Spectral Modeling and Virtual Restoration on a Polychrome Medieval Sculpture

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ABSTRACT

The presented work is led in the framework of a general collaboration between three academic labs, industrial partners and Cultural institutions (Centre des Monuments Nationaux, Louvre museum). Such a pluridisciplinary work always in progress at Ecole Centrale Paris deals with 3D digitization, simulation, rapid prototyping, virtual restoration applied on a french medieval sculpture. The main purpose is to virtually represent a polychrome statue of the XIIIth century in high quality spectral rendering, to simulate its visual and original appearance at that period. The complete process used throughout all the phases of the project mainly involves optical devices that ensure no physical contact with the museum object. This article describes the complete chain of engineering resources and the main models we used for accomplishing our objective. From 3D capture without contact to plaster replica, the complete process will be described and illustrated with images and objects during the conference. Some sequences extracted from the didactic and scientific movies produced will also be presented.

Keywords Ray-tracing ; spectrophotometry ; 4-fluxes model ; sculpture ; virtual restoration ; gilding ; painting.

Figure 1: The recumbent statue of Philippe Dagobert in Saint-Denis Basilica (13th century AC).

1 INTRODUCTION

A collaboration with the "Centre des Monuments Nationaux" (CMN) helps us to elaborate a new project : the study of a polychrome medieval statue, the recumbent statue of Philippe Dagobert de France (circa 1222 - 1232 AC) in the Saint-Denis Basilica (Fig. 1), the royal necropolis in the north of Paris.

The presented work takes place in a previously defined general framework known "OCRE method" (Optical Constants for Rendering Evaluation) [CALLET, 1998], developed by Patrick Callet and describing the optical behaviour of materials on the basis of their fundamental properties.

For metals and alloys we naturally used their complex indices of refraction, either computed or carefully measured, but, in any case always validated by measurements. These indices were measured or validated by spectroscopic ellipsometry. These previous important results are very useful for the rendering of the golden parts of the statue.

Moreover, the last study about the visual influence of the under-laying paint, called "bole", on a gilded surface has shown the importance of the transparency of a thin metallic gold film and of the associated small cracks due to the application process (called "burnishing")[DUMAZET et al., 2007].

We deal with modelling and representing the paint materials using spectrophotometry and extrinsic physical parameters such as the paint film thickness and the concentration of each species of pigments. To achieve this goal, we used an extension of the Kubelka and Munk theory : a four-fluxes approach of the multiple scattering of light for the simulation.

We were helped by the "Centre des Monuments Nationaux" (National Monuments Centre) and its knowledge of the painted stones during the middle-age in order to study the pigments and binders that were probably used by the artist.

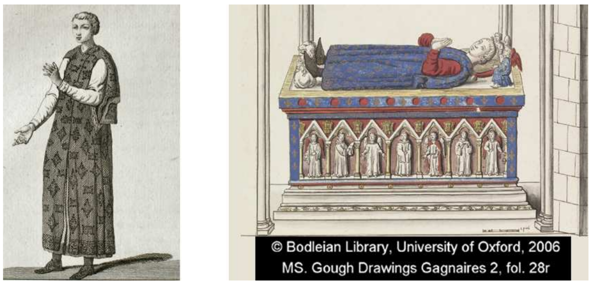

Figure 2: Philippe Dagobert - (left) Stained glass window drawing, (right) Drawing of the tomb with its recess.

2 HISTORICAL CONTEXT

This recumbent statue has been chosen for its remaining polychrome traces (Fig. 1) and also because it was representative of the fine middle XIIIth century funeral sculpture. It concerns the Philippe-Dagobert's tomb, a young Saint-Louis's brother born in 1222 and dead in 1232 and buried in Royaumont (North of Paris).

This statue has encountered several dramatics events. With the french revolution, it has been transferred in Saint-Denis in 1791, then damaged in 1793-94 and deserted until 1796 when it has been preserved in the *Musée des monuments français*. In 1816 it returns to Saint-Denis. In 1820, the architect François Debré order a restoration campaign but, as Baron de Guilhermy has published [de Guilhermy, 1848], this works was already severely damaged by the time in 1848. Then the famous architect Viollet-le-Duc, between 1860 and 1867, inspired by the original pieces, rebuilt entirely the sarcophagus.

Luckily, Millin could study and draw the Philippe-Dagobert tomb in Royaumont as it was in 1790, just before it was transferred to Saint-Denis [Millin, 1791]. Most of his observations were confirmed by a drawing made in 1694 for Roger de Gaignières, which represents a missing stained glass window in the Royaumont abbey-church with the young prince up and wearing the same clothes than his recumbent statue (left figure 2). There is also an other reproduction of the original coloured drawing conserved in the Bodleian Library in Oxford (right figure 2).

As attested on the picture (Fig. 1) there are still some traces of paint and gilding on the stone at the contrary of the other graves gathered in the basilica. First we have tried to retrieve the used pigments with a method based on spectrophotometry without any contact. Then it was decided to take some samples for discriminant chemical analyses.

3 DATA ACQUISITION

Two kinds of data are necessary for the visual reconstruction of the statue. Shape and spectrophotometric informations are required. The first one is more frequently used in computer graphics while the second is generally replaced by a trichromatic description of the actual colors.

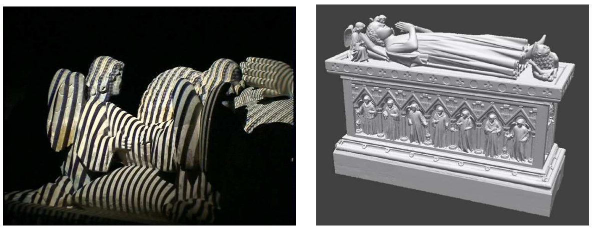

Figure 3: (left) 3D digitization using structured natural white light. (right) 3D colorless model displayed with Catia software (Dassault System).

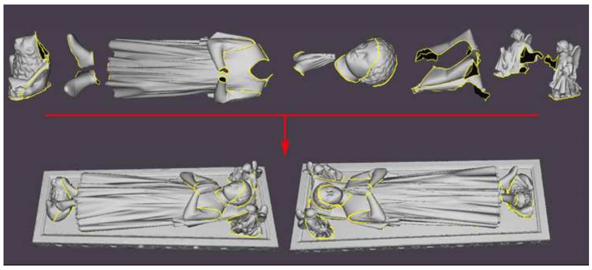

Figure 4: 3D model cut up not completely decomposed in color parts.

3.1 3D shape acquisition and virtual reconstruction

Like previous projects, the 3D digitization has been performed *in situ* (in Saint-Denis Basilica) after public hours, without contact using an optical system based on structured light projector and a camera (Fig. 3) from Breukmann corp. A regular light and shadow grid is projected onto the object surface and the distortion of the boundaries between light and shadows on the surface is recorded by the camera and analysed, in order to extract a cloud of 3D points with a good accuracy. This gave us 167 clouds of 3D points used for reconstructing the surface with RapidForm, a software from Inus Technology corp. The final 3D shape of the tomb (recumbent statue on the sarcophagus) in full resolution represents about 7,5 Million triangles (Fig. 3).

3.2 Spectrophotometric data acquisition

The portable spectrophotometer (USB 2000 series from Ocean Optics corp.) consists in a set of optical fibers guiding a calibrated light source (a halogen lamp for CIED65 illuminant) surrounding the backscattered light guiding fiber. The latest is connected to a spectrometer where a 1024 photodiodes array transforms the received light in a digital signal. We then obtain the diffuse reflectance factor of the paint according to the visible wavelengths domain (from 380*nm* to 780*nm*). Also *in situ*, we captured spectral information on the remaining traces of paint thanks to a spectrophotometer with optical fiber useful for further analyses and material characterization.

 ISBN 978-86943-15-2

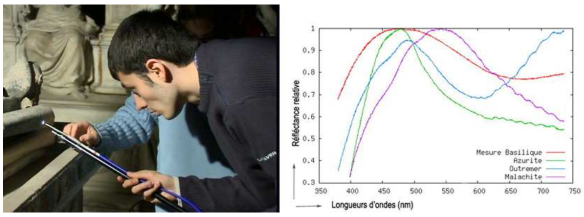

Figure 5: Spectral data acquisition *in situ*, reflectance factors layout and analysis.

Figure 6: Medieval paint samples preparation.

The portable spectrophotometer and the recorded refectance factors are shown in Fig. 5.

As we need to compare and validate permanently our choices and models, we also used pigments and binder samples prepared in the laboratory. We also made our spectral measurements on these samples (Fig. 5).

3.3 Painting materials

During the Middle Age and more particularly during the thirteenth century, we already know, [Escalopier, 2004] what pigments are the most commonly used and also how the paints were applied on the sculpture.

First, the artists start with two or three layers of ceruse, made out of water and white lead. This step enables to waterproof the stone but also enables the painters to rectify the surface defects of the stone by coating. Then they apply several layers of paint, consisting of pigments embedded in a binder like egg (tempera technique), animal protein or gum-resin. The most frequently encountered pigments are:

- White : White Lead, White Chalk, Lime;
- Black : Wood Smoke, Black Vine;
- Blue : Azurite, Lapis-Lazuli, Indigo;
- Red : Vermilion, Red Lead, Red Ochre;
- Yellow : Yellow Ochre, Massicot;
- Green : Green Clay, Malachite, Verdigris.

Which of them were used in the specific case of Philippe Dagobert's recombent statue? The recorded diffuse reflectance factors, obtained by the only possible analysis involving a non-destructive method allow us to compare them to the results obtained by the chemical analysis for validation and simulation.

We also need to specify which part of the global reconstructed shape would be associated with each kind of materials (pigments and binder or/and gilts) (Fig. 4).

4 THE KUBELKA-MUNK MODEL EXTENDED TO FOUR-FLUXES THEORY

Many works are related to spectral representation of colours ([Sun et al., 2000], [Rougeron and Peroche, 1997], [Devlin et al., 2002], [Gondek et al., 1994]) and pigmented materials in computer graphics ([Haase and Meyer, 1992], [Baxter et al., 2004]) using the Kubelka-Munk theory. Our first concern here was, to use an extended Kubelka-Munk model to four fluxes to better fit a modeling of paintings and to realize our calculations based on 81 wavelengths bands of 5nm over the visible spectrum [380;780]nm. Our second concern was in our entire process to constantly go back and forth between our theoretical models and our measurements on real materials. And above all, helped with our collaborations to apply these concepts in an archeological, and historical approach. The Kubelka-Munk model is almost well suited for the description of pigmented materials. These are described as scattering media, laying on a scattering background. The system is illuminated by a diffuse orthotropic incident light. It can be demonstrated ([Volz and Teague, 2001, CALLET and ZYMLA, 2004]), that an incident orthotropic flux of light on such a film of thickness h can be considered equivalent to a collimated directionnal and normal incident flux on a paint film of thickness $2h$. This model gives the reflected and the transmitted fluxes from an incident light, normal to the layers across a paint film of thickness h laid on a substrate. The plain Kubelka-Munk theory [KUBELKA and MUNK, 1931] is then reduced to an equivalent 2-fluxes theory involving two directional fluxes of light and, according to the previous remark above, one going downward L^+ and the other upward L^-. The layer of paint is a macroscopic scattering and absorbing medium so we consider these two coefficients: S and K, respectively the scattering and absorption ones. These last coefficients depend on the pigments diameters and consequently on the granulometry distribution function. A fine grinding involves an important multiple scattering inside the paint film and gives a desaturated colour obtained without any addition of white pigments. This is a

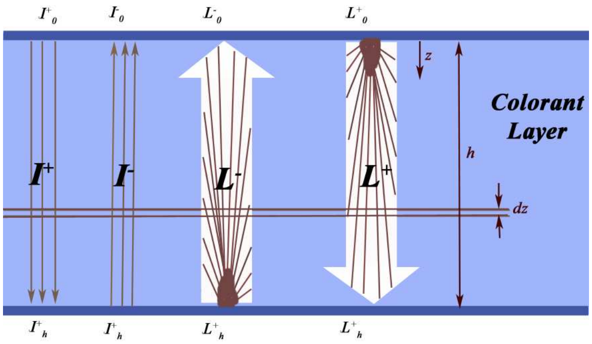

Figure 7: Four-Fluxes theory: the directional (l^+,l^-) and diffuse (L^+,L^-) fluxes, together with the boundary conditions.

physical whitening only. Further works will use the micro-stratigraphy images for extracting a best estimation of the granulometry distribution function. We saw that the 2-fluxes model work on a scattering layer deposited on a scattering background but there are cases where we have a paint layer on a gold leaf. That's why we need the extension of the model to 4-fluxes and, conveniently we use 2 additional, directional, fluxes of light l^- and l^+. These two fluxes are added to the model with the two previous diffuse fluxes, one downward L^+ and the other upward L^- (as described in [Volz and Teague, 2001]).

The figure 7 shows the principle of this model. The incident light is then decomposed in a remaining directional reflected light and an additional scattered light due to volume and surface scattering from the substrate. Two specular components are then added to the classical Kubelka-Munk model.

4.1 Getting the model parameters

The model need five parameters, K, S, k', s^+ and s^-, respectively the absorption and the scattering coefficient for the diffuse light, the absorption, forward scattering and the backward scattering coefficients for directional light. These parameters are fixed after a local radiative balance by the four following equations. Note that all the involved terms are wavelength dependent.

$$dl^+ = -(k+s^+ + s^-)l^+ dz \quad (1)$$

$$-dl^- = -(k+s^+ + s^-)l^- dz \quad (2)$$

$$dL^+ = s^+ l^+ dz + s^- l^- dz - (K+S)L^+ dz + SL^- dz \quad (3)$$

$$-dL^- = s^- l^+ dz + s^+ l^- dz - (K+S)L^- dz + SL^+ dz \quad (4)$$

To get the parameters we start by computing the K and s parameters by setting the fluxes l^+ and l^- equal to zero. Then the solution of the above system of equations, leads to the plain 2-fluxes Kubelka and Munk expressions. At this step we need some measurements on some samples of the material we want to characterize. To get them, we need to prepare three samples:

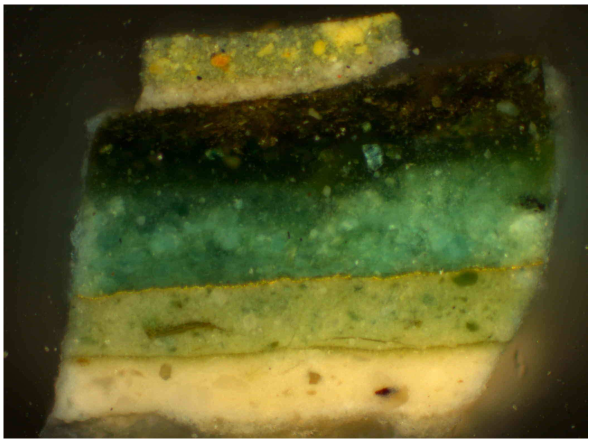

Figure 8: Microstratigraphy of a paint scale taken on the green bottom cushion. Each successive layer gives information about its mean thickness and the granulometric distribution of the embedded pigments. Magnification x20.

- a sample with the substrate only. It's reflectance will be noted R_g ;
- a sample with a totally opaque layer of the considered paint. It's reflectance will be noted R_∞ ;
- a sample of a non-opaque layer of pigments on the substrate with reflectance R and thickness h.

These measurements are made with a spectrophotometer for the reflectances and we get the thickness h with micro-stratigraphic images (Fig. 8).

Next, we deduce the reflectance R_0 of a layer with a thickness h on a perfect black background:

$$R_0 = \frac{R_\infty(R_g - R)}{R_g - R_\infty(1 - R_g R_\infty + R_g R)} \quad (5)$$

Then we get S, K the macroscopic scattering and absorption coefficients:

$$S = \frac{1}{h}\frac{R_\infty}{1-R_\infty^2}\ln\frac{R_\infty(1-R_0R_\infty)}{R_\infty - R_0} \quad (6)$$

$$K = \frac{1}{2h}\frac{1-R_\infty}{1+R_\infty}\ln\frac{R_\infty(1-R_0R_\infty)}{R_\infty - R_0} \quad (7)$$

The four-fluxes model requires the specification of all the terms k, s_i, s_j from the optical coefficients. Let:

$$a = 1 + \frac{K}{S}, \qquad b = \sqrt{\frac{K}{S}\left(\frac{K}{S}+2\right)}$$

$$x = bhS, \qquad A = asinh(x) + bcosh(x)$$

from which we deduce the following parameters

$$\tau = \frac{b}{A}, \qquad \rho_1 = a - b, \qquad \rho_2 = \frac{sinh(x)}{A}$$

 ISBN 978-86943-15-2

We then obtain from the previous relationships:

$$T_r = \left(\frac{R - R_0}{R_g}\right)^{\frac{1}{2h}}$$

and:

$$p = \frac{R_\infty(1 - \tau T_r) - R_0}{\rho_1(1 - \tau T_r) - \rho_2}$$

$$q = \rho_1 p - R_\infty$$

Next, inverting p and q equations leads to the scattering coefficients:

$$\left\{ \begin{array}{l} s_i = p(\mu - aS) + qS \\ s_j = pS - q(\mu + aS) \end{array} \right\}$$

where $\mu = \frac{1}{h} \ln \frac{1}{T_r}$

Finally we can deduce the absorption coefficient for a very thin layer:

$$k = \mu - (s_i + s_j)$$

4.2 Computing the reflectance

With the five parameters we can compute the reflectance of a paint layer with any thickness h and over any background if we have the substrate reflectance R_g. Let $R_{gs}(\theta_i, \theta_r)$, the substrate specular reflectance function and R_g, the substrate diffuse reflectance. θ_i and θ_r are respectively the incident and the reflection angles. The computation of the diffuse part, R_d, uses the Kubelka and Munk formulas. We start by the calculation of the R_∞ term:

$$R_\infty = 1 + \frac{K}{S} - \sqrt{\frac{K^2}{S^2} + 2\frac{K}{S}} \tag{8}$$

Then we can compute R_d:

$$R_d = \frac{(1/R_\infty)(R_g - R_\infty) - R_\infty(R_g - 1/R_\infty)e^{Sh(1/R_\infty - R_\infty)}}{(R_g - R_\infty) - (R_g - 1/R_\infty)e^{Sh(1/R_\infty - R_\infty)}} \tag{9}$$

For the directional part, R_s, we have to compute the fluxes l^+ and l^-. The forward part l^+ is computed with (1). We get:

$$dl^+(z)/dz + (k + s^+ + s^-)l^+(z) = 0 \tag{10}$$

and the solution to this differential equation is:

$$l^+(z) = l^+(0)\, e^{-(k+s^+ + s^-)\, z} \tag{11}$$

We obtain a similar formula for l^- from the equation (2). The absorption term $A(x)$ is:

$$A(x) = e^{-(k+s^+ + s^-)\, x} \tag{12}$$

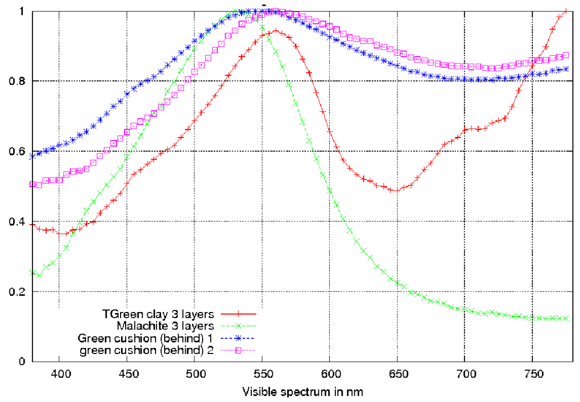

Figure 9: Comparison of the *in situ* measured diffuse reflectance factors with those of their corresponding handcrafted samples.

where x is the effective geometrical thickness of the paint layer (equal to h at normal incidence). We can deduce R_s:

$$R_s(\theta_i, \theta_r) = A\left(\frac{h}{\cos\theta_i}\right) A\left(\frac{h}{\cos\theta_r}\right) R_{gs}(\theta_i, \theta_r) \tag{13}$$

Finally we can compute the final reflectance:

$$R(\theta_i, \theta_r) = R_d * \cos(\theta_i) + R_s(\theta_i, \theta_r) \tag{14}$$

We use here a lambertian model for the diffuse contribution.

5 SPECTRAL SIMULATION AND VIRTUAL RESTORATION

To achieve our goal, the optical properties of the paint compounds are required. As we cannot alter the painting in any way, we have to get the parameters from methods that don't require any contact. We used a methodology that rely on spectrophotometry and spectral sample matching. So we create some samples (Fig. 6) of paint layers and tried to match their reflectance factor with the ones measured on the statue. These samples were created with pigments known to be used when the statue was created. For example, the spectral measurements helped in differentiating a lapis-lazuli from an azurite film (Fig. 10). The first one has two peaks, one in the blue wavelengths near 455nm and the other in the red and the near IR wavelengths region (780nm). Only one peak appears for the azurite pigments. The Lapis-lazuli and its subtle reddish component is known as natural ultra-marine blue, very efficient for reflecting infra-red radiations (ideal paint used for the shutters in the mediterranean countries). We can also notice that the two pigments have different maxima in the blue region of the visible spectrum. One exhibits a relatively narrow peak while an other has a more flat and extended peak tending to cover the green shades region. Sometimes, a small amount of malachite

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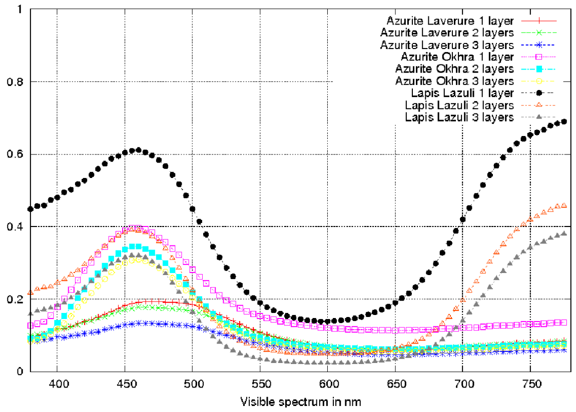

Figure 10: Reflectance factors of the blue pigments, azurite and lapis-lazuli depending on the paint film thickness.

in natural azurite can explain the reflectance curve aspect and also the greenish color observed on the sample. Now that we have the reflectance factors of the pure pigments, we want to compare them to the ones we measured in the basilica on the recumbent statue itself. We have identified some of the pigments used on the statue. It seems that the pigment used by the artist was identified as green clay, according to the reflectance curves. We have made this kind of comparison for each *in situ* reflectance factor. The blue areas seem to have been painted with azurite. The reflectance factor of the red cushion approaches that of the red lead. We can notice that the recorded spectra in the basilica have a weaker amplitude between their maxima and minima than those of the samples. This phenomenon expresses the fact that the colors of the sculpture are now very desaturated. There are several reasons for that whitening. The paint layers are old and dirty so that the deposited dust on the surface increases the whitening by surface scattering of all incident light. More studies were necessary to exactly determine the composition of each paint used to achieve this sculpture. Many samples of different pigments have been elaborated. Therefore we virtually created mixtures of pigments and compared them to the *in situ* recorded reflectance factors, according to the robust color matching methods.

5.1 Samples measurements

The samples give also the parameters used by the Kubelka and Munk extended to four fluxes. Indeed, for several pigments (green clay, malachite, black vine, vermilion, and lemon ochre), the reflectance factors for two and three layers tends to be quite similar, so we will consider that the diffuse reflectance factor for the three layers samples gives the term R_∞ to determine the coefficients K and S of these pigments. We measure also the thickness of the layers using some micro-stratigraphic images of these samples.

Figure 11: First results obtained with only spectrophotometric data and physical samples preparation. (left) Philippe-Dagobert's *cappa magna* rendered in lapis-lazuli blue pigment by the radiosity software Candelux. (right) Philippe-Dagobert's sleeves - vermilion pigment rendered with Candelux, opaque gold leaf covered lion rendered in ray-tracing with Virtuelium. CIE D65 illuminants.

5.2 Material compositing

The main goal of this model is to permit the computation of multi-layered materials. As we can see on figure 8 a lot of paint layers often 3 or 4 are encountered. The overall reflectance is obtained by computing the reflectance bottom-up, by using the previous reflectance as substrate reflectance. Associated with the method presented in [DUMAZET et al., 2007] we can compute any multi-layered paint system where we dispose of the K, S, k, s^+ and s^- parameters. Moreover, an approximation can be used for pigments mixing in the Kubelka and Munk theory. The approximation of the resulting K/S of a such mixing can be computed by :

$$\frac{K}{S} = \frac{\sum_i C_i K_i}{\sum_i C_i S_i} \tag{15}$$

where C_i is the volumetric concentration of the i-th component in the mixing. Therefore, we haven't done yet the studies on the validity of using a such composition law on the k, s^+ and s^- parameters.

5.3 The metallic film influence

Some part of the recumbent statue are "just" gilded. So to make good simulations we have to be able to render metallic materials. For this we use the real part and the imaginary part of the complex indices of refraction of the metals or alloys. Then we introduce them into the Fresnel formulas in the rendering pass. These indices can be measured on real polished plates by *spectroscopic ellipsometry*, an optical method based on Fresnel formulas and the analysis of the amount of

 ISBN 978-86943-15-2

the reflected and polarized light at large incidence on a smooth surface. The whole theory of reflection of light by a metallic surface is available in optics books such as Born and Wolf's [BORN and WOLF, 1975] and for computer graphics and lighting in [CALLET, 2006, CALLET, 2007] including (rare) quantitative data.

5.4 Spectral simulations

With a correct characterisation of all materials and their state of surface (waviness, roughness, paint-film thickness), the lighting conditions and a standard colorimetric observer (CIE 1964 10°), we can compute the restored visual aspect of the museum artifacts[Pitzalis et al., 2007]. In order to produce the rendering we use our multithreaded raytracer, *Virtuelium*. It is a multispectral raytracing software using generally 81 spectral bands of 5 nm width, ranging from 380 to 780 nm. It can also handle light polarisation and rely on CIE standard illuminants and colorimetric observers. All the materials are not define by radiative properties but by there intrinsic properties. So we use the complex indices of refraction for the metals or the dielectric materials. We use four fluxes parameters for paints. The algorithm, itself, used here is a simple ray tracing with just direct and specular illumination. We use an octree to speed up the computation and take benefit from multi-processor/core computer. According to the physical parameters extracted from microstratigraphy and spectrophotometric measurements, we have computed with the 4-fluxes model and Virtuelium, the image presented in Fig. 12. Thus, the successive layers with their thickness and concentration as plausible for the XIIIth century can appear. We have made many simulations with different standard illuminants and with characterized real light sources. Some parts of the sculpture as the prince's face or the angel head were probably painted with a cinnabar and vermilion mixture. No visible traces permitted to confirm that point and we decided to render the corresponding parts with the only white lead in a totally opaque layer.

We computed the virtual restored aspect of the medieval sculpture with the following materials :

- angel tunic: lapis-lazuli layer on azurite ;
- angel wings: 50 % red lead and 50 % cinnabar ;
- Philippe Dagobert hair: 200 nm thick gold leaf on yellow ochre substrate ;
- upper cushion: 50 % red lead and 50 % cinnabar ;
- lower cushion: not completely opaque layer of malachite pigments.

Figure 12: The actual most plausible colors of the medieval polychrome recumbent statue of Philippe Dagobert. Rendering in spectral ray-tracing by Virtuelium on an INTEL Pentium4 3GHz dual processors computer in 8 hours for a 1200x1200 resolution, 1Gb RAM and running under the operating system GNU/Linux - Fedora 5.

6 CONCLUSIONS

We went one step ahead in our knowledge and know-how of cultural heritage engineering and physical models for materials rendering in optical simulation. Till now we worked with standard illuminants such as CIE A, D65 or E illuminants. We shall need for these studies a better knowledge on natural lighting by stained glass windows and all other anthropogenic lightings used in the medieval era. For also improving the rendering of the complete sculpture, the canopy and all the gilded ornaments will be added to the 3D model along with photon mapping and texturing. The second part of that project will gather all the acquired new results and will also produce a new video movie translated in many languages.

ACKNOWLEDGEMENTS

In this pluridisciplinary project the authors were greatly helped by the following colleagues, in Ecole Centrale Paris by François-Xavier de Contencin for 3D digitization, Anna Zymla for material analysis, Philippe Denizet and Marie-France Monanges for video assisted by many students. Colleagues in Laboratoire de Recherche des Monuments Historiques, Vincent Detalle, Olivier Rolland and Annick Texier worked on microstratigraphy, paint analysis and gilding. The Centre des Monuments Nationaux and the Saint-Denis basilica scientific and history team with Jacqueline Maillé, Georges Puchal, Robert Lequément, Alain Erlande Brandenbourg, Françoise Perrot and Serge

Santos. In Louvre museum, Pierre-Yves Le Pogam gave access to an essential and original colored piece while Nicolas Hueber at Ecole Nationale Supérieure d'Arts et Métiers realized the physical replica assisted by the sculptor Brigitte Bonnet. Gilles Raffier from AXIATEC facilitated the physical realization and sponsorship.

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