Noise Filtering of Images Using Generalized Singular Spectrum Analysis

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ABSTRACT

There is a noise filtering method for images using Singular Value Decomposition (SVD). This is a method in which pixel values of an image are regarded as elements of a matrix, the image is separated into rough parts and detailed parts by SVD of the matrix and the detailed parts of the image are regarded as noise and removed.

Generalized Singular Spectrum Analysis (GSSA) is a method that generalizes SVD to treat more generalized data structures than in SVD. In this research, we present noise filtering methods of images using GSSA.

Keywords

Generalized Singular Spectrum Analysis (GSSA), Singular Value Decomposition (SVD), Noise Filtering of Image.

1. Introduction

There is a noise filtering method of images using Singular Value Decomposition (SVD) [1, 2]. This is the method in which the noise filtering of signal processing is applied to noise filtering of image processing. We assume that pixel values of a column in an image are a signal vector at a time and the direction of a row in an image is the time instant direction at a time series. We can interpret that the pixel values vector of a column in an image moves along the row direction as time passes. By taking autocorrelation of an image and doing eigenvalue decomposition of the autocorrelation matrix, spectrum decomposition of the image is performed. In general, since white noise is uniformly distributed in the frequency domain, relatively large noise exists over parts with small eigenvalue. By filtering at predictable parts with much noise, a certain amount of noise can be removed. However, the method using SVD cannot utilize enough 2D properties of images, because this is a simple method that extends time processing into 2D.

On the other hand, Generalized Singular Spectrum Analysis (GSSA) [3] is a method that generalizes SVD to treat more generalized data structure than in SVD. This method using GSSA is different to forcedly apply a method of time processing into a method of image processing as the method using SVD and performs spectrum decomposition of images by natural extension. In this research, we present noise filtering of images using this GSSA.

2. Spectrum Decomposition of Images

In GSSA, we need to make a trajectory matrix corresponding to the data structure treated. Next, by doing singular value decomposition for the trajectory matrix and doing inverse transformation of GSSA, we realize spectrum decomposition of targets. Refer to the paper [3] for the detailed procedures.

Subdivision of an original image

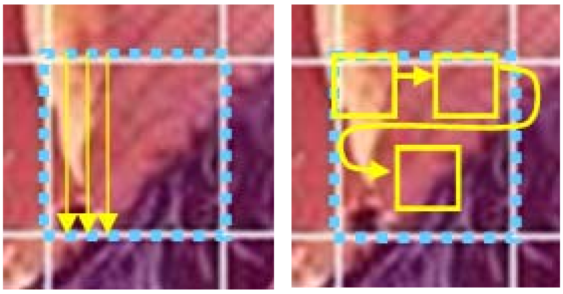
In this paper, Figure1(a) is an original image. The size of the image is 256×256 pixels. If the size of an image is too large, we need to partition the image into sub-images. Figure1(b) is a partitioned image of Figure1(a). We performed spectrum decomposition for each sub-image and remove noise from the result.

(a) original image (b) subdivision of the image

Figure 1. Original image.

Trajectory Matrix

If existing SVD is used, a trajectory matrix whose elements correspond directly to pixel values of the image as shown in Figure2(a) is made. Concretely speaking, it is as shown in Figure3. If GSSA is used, a certain window is defined and a trajectory matrix is made by moving the window all over the image. Concretely speaking, it is as shown in Figure4 and the size of the window is 2×2. In the case of SVD, we take autocorrelation among every column. In the case of GSSA, we take autocorrelation between each window pair. In this paper, we call the existing method using SVD as "type SVD" and the new method using GSSA as "type GSSA".



(a)SVD (b) GSSA

Figure2. Order of pixel values of an image for trajectories.

a_{00}	a_{01}	a_{02}	a_{03}	a_{04}	a_{05}
a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_{20}	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}
a_{30}	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}
a_{40}	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} & a_{04} & a_{05} \\ a_{10} & a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{20} & a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{30} & a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{40} & a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \end{bmatrix}$$

(a) pixel values of an image (b) trajectory matrix

Figure3. Pixel values of an image and a trajectory matrix in SVD method.

a_{00}	a_{01}	a_{02}	a_{03}	a_{04}	a_{05}
a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_{20}	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}
a_{30}	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}
a_{40}	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}

(a) pixel values of an image

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} & a_{04} & a_{10} & a_{12} & \cdots & \cdots & \cdots \\ a_{01} & a_{02} & a_{03} & a_{04} & a_{05} & a_{11} & a_{13} & \cdots & \cdots & \cdots \\ a_{10} & a_{11} & a_{12} & a_{13} & a_{14} & a_{20} & a_{22} & \cdots & \cdots & \cdots \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{21} & a_{23} & \cdots & \cdots & \cdots \end{bmatrix}$$

(b) trajectory matrix

Figure4. Pixel values of an image and a trajectory matrix in GSSA method.

Spectrum Decomposition of Image

Figure9(a)(c) shows the results of spectrum decomposition for the original image and Figure9(b)(d) shows the differences between the original image and (a)(c). *r* is defined as a number whose sets of singular value and base vector are added in reconstruction of the image. If *r* = (number of decompositions), the decomposed image is completely reconstructed into the original image.

If Figure9(a-4) is compared with (c-4), the image in (c-4) is more smooth than (a-4) to the eye. These result shows that the image using "type GSSA" is reconstructed into a more similar image to the original image than the image using "type SVD". This is because "type GSSA" can take enough 2D autocorrelation for both column and row directions, on the other hand, "type SVD" can take only 1D autocorrelation for each columns.

As for other properties, if Figure9(b-3)(b-4) are compared with (d-3)(d-4), edges in (d-3)(d-4) can be extracted more clearly than (b-3)(b-4) by the same reason.

From these results, we confirm that "type GSSA" can perform spectrum decomposition better than "type SVD" in treating 2D image.

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3. Noise Filtering of images

Figure5(a) is the original image and Figure5(b) is an image that embeds normal random number of average 0.0 and standard deviation 26.66 as noise.

(a) original image **(b) noisy image**

Figure5. Original image and noisy image.

Discrete Cosine Transform (DCT)

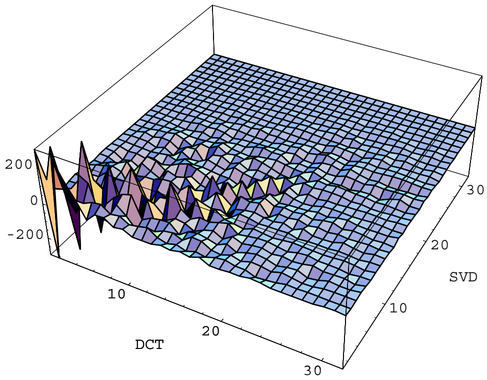
Figure6 show the results that DCT for base vectors are performed and the corresponding singular value is multiplied after spectrum decomposition of “type SVD” or “type GSSA”. Since the base vectors are arranged in descending order, there is a peak around the origin in Figure6.

If comparing Figure6(a) and (b) or (c) and(d), though we can see noise in (b) and (d), their rough trends are similar.

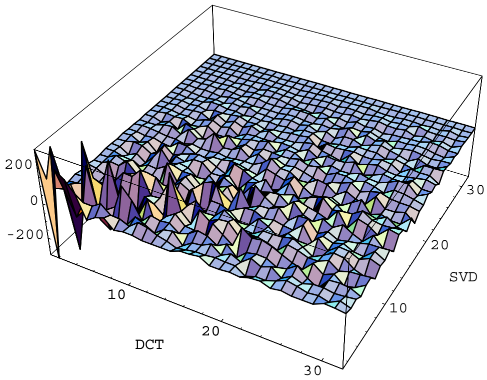
Additionally, since “type GSSA” is regarded as the method to do “type SVD” in both of the column and row directions, we can see that Figure6(a)(b) appear in a reticular pattern and in sequence in (c)(d).

Two Noise Filtering

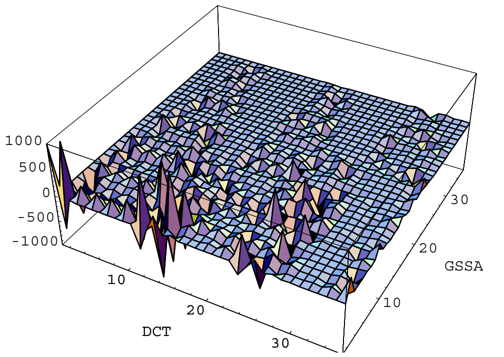
We define two noise filtering using spectrum decomposition, “Filter1” and “Filter2”. “Filter1” is the method that an image is reconstructed using *r* sets of singular values and base vectors in descending order after spectrum decomposition using “type SVD” or “type GSSA”. “Filter2” is the method where an image is reconstructed using a half number of coefficients in descending order of DCT for a base vector after “Filter1”. In Figure7, “Filter1” is the method reconstructed using the lower *r* columns and “Filter2” is the method reconstructed using the left half of “Filter1”.



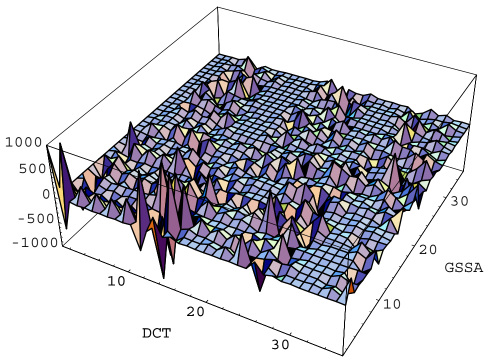
(a) DCT after SVD of an original image



(b) DCT after SVD of a noisy image



(c) DCT after GSSA of an original image



(d) DCT after GSSA of a noisy image

Figure6. DCT after SVD or GSSA.

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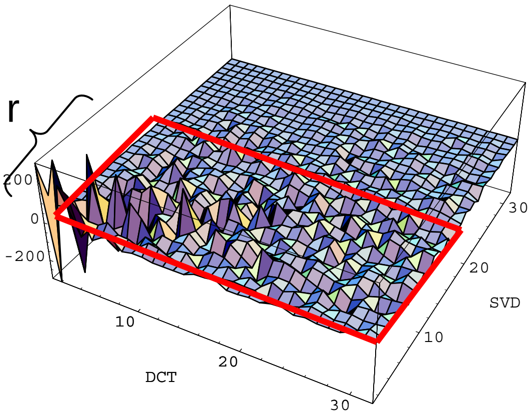
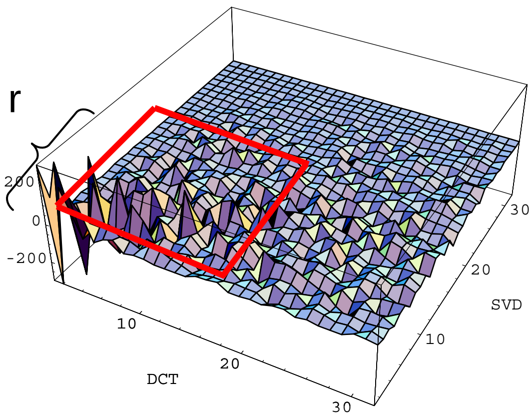


(a) Noise filtering 1



(b) Noise filtering 2

Figure7. Two noise filtering.

Table1. RMS between an original image and a noisy image.

r	SVD		GSSA	
	Filter1	Filter2	Filter1	Filter2
4	64.19	56.75	43.53	45.63
8	63.22	53.20	44.31	45.00
16	55.76	54.68	48.30	48.25

Table1 is root-mean-square (RMS) between the original image and the reconstructed images using "Filter1" and "Filter2" for images embedding white noise. If comparing "type SVD" and "type GSSA", we can see that "type GSSA" has more effects of noise filtering by "Filter1" than "type SVD". Additionally, we can see effects of noise filtering by "Filter2" in "type SVD" and no effects of noise filtering by "Filter2" in "type GSSA". This is why "type GSSA" with 2D autocorrelation has much information of the image at bases with large singular values and "type GSSA" has less noise over bases with large singular values than "type SVD".

Figure10 shows images after noise filtering using "type SVD" or "type GSSA" and differences between the original image and them. If comparing their images, we can see that "type GSSA" can remove noise better than "type SVD". But we cannot see much difference between "Filter1" and "Filter2". From this result, we find that the effect of "Filter1" is larger than the effect of "Filter2".

4. Conclusion and Future Work

In this paper, we present new spectrum decomposition for images and apply it to noise filtering. This is the method that generalizes exiting SVD by taking 2D autocorrelation for both of column and row directions. We have been able to find more detail properties of the image by GSSA.

Finally, there is an improvement point. In this paper, we do noise filtering for each sub-image after partitioning the image into some sub-images. Therefore, the sub-images are connected non-smoothly on boundaries. To solve this problem, we are considering the method using larger windows than sub-image sizes as shown in Figure8, spectrum decomposition in this larger window is done and weighted interpolation over the overlapping regions is performed. Figure8 shows the appearance of a cubic spline interpolation over the overlapping regions.

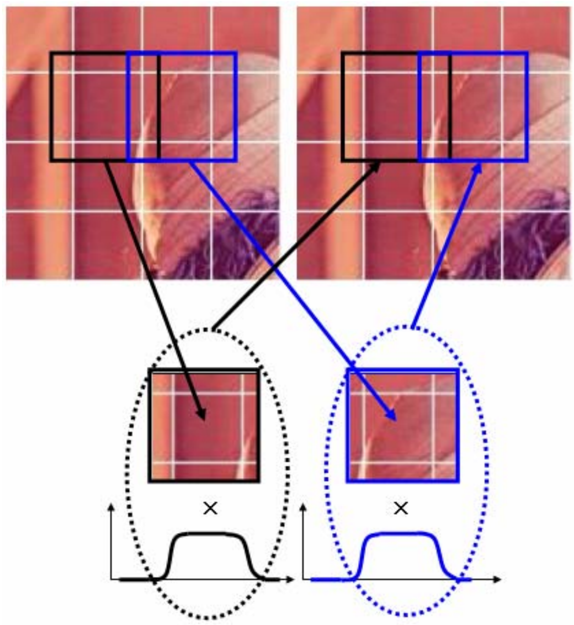


Figure8. Smoothing on boundaries.

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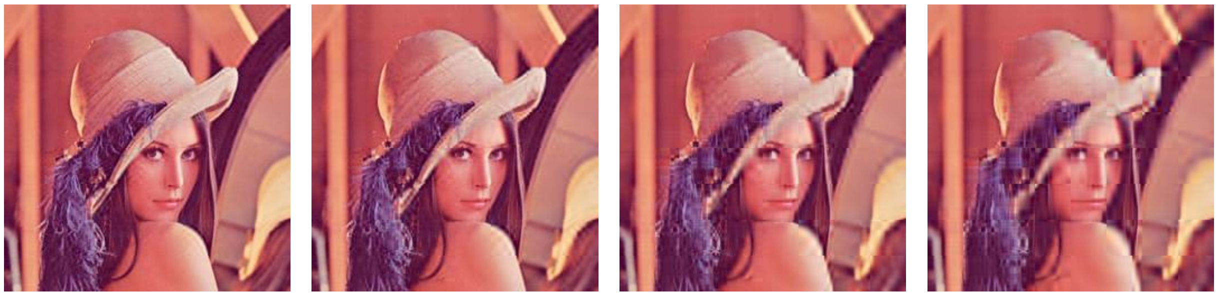
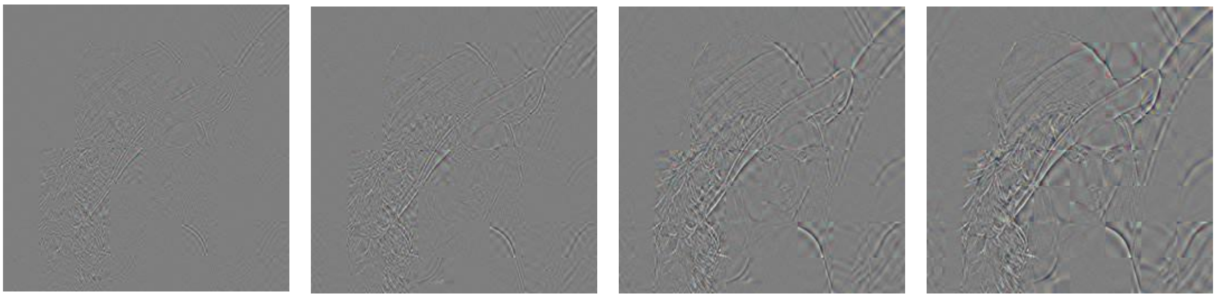


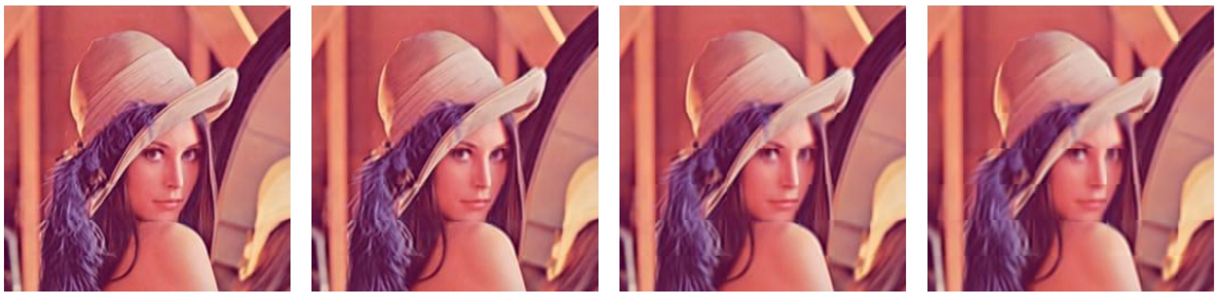
(a-1) r=8 (a-2) r=6 (a-3) r=4 (a-4) r=3

(a) Spectrum decompositions using SVD of an original image



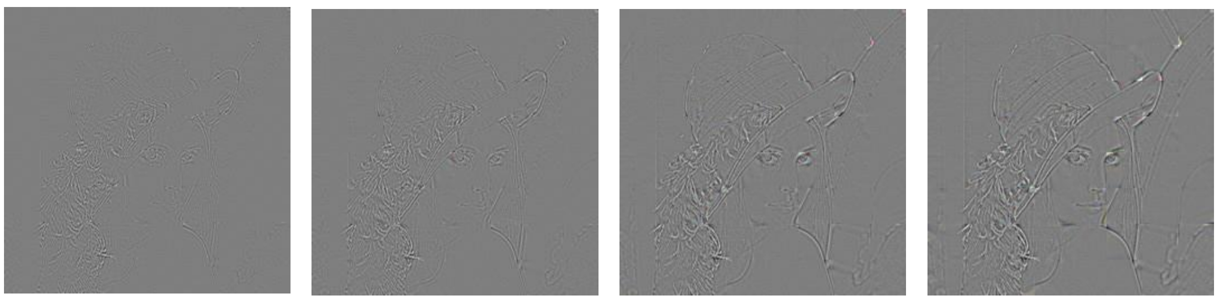
(b-1) r=8 (b-2) r=6 (b-3) r=4 (b-4) r=3

(b) Differences using SVD between an original and (a)



(c-1) r=8 (c-2) r=6 (c-3) r=4 (c-4) r=3

(c) Spectrum decompositions using GSSA of an original image



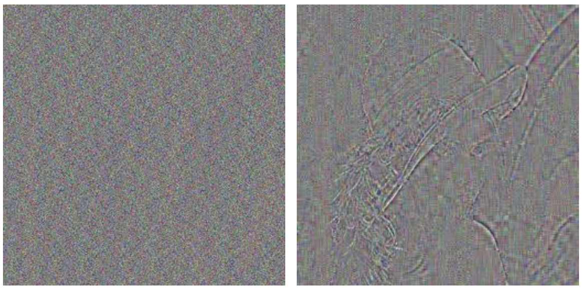
(d-1) r=8 (d-2) r=6 (d-3) r=4 (d-4) r=3

(d) Differences using GSSA between an original and (c)

Figure9. Spectrum decompositions of an original image and differences between an original and them.

(a-1) r=16 (a-2) r=4

(a) Noise filtering 1 using SVD.

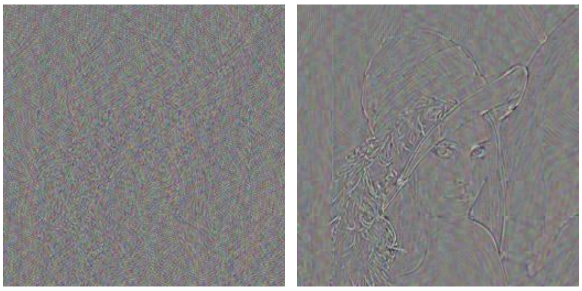


(b-1) r=16 (b-2) r=4

(b) Differences between an original image and (a).

(c-1) r=16 (c-2) r=4

(c) Noise filtering 1 using GSSA.

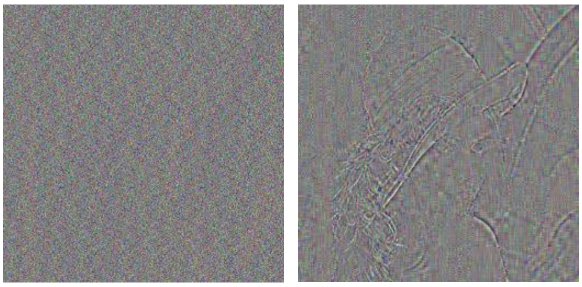


(d-1) r=16 (d-2) r=4

(d) Differences between an original image and (c).

(e-1) r=16 (e-2) r=4

(e) Noise filtering 2 using SVD.

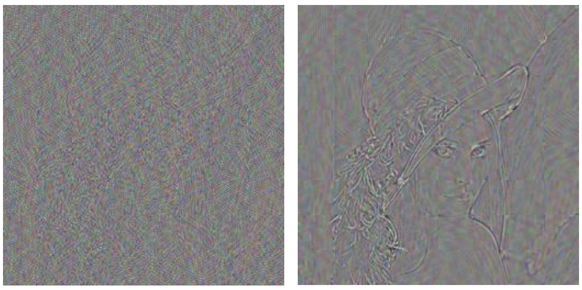


(f-1) r=16 (f-2) r=4

(f) Differences between an original image and (e).

(g-1) r=16 (g-2) r=4

(g) Noise filtering 2 using GSSA.



(h-1) r=16 (h-2) r=4

(h) Differences between an original image and (g).

Figure10. Images after Noise filtering and differences between an original image and them.

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