and builds a pathline, thus making sure that all the vectors of the flow field along the pathline way are taken into account. This frees the user of the sometimes difficult decision of which integration time step to use, especially in cases of very high velocities. The resulting datasets consisted of up to 10000 pathlines and their attributes (5 scalar and 11 time series attributes).

4.1 Exhaust Manifold

Every internal combustion engine has an exhaust manifold. It is generally a simple unit that collects engine exhaust gases from multiple cylinders and delivers them to the exhaust pipe. Each cylinder has its own exhaust head-pipe, and they usually converge into one tube called a collector.

When an engine starts its exhaust stroke, the piston moves up the cylinder bore, decreasing the total chamber volume, which increases the pressure in the cylinder, and when the cylinder's valve opens, the high pressure exhaust gas exits into the exhaust manifold, creating an exhaust pulse comprising three main parts. They are: high-pressure head, medium-pressure body, and the low-pressure tail component. The momentum of the high and medium pressure components reduces the pressure in the combustion chamber to a lower than atmospheric level. This relatively low pressure helps to extract all the combustion products from the cylinder and induct the intake charge during the overlap period when both intake and exhaust valves are partially open. The effect is known as scavenging. Length, cross-sectional area, and shape of the exhaust ports and pipeworks influence the degree of scavenging effect, and the engine speed range over which scavenging occurs.

Selecting the length and diameter of the primary tubes must be done very carefully depending on what we want to accomplish (more power, lower fuel consumption, ...).

4.2 Identifying Areas of Interest

To get a better overview over the time series attributes some first order statistics (mean, max, ...) were computed. These newly computed scalars were depicted using scatterplots, parallel coordinates, and histograms. Also we had the 3D view active showing pathlines with color mapping [13] (mapping of the colors to the values) enabled, so we could see if the features found in other views as interesting are the result of the interesting behavior of the pathlines. For color mapping we use time, particle velocity, or any other time series attribute of the ones listed in Table 1. The flow field features stored as time series along the pathline proved to be very useful, since they give us an insight into what made pathlines behave as they do.

One such example can be seen in Figure 2. In the scatterplots (lower-left views), depicting relative start-

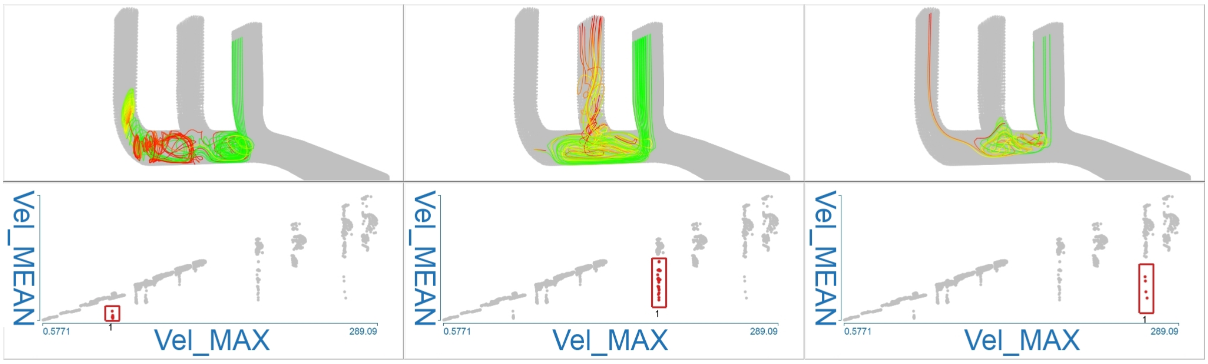

Figure 4: One more example of using scalar attributes to identify areas of interest. Attributes depicted in scatter plots are particle velocity mean and maximum. We can clearly see how brushing different clusters in the scatter plot views (bottom views) results in pathlines of different behavior being selected (top views). Also similarities within the groups are very obvious.

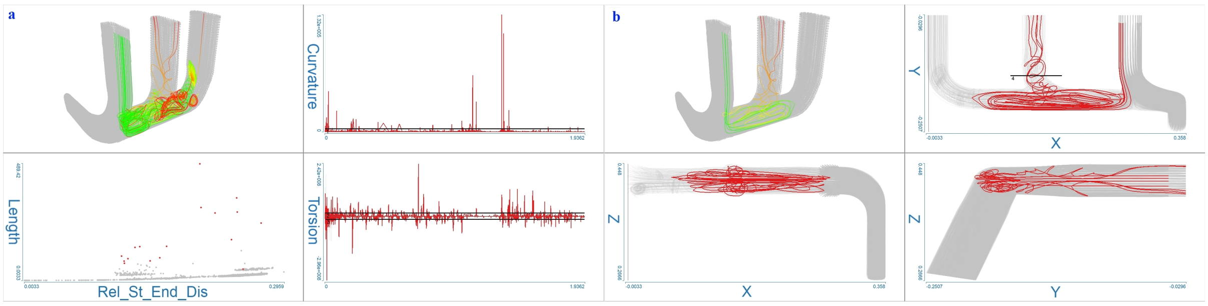

Figure 5: **a**: Finding "problematic" pathlines using composite brushing in two curve views depicting curvature and torsion. High curvature and torsion, as a stable indicator of pathlines forming a vortex somewhere along its way, are brushed. This way almost the same pathlines are selected as in previous two cases, but with less control (there are no clusters, i.e., groups are not as easy to identify). **b**: using pre-configured view arrangement for projections to brush the pathlines going to the middle tube. We add one more brush to the composite brush from a, thus getting better control over what is selected.

end distances and length attributes, clusters are clearly recognizable, so we investigated what we get when we brush some of them. The most interesting proved to be clusters with relatively high length values. We have selected pathlines with high length, and medium (Figure 2a) and high (Figure 2b) distance between start and end. There are other pathlines that traveled the same distance (gray points in the scatterplots below the brush), but they have shorter length. The higher length of the brushed pathlines is the result of vortices along their way, and this is confirmed when looking at the parallel coordinates view with relatively high maximums of torsion, curvature, vorticity magnitude, and swirling strength values. As, in the case of an ideal exhaust manifold, gases would travel as fast as possible to the exhaust, this is certainly an unwanted behavior.

Continuing the exploration, we investigate other attribute combinations which support the analyst in getting insight into the flow. Having multiple ways to identify possible problems gives us a better chance to identify all of them. Since different attribute combinations (both scalar and time series) are used in these approaches, it also gives us a better understanding of the attributes. We can find correlations between them, and thus get better insight into the dataset.

Another such example, using scalar attributes, but in a different combination, can be seen in Figure 4. We use average and maximum particle velocity attributes. As can be seen, using a different combination does result in a different clustering. In this case, clusters are even more easy to notice and brushing them reveals groups of pathlines, with the behavior of the pathlines being very similar within a group, which was not always the case in the previous approach, where these three groups were mixed into two. So it seems that we have found a better way (that gives us better control) for finding pathlines of interest.

Using time series attributes we get a similar result, as can be seen in Figure 5a. Two curve views are used to depict time series attributes, which are in this case curvature and torsion. Again almost the same pathlines are selected, but in this case we do not have as good control over what is selected as in previous cases. To gain better control we can use the projection views with their direct pathline brushing ability, as can be seen in Figure 5b.

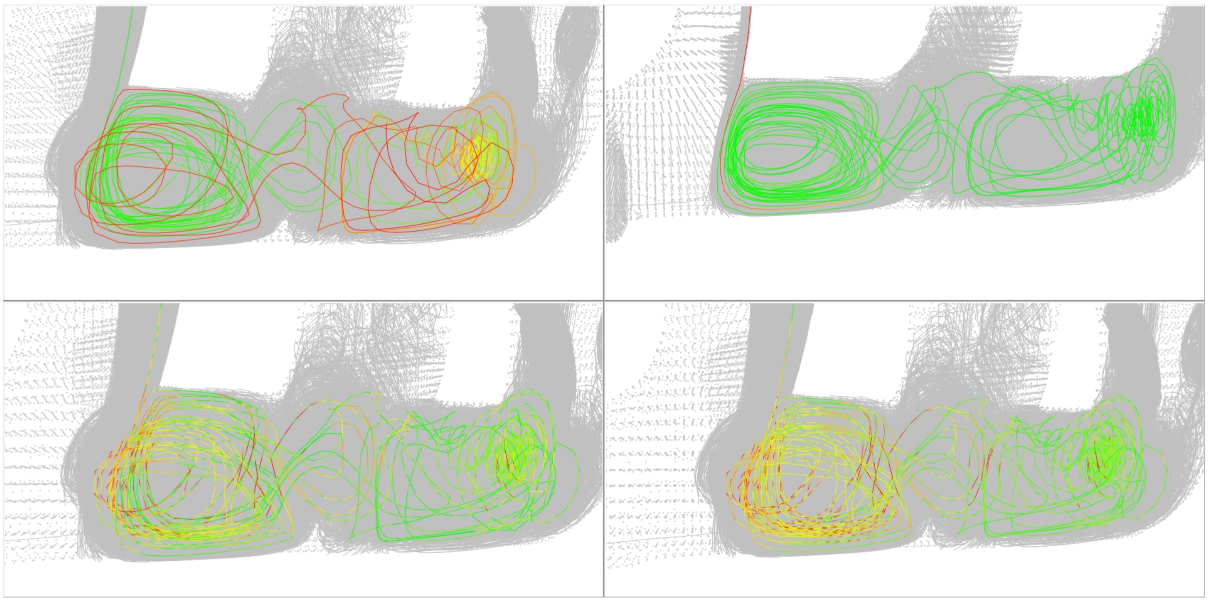

Figure 6: Analysis of the particle velocity along the pathline. We use one pathline with four attributes color mapped: time (top-left), particle velocity (top-right), swirling strength (bottom-left), and vorticity magnitude (bottom-right). Color mapping is done from green to red, green representing minimum value and red representing maximum value.

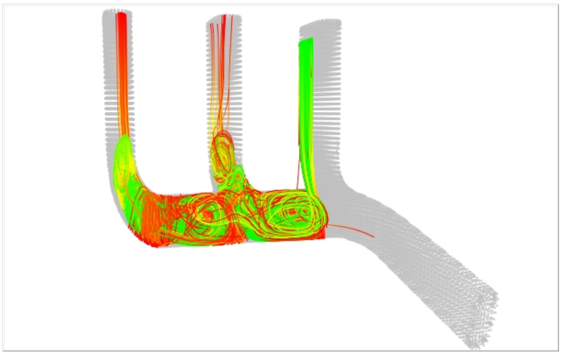

Figure 7: Result of the higher resolution simulation for the seed cells of pathlines brushed in Figure 2b. Immediately it is obvious that, unlike in lower resolution case, here we have some pathlines ending in the middle pipe.

What can be seen in all cases is that none of the brushed pathlines goes out through the expected pipe, but instead they finish in the left or middle pipe, which is undesirable. Also, almost all of these pathlines are seeded or pass near the edges of the pipes. The according histogram views show us that this happens only in few time steps, and for a small amount of cases, but still this could be an indication of a problem in the exhaust manifold. To gain a better understanding why is this happening we use the finer (higher resolution) dataset for those time steps and seeding points and then do a more detailed analysis to see what can be done, if anything, to reduce them even more.

4.3 Detailed Analysis

For the detailed analysis based on the high-resolution dataset, pathlines are computed only in the spatio-temporal areas of interest identified in the previous step. What becomes immediately obvious, when looking at Figure 7, showing a 3D view depicting pathlines computed in the higher resolution dataset, but only for the seed cells of pathlines brushed in Figure 2b, is that, unlike in lower resolution case, now we have pathlines ending in the middle pipe also. This shows how complex the flow is and that very small changes can drastically change the result.

Next we focus on one interesting pathline from the pathlines shown in Figure 7. Using color mapping with time series attributes and also different types of views with attributes we try to discover as much as we can about the behavior of that pathline, i.e., the behavior of the particle in the flow. Figure 6, which shows four 3D views depicting the same pathline, but with different attributes color mapped, shows an example of such an analysis. Our focus here was on particle velocity (color mapped in the top-right view in the figure). We can see that the particle velocity dropped very rapidly once it started changing direction, and the reason that it did change direction is because it entered areas of high swirling strength and vorticity magnitude which can be seen in the two bottom views.

With this we have shown an example of how our tool can be used in the analysis of the exhaust manifold dataset. The domain expert found it very interesting, and thinks it has a great potential, especially when used in combination with other tools, and we will continue our collaboration.

5 CONCLUSION AND FUTURE WORK

In this paper we have shown how pathlines in combination with a carefully designed tool can give a user new abilities in analyzing the flow field. Pathlines and their attributes are used to help identify possible problems in the exhaust manifold which can cause loss of engine power and increase of fuel consumption.

We show how computing power limitations can be avoided by first using lower resolution to identify spatio-temporal areas of interest and then using higher resolution for detailed analysis of those areas. Furthermore, direct pathline brushing by using projections is introduced. Also, usage of pre-configured view arrangements, to help with screen space organization, is proposed.

The domain expert gave us positive feedback and we plan to continue our work on this subject in the future, specially focusing on the selection of the attributes available and combining our tool's results with other tools. We think that our tool is a step forward and can lead to further developments in this area.

6 ACKNOWLEDGMENTS

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