Fuzzy mathematical morphology and its applications to colour image processing

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ABSTRACT

In image analysis and pattern recognition fuzzy sets play the role of a good model for segmentation and classifications tasks when the regions and the classes cannot be strictly defined. One of the most widely used fuzzy approaches in image processing is fuzzy mathematical morphology because of the underlaying structure of complete lattices. Thus, mathematical morphology is used mainly in processing of binary or single - valued intensity (grey - scale) images for which a partial ordering, hence a lattice structure, is apparent. However, the problem of morphological image processing of colour images is that it is not naturally clear how to define ordering in a colour space. This paper shows a possible way for solving this problem. It is based on the usage of YCrCb colour space as physically intuitive and easy to compute.

Keywords: Complete lattice, fuzzy sets, colour models, morphological operations.

1 INTRODUCTION

Fuzzy mathematical morphology has been developed to soften the classical binary morphology so as to make the operators less sensitive to image imprecision. It can also be viewed simply as an alternative grey-scale morphological theory. The use of morphological operations require ordering of 3 dimensional colour space, which is hard to be done in natural way, because one should have to order the colours. A way to this is to refer to psychophysiological experiments showing the level of stimulation of eye retina. In our work we use the YCrCb colour model for its simplicity and efficient computation. We obtain adjoint fuzzy erosions and dilations necessary for the construction of efficient filters for colour images denoising without changing the natural colours by subdivision of the CrCb square. This subdivision guarantees the result in particular in the presence of data uncertaincies in vision systems.

2 MATHEMATICAL MORPHOLOGY – BACKGROUND

Serra [Serra88] and Heijmans [Heijmans94] have shown that morphological operations can be formulated on any complete lattice. A set $\mathscr{L}$ with a partial ordering "$\leq$" is called a *complete lattice* if every subset $\mathscr{H} \subseteq \mathscr{L}$ has a supremum $\bigvee \mathscr{H} \in \mathscr{L}$ (least upper bound) and infimum (greatest lower bound) $\bigwedge \mathscr{H} \in \mathscr{L}$.

An operator $\varphi : \mathscr{L} \mapsto \mathscr{M}$, where $\mathscr{L}$ and $\mathscr{M}$ are two complete lattices, is called dilation if it distributes over arbitrary suprema: $\varphi(\bigvee_{i\in I} X_i) = \bigvee_{i\in I} \varphi(X_i)$, and erosion if it distributes over arbitrary infima. Erosions and dilations are increasing operations [Heijmans94]. An operator $\psi : \mathscr{L} \mapsto \mathscr{L}$ is called a closing if it is increasing, idempotent ($\psi^2 = \psi$) and extensive ($\psi(X) \geq X$). An operator ψ is called an opening if it is increasing, idempotent and anti-extensive ($\psi(X) \leq X$) [Heijmans94]. A pair of operators (ε, δ), $\delta : \mathscr{L} \mapsto \mathscr{M}$, $\varepsilon : \mathscr{M} \mapsto \mathscr{L}$, is called an adjunction, if for every two elements $X \in \mathscr{L}, Y \in \mathscr{M}$ it follows that $\delta(X) \leq Y \iff X \leq \varepsilon(Y)$. In [Heijmans94] it is proved that if (ε, δ) is an adjunction then ε is erosion and δ is dilation. If (ε, δ) is an adjunction, then the composition $\varepsilon\delta$ is a closing in $\mathscr{L}$, and $\delta\varepsilon$ is an opening in the lattice $\mathscr{M}$. As an example, let us consider the lattice $\mathscr{L}$ with elements the subsets of a linear space E. Then every translation-invariant dilation is represented by the standard Minkowski addition: $\delta_A(X) = A \oplus X = X \oplus A$, and its adjoint erosion is given by Minkowski subtraction: $\varepsilon_A(X) = X \ominus A$ [Heijmans94]. Then closing and opening of A by B are defined as $A \bullet B = (A \oplus B) \ominus B$, $A \circ B = (A \ominus B) \oplus B$. These operations are referred to as *classical* or *binary morphological operations*. Openings and closings are generally used as filters for denoising of binary images.

3 FUZZY MORPHOLOGICAL OPERATIONS

Consider the set E called the universal set. A fuzzy subset A of the universal set E can be considered as a function $\mu_A : E \mapsto [0,1]$, called the membership function of A. $\mu_A(x)$ is called the degree of membership of the point x to the set A. The ordinary subsets of E, sometimes called 'crisp sets', can be considered as a particular case of a fuzzy set with membership function taking only the values 0 and 1. This definition leads to two possible interpretations:

– in image representation the value of the membership function $\mu_A(x)$ at a point x may be interpreted as the grey level value associated with that point of the image plane,

– in pattern recognition, the value $0 \le \mu_A(x) \le 1$ indicates the probability that the point x is in the foreground of an image.

The usual set-theoretical operations can be defined naturally on fuzzy sets: Union and intersection of a collection of fuzzy sets is defined as supremum, resp. infimum of their membership functions. Also, we say that $A \subseteq B$ if $\mu_A(x) \le \mu_B(x)$ for all $x \in E$. The complement of A is the set A^c with membership function $\mu_{A^c}(x) = 1 - \mu_A(x)$ for all $x \in E$. Further, for simplicity, if there is no confusion we will write $A(x)$ instead of $\mu_A(x)$.

If the universal set E is linear, like the n-dimensional Euclidean vector space $\mathbf{R}^n$ or the space of integer vectors with length n, then any geometrical transformation, like scaling, translation etc., of a fuzzy set can be defined by transforming its $\alpha-$cuts [Nguyen00].

Further we consider a simple way to generate adjoint fuzzy morphological operations.

Say that the function $c(x,y) : [0,1] \times [0,1] \mapsto [0,1]$ is conjunctor if c is increasing in the both arguments, $c(0,1) = c(1,0) = 0$, and $c(1,1) = 1$.

Say that the function $i(x,y) : [0,1] \times [0,1] \mapsto [0,1]$ is implicator if i is increasing in y and decreasing in x, $i(0,0) = i(1,1) = 1$, and $i(1,0) = 0$.

Say that the conjucnctor - implication pair is adjoint if $c(b,y) \le x$ is true if and only if $y \le i(b,x)$. Then for fixed b function $f(x) = i(b,x)$ is an erosion, and its adjoint dilation is $g(y) = c(b,y)$.

Then having an adjoint conjunctor - implicator pair, as proposed in [Heijmans00], we can define an adjoint pair of fuzzy erosion and dilation:

$$\delta_B(A)(x) = \sup_y c(B(x-y), A(y)),$$
$$\varepsilon_B(A)(x) = \inf_y i(B(y-x), A(y)).$$

Heijmans [Heijmans00] has proposed a number of following conjunctor - implicator pairs to construct morphological operations. Here we give examples of two of them:

$$c(b,y) = \min(b,y),$$
$$i(b,x) = \begin{cases} x & x < b, \\ 1 & x \ge b \end{cases}.$$

These operations are known as operations of Gödel-Brouwer.

$$c(b,y) = \max(0, b+y-1),$$
$$i(b,x) = \min(1, x-b+1).$$

These operations are suggested by Lukasiewicz.

Most often the first conjunctor - implicator pair is used. The respective dilation has the form $(\delta_B(A))(x) = \sup_b \min(A(b), B(x-b))$. In this case we can denote $\delta_A(B) = \delta_B(A) = A \oplus B$, because this operation can be obtained also directly from the binary Minkowski addition using the *extension principle*.

Note that in these cases the conjunctor is symmetric, i.e. it is a t-norm [Nguyen00], and therefore we have $\delta_A(B) = \delta_B(A)$.

4 MORPHOLOGICAL OPERATIONS FOR COLOUR IMAGES

We saw that the basic morphological operations are expressed as products of the suprema and the infima of the lattice under study. When we deal with colour images, we work in fact in a multidimensional space (usually $\mathbf{R}^3$ or $\mathbf{Z}^3$), where a natural ordering of the elements cannot be achieved. Therefore we try to introduce some heuristics and to compromise with the accuracy at acceptable level to guarantee the lattice properties and therefore to ensure idempotent opening and closing filtering.

An useful implementation of a basic subjective colour model is the HSV (hue, saturation, value) cone [Rogers98] and its slight modification HLS [Hanbury01]. It was created by A. R. Smith in 1978. It is based on such intuitive colour characteristics as tint, shade and tone (or family (hue), purity (saturation) and intensity (value/luminance)). The coordinate system is cylindrical, and the colours are defined inside a cone for the case of HSV and a double cone in the case of HLS. The hue value H runs from 0 to 2π. The saturation S is the degree of strength or purity and varies from 0 to 1. The saturation shows how much white is added to the colour, so S=1 makes the purest colour (no white). Brightness (value) V also ranges from 0 to 1, where 0 is the black. We could experience that in general the value/luminance component contains most of the information about the texture and boundary location. The hue component is usually the most homogeneous (with small variance). The saturation component allows to differentiate between different shades of the same colour. The hue is measured as the polar angle in a plane through the saturation segment and orthogonal to the cone axis. This angle is measured counterclockwise from red. Instead of $[0, 2\pi)$ we may use the interval $[0,1)$ dividing the angle by 2π. If S=0 the hue is undefined, and therefore the colour is

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achromatic, namely a shade of grey. The fully saturated primary colours occur when S=1.

However, when we use the HSV or HLS model, the main obstacle is the fact that the hue is measured as an angle, and it is not defined for the levels of grey. In [Hanbury01] a circular 'ordering' of the hue modulo 2π is defined, however this ordering does not lead to a complete lattice. Therefore in this case the obtained operators provide good results for some segmentation tasks, but the usage of openings and closings for denoising and filtering is risky. In general, there are no clear mathematical reasons for hue ordering. However, from psychophysiological point of view one may order the colours in the following way - red, magenta, blue, yellow, cyan, green, based on the way humans perceive the hue of the colour. Red is considered to be the smallest, since it stimulates the eye less than the other colours. Contrary, green mostly stimulates the eye [Louverdis02]. In [Hanbury02] an interesting approach for creating colour morphology is presented based on L*a*b* (CIELAB) colour space representation (the two models are related by the fact that $H = \arctan\left(\frac{b^*}{a^*}\right)$). There the authors divide this space into equipotential surfaces. Unfortunately, the best order for the colour vectors in the same equipotential surface is not obvious. In order to obtain a complete ordering of the colour vectors, they make use of the lexicographical order. Therefore in this case we have idempotent closing and opening filters. However, the transformation from RGB to L*a*b* is non-linear and time consuming. Moreover, there exists no unique inverse transform. The inverse transform depends on the way we characterize the white point. If one knows the illumination conditions used when acquiring the image, then the specification of the white point is simple. However, if the illumination conditions are unknown, a heuristic hypothesis should be made.

An alternative approach is to use the YCrCb colour model [Rogers98]. To obtain the parameters Y, Cr and Cb we use a simple linear combination of R,G, B values. Note, that Y represents the *lightness* , and should not be mistaken with the yellow colour in the RGB model notation. The parameter Cr encodes the red-cyan sensation, with value ≈ 0 for the cyan colour and ≈ 1 for the red. The parameter Cb encodes the yellow-blue sensation with ≈ 0 indicating yellow and ≈ 1 indicating blue. Without lack of generality we can assume that R,G and B values are represented as points in an unit cube, namely $0 \le R, G, B \le 1$. The YCrCb colour space is also a unit cube with transformation formulas:

$$Y = 0.299R + 0.577G + 0.114B, \quad (1)$$

$$Cr = 0.5R - 0.411G - 0.081B + 0.5, \quad (2)$$

$$Cb = 0.5B - 0.169R - 0.325G + 0.5\,. \quad (3)$$

Henceforth it is clear that the transformation between RGB and YCrCb models is linear and easy to compute. In colour image processing we give priority to the luminance parameters (V in HSV, L in HLs and Y in YCrCb), since if anyone looks at them, he can usually distinguish the different objects on the image as looking on a black -and- white TV. In general, in colour image processing less priority is usually given to the chrominance maps - for instance H and S in HSV model, or Cr and Cb in YCrCb. When working with HSV model, the second priority is given to the hue, because it contains mostly the colour information, and the least priority is given to the saturation because it is correlated with the other two components and its role as a parameter in image processing tasks is sometimes criticized – one can refer for instance to the work [Hanbury01]. This is another reason to prefer the YCrCb model, where the components Cr and Cb have equal weights.

Let us divide the interval $[0,1]$ into N equal pieces $I_i = \left(\frac{i-1}{N}, \frac{i}{N}\right]$. $(0 \in I_1)$. Let us also suppose that for a pixel x $Cb(x) \in I_j$ and $1 - Cr(x) \in I_i$. Note that we use the negation of Cr to obtain an ordering closer to the one presented in [Louverdis02]. Thus we code approximately with accuracy $1/N$ the Cr and Cb values by the number of the step at which we visit the respective cell. Let us consider the cell (i,j) and let $n = \max(i,j)$ and $m = \min(i,j)$. Then for the number of the step we can prove by induction that

$$T(i,j) = \begin{cases} n^2 - m + 1 & (*) \\ n^2 - 2n + m + 1 & (**) \end{cases}$$

The case (*) means that either n is even and $n = i$, or n is odd and $n = j$. The case (**) is the opposite one. Further on, for simplicity, for any pixel x we will denote its respective colour integer code by $T(x)$. Then if given a colour image X, we define the transformation

$$(\chi(X))(x) = \frac{N^2[(N^2-1)Y(x)] + T(x) - 1}{N^4 - 1},$$

which is a real number between 0 and 1. Then it is clear that having $\chi(X)$, we can find $Y(x)$ with accuracy $1/N^2$ and $Cr(X)$ and $Cb(x)$ with $1/N$ simply by taking the quotient and the reminder of the division of $[\chi(X)(N^4-1)]$ by N^2. Here $[t]$ means the integer part of t. The last transformation we denote by χ^{-1}. Then it is obvious that $\chi^{-1}(\chi(X))$ gives an approximation of the original colour image X , while for any grey-scale image $\chi(\chi^{-1}(Y)) = Y$.

Then we could order the colour images, namely say that $A \prec B$ if $\chi(A) \le \chi(B)$. In this case, if $A \prec B$ and $B \prec A$ doesn't mean that $A = B$, but means that they are close enough and lie in the same equivalence class. Then we can give correct definition of colour fuzzy morphology, simply when we are given an adjoint conjunctor - implicator pair we can define dilation-erosion adjunction as:

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$$\delta_B(X)(x) = \chi^{-1}\left[\bigvee_y c(B(x-y),(\chi(X))(y))\right], \quad (4)$$

$$\varepsilon_B(X)(x) = \chi^{-1}\left[\bigwedge_y i(B(y-x),(\chi(X))(y))\right]. \quad (5)$$

Here B can be any fuzzy structuring element, i.e. an image which pixel values are real numbers between 0 and 1.

To compute easily χ^{-1} we represent the function $T(i,j)$ by a $N^2 \times 2$ table in which the number of the row s means the current step of the zigzag line, while its columns hold the numbers of the intervals i and j such that $T(i,j) = s$. In the examples shown in the next section we use value $N = 16$.

5 EXPERIMENTS AND FURTHER RESEARCH

In this work a new approach for construction of morphological filters for colour images is presented. It is based on interval approximation of the colour space. Thus we meet the requirements for image quality and we control the colour accuracy. The same approximation can be applied also in fuzzy algorithms for image enhancement and binarization.

On figure 1 one can see a colour picture of flowers. Next its dilation, erosion, opening and closing by 3×3 flat square structuring element are presented. Note that for flat structuring elements the choice of the t-norm - implicator pair is not essential, which follows easily from the properties of fuzzy t-norms and the uniqueness of the adjoint erosion [Popov00]. The second application of the opening (closing) operation on the opened (closed) image does not affect it due to the idempotence of thus generated opening and closing filters. Thus in our case we check experimentally our theoretical result for the opening and closing idempotence. On figure 2 we show the opening and closing top-hat transforms and the gradient of a picture of a traditional Bulgarian table on Easter holiday. The structuring element B is a centrally symmetric 'pyramide':

$$B = \begin{pmatrix} 0 & 0.7 & 0 \\ 0.7 & 1 & 0.7 \\ 0 & 0.7 & 0 \end{pmatrix}.$$

Remind that the opening top-hat transform is defined as the difference between the original and the opening, the closing top-hat is the difference between the closing and the original, and the gradient is the difference between the dilation and erosion [Soille02]. However, operations with non-flat structuring elements are rarely used in image filtering, because it is not clear a priori how an incremental operation will affect the colours. Further experiments with non-flat operators will be made to show the efficiency of our approach in texture analysis.Also we can study the convexity and connectivity of objects on colour images modifying the known approaches for grey - scale images presented in [Bloch93], [Popov97] and [Popov00]. Fuzzy colour morphology operators could be employed in biometric applications incuding fingerprint identification and removing textured background imprints on bank checks.

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Figure 1: **From top to bottom and left to right: original, dilation, erosion; opening, closing**

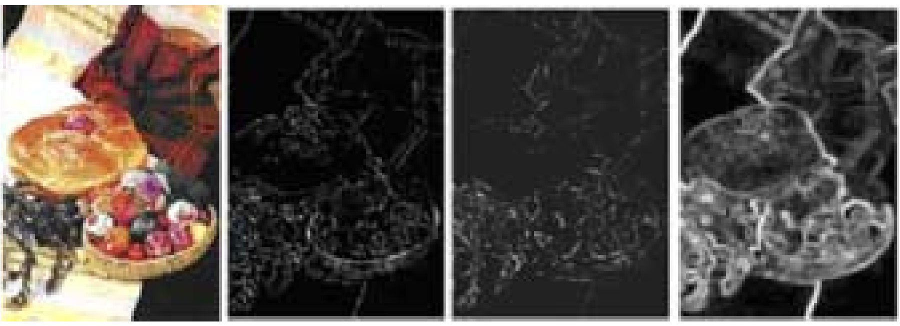

Figure 2: **The original colour image, its opening top-hat, closing top-hat and morphological gradient**

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