# Associating 6 DoF sensor data to 3D scan view registration

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## ABSTRACT

To make 3D scanning an attractive tool for incidental or inexperienced users, the process of scan view registration should be avoided or significantly simplified. If a 6 DoF sensor is attached to the object to be scanned, additional data about each of the 3D views can be supplied to the registration software as to provide initial relative placements of pair of views, thus making automatic matching feasible. This releases the user from the tedious manual registration process. A method to apply a 6 DoF device to 3D scanning is in development. To calibrate the sensor to the scanner, the equation of similarity matrices needs to be solved. We verified numerically that this leads to ambiguities if only one sensor-to-scanner association is measured. A method based on a geometric treatment is proposed to achieve an unambiguous association between the two devices. Initial numerical results are presented.

### Keywords

Conceptual shape design, 3D scanning, calibration, registration, 6 DoF sensor

## 1. INTRODUCTION

In industrial design engineering 3D scanning is increasingly used to provide a starting point in conceptual shape design. Rather than creating a new product's shape from scratch with a CAD system, designers may chose to first create a physical model manually (e.g. made out of clay), obtain a surface mesh from the model using 3D scanning and then import the surface mesh into a CAD modeling system for further modification and refinement. Also the redesign of manufactured parts or the reuse of existing product shape features is becoming routinely applied in product design [Smyth 2000, Vergeest 2001, Song 2005].

To generate a CAD surface or solid model from a physical object, the designer (or user, in general) needs to take the following steps. First the object's surface is digitized using some digitization instrument. We use the Minolta Vivid 700 scanner, which produces almost instantly, a matrix of (at most) 200×200 three-dimensional data points of the part of the surface which is orientated toward and visible by the scanner, from a particular viewing direction. Multiple views should be taken from other directions, until the entire surface has been recorded. In case consumer products are digitized, which have a diameter typically in the range between 5cm to 50cm, the user should displace and/or rotate the product between subsequent shots. In most cases, taking the scanning views can be routinely performed by people without experience of using the scanner.

The different data sets, or point clouds, one for each scan view, need to be assembled into one set, in a common coordinate system. This process is called scan view registration. Normally, the user selects two views which are partly overlapping. Graphically supported by a software tool belonging to the scanning system, the user must approximately designate corresponding points in each of the two views, in the overlap region. With three or more of such point pairs, the software derives an initial relative positioning of the two views, and an algorithm will then search for the pose of maximum matching of the point sets in their overlap region. If the algorithm fails to find the optimal pose, the user is prompted to designate new or additional point pairs. If the algorithm succeeds in finding a good match, the next scan view is considered, which should be



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matched with the views already processed. When all scan views are thus processed, they are defined (at least approximately) in the same coordinate system. Finally, the scanner's software performs a global registration, in which all scan views are considered collectively in order to obtain the best achievable placements of the scan views relative to each other.

The resulting point set may then be converted into a surface mesh, from which a CAD surface model can be derived, or in case the mesh is consistent with a topological shell structure, a CAD solid model. From then on, the model is in a format to which designers are accustomed.

The major bottleneck in this process is the scan view registration. For an inexperienced user the designation of corresponding points in different views is difficult and the results of the registration software is often unpredictable. As a result, 3D scanning is perceived as unpractical to (*e.g.*) shape designers. This problem disappears when the scanned object is stationary and the scanning device's position and orientation is tracked, for example using a mechanical arm or another tracking system. Then the registration procedure can be practically automated based on the tracking data. However, this method is designed for hand-held scanners and large objects, like cars or statues, and the devices are relatively costly. To scan small objects, a rotation platform can be used to bring the object in different orientations. Since the rotation axis is mechanically very stable, and sometimes the amount of rotation is known to the matching software as well, automatic registration can be achieved. However, the use of a rotation table limits the orientation of the object relative to the scanner, which is unwanted when the entire surface needs to be digitized.

We experienced that designers find it quite natural to hold an object (for example a clay model they produced manually) in front of the scanner in different orientations, thus collecting scan views from the entire surface. However, as mentioned, the next step, which is registration of the scan views into a single surface mesh, is perceived as too complex and too tedious.

We have developed a solution to this problem, based on additional data from a 6 degrees-of-freedom (DoF) positioning sensor attached to the object being digitized. To correctly associate the 6D placement data from the sensor with the required matching transformation of the 3D scan views, the sensor should be calibrated to the scanning device. We can derive the calibration from a couple of scan views, such chosen that they can be matched easily. The matching transformations and their corresponding 6D displacements (from the sensor) form, pair wise, so-called similar matrices. The calibration transform is then equal to the quotient from any such pairs. This type of calibration is different from calibrations based on position/orientation information from each of the two reference frames, as is the case, for example, in calibration problems for Augmented Reality devices [Grasset 2001], [Kato 1999], [Wheeler 1998]. Our problem would be similar to those when we would base the scanning information on explicit feature points on or near the sensor. In our proposed method however, we neither need to rely on feature recognition nor to make assumptions about the sensor's centre relative to its housing. We only need to perform some relatively simple scan view matchings.

In this paper we present in Section 2 the mathematical adaptation of the sensor data to the registration algorithm as a minimization problem and we analyze the problem using eigenvectors of the transforms. In section 3 we verify numerically how the displacement of the 6 D0F sensor. should be associated to the scan registration matrix. The derivation of the calibration matrix is presented in Section 4. In Section 5 we sketch a method to apply the 6 DoF sensor data to the registration algorithm. Conclusions are drawn in Section 6.

## 2. ASSOCIATING THE 6D DATA TO THE REGISTRATION TRANSFORM

The sensor is a device that measures its own position and orientation relative to a magnetic transmitter, which is stationary in the laboratory. In our application, also the 3D scanning device is stationary relative to the laboratory. Therefore, if the sensor is rigidly connected to the object being digitized (see Fig. 1), it should be possible to determine the placement of the object relative to the scanner as a function of time.

The sensor produces 6-tuples ($x(t)$, $y(t)$, $z(t)$, $\alpha(t)$, $\beta(t)$, $\gamma(t)$), its position coordinates and orientation angles, as a function of time, relative to the magnetic transmitter. Obviously a calibration step is required to define a "starting" placement of the sensor at time $t_0$ relative to the scanner. All placements at $t>t_0$ are then defined relative to the starting placement. We need to determine the coordinate system $T$ of the magnetic transmitter relative to the coordinate system $S$ of the scanner, *i.e.* we need to find ${}^{S}T$ (the upper index denoting the reference frame).

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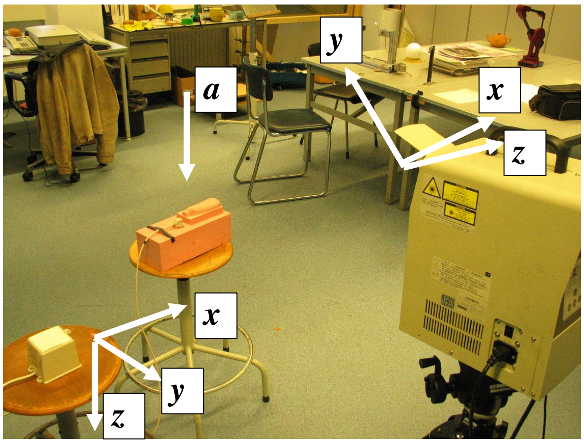


**Figure 1. Calibration setup: scanner and its local frame *S* on the right-hand side, transmitter of 6 DoF sensor and its frame *T* on the left and scanned object in the center. The 6 DoF sensor (wire coming out) is attached to the object during (at least) two subsequent scan views taken.**

A 6-tuple delivered by the magnetic sensor defines, by convention, an equivalent 4×4 matrix ${}^{T}F$ which can be interpreted as the local frame of the sensor measured relative to frame *T*. If two frames ${}^{T}F(t_0)$ and ${}^{T}F(t_1)$ are measured, $t_1 > t_0$, then the 6-dimensional displacement of the sensor over the time interval is ${}^{T}F(t_1)\ ({}^{T}F(t_0)\ )^{-1}$. Equivalently, the transformation to bring ${}^{T}F(t_1)$ back to ${}^{T}F(t_0)$ is defined as $N = {}^{T}F(t_0)\ ({}^{T}F(t_1)\ )^{-1}$. Now we need to relate these measurements to measurements relative to the scanner. A problem is that the scanner does not deliver quantities like ${}^{S}F(t_0)$. All we get from the scanner is a set of 3D data points defined relative to *S*, but there is no explicit information about *S*. However, if we apply the registration procedure of the scanner to two point sets, one from the object (including the sensor) at $t_0$ and one at $t_1$, then the outcome of the registration is the displacement *M*, where $M = {}^{S}F(t_0)\ (\ {}^{S}F(t_1)\ )^{-1}$. If transformation *M* is applied to the points obtained at $t_1$ then the transformed points will be in accordance with the points obtained at time $t_0$. The matrices *N* and *M* describe the same displacement, defined relative to *T* and to *S*, respectively. Such matrices are called *similar matrices*, for which exists a similarity transformation *X* such that $XNX^{-1} = M$. *X* specifies coordinate system *T* relative to *S*, or $X = {}^{S}T$, exactly the quantity we were looking for. This can be verified as follows:

$$\begin{aligned} XNX^{-1} &= {}^{S}T\ {}^{T}F(t_0)\ {}^{F(t1)}T\ {}^{T}S \\ &= {}^{S}F(t_0)\ {}^{F(t1)}S\ = {}^{S}F(t_0)\ ({}^{S}F(t_1))^{-1} = M. \quad (1) \end{aligned}$$

The calibration comes down to finding *X* for given *M* and *N*. If the sensor placements could be measured with infinite precision and if the scanning registration would be perfect, equation (1) could be solved for *X* based on a single observation of *M* and *N* (however, we will show that the solution is not unique). In practice we measure a set of *n* pairs ($M_i$ , $N_i$) and search for the 6-tuple $X' = (x_X, y_X, z, \alpha_X, \beta_X, \gamma_X)$ which minimizes

$$d = \sum_{i=1}^{n} |\ XN_i X^{-1} M_i^{-1}\ |^2, \qquad (2)$$

where *X* denotes the placement matrix derived from *X'*. The norm in equation (2) can be defined as a function of the principle rotation angle and displacement component of the 4×4 matrix. In the next section we will numerically verify that equation (2) has no unique solution for $n$=1, and we will present a solution to it for small *n*, $n$>1.

## 3. NUMERICAL QUANTITIES

We consider two placements of the object shown in Fig 1. The 3D scanner (in this setup a Minolta Vivid 700 [Min2006]) and the 6 DoF sensor (Flock of Birds, or FOB, from Ascension [Ase2006]) attached to the object are shown. The placement matrix of the FOB sensor is measured relative to the magnetic transmitter, shown in Fig.1 together with an indication of its coordinate system. The difference between the two placements of the objects was chosen to be roughly an anti-clockwise rotation (as viewed in Fig.1) of the object about a vertical axis near the center of the object. The approximate axis direction is indicated by vector *a* in the picture. The placement difference can be viewed in Fig. 2, which shows the original scan data views $V_1$ and $V_2$. The RapidForm software [Rap 2006] was used to create the pictures and to perform scan view registration. The registration was implemented as a transformation *M* to displace $V_2$ back to $V_1$, hence, roughly, a clockwise rotation is applied to $V_2$, which is equivalent by rotation in positive direction about axis *a*, directed vertically downward.

The matrix *M* producing the best match of $V_2$ to $V_1$ using RapidForm's "Fine Registration" is

$$M = \begin{pmatrix} 0.5828 & 0.4963 & -0.6434 & -820.36 \\ -0.5116 & 0.8393 & 0.18387 & 236.7886 \\ 0.63128 & 0.22205 & 0.74309 & -325.2566 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

where we use the common notation of 3×3 rotation submatrix being in the topleft part of *M* and the translation specified by the 4th column (length units in mm).

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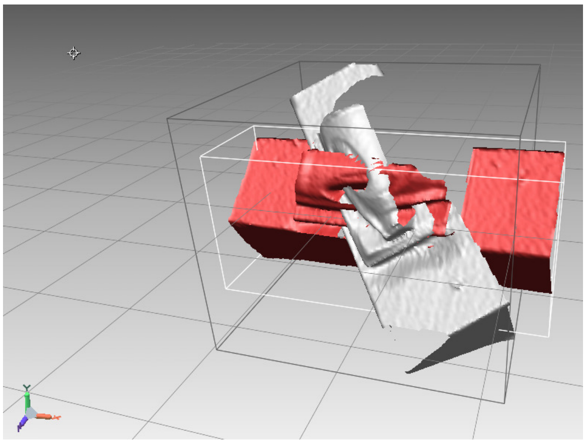

**Figure 2. Two scanned views of the object shown in Figure 1, referred to as $V_1$ and $V_2$ . View $V_2$ is the one which is roughly aligned with the scanner's *x*-axis.**

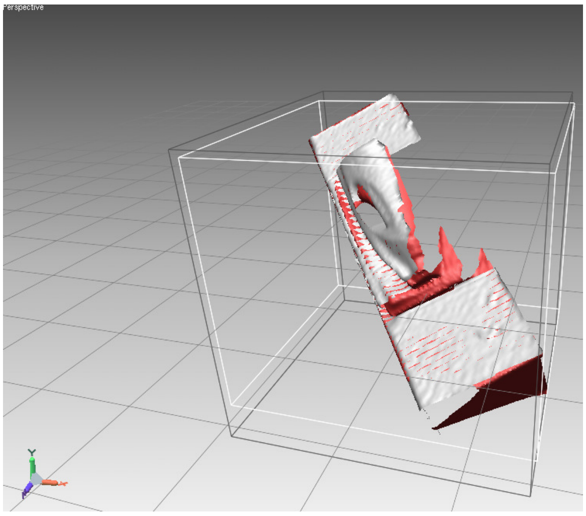

**Figure 3. View $V_2$ is matched to $V_1$ using "Fine Registration" of RapidForm.**

The matrices $F_1$ and $F_2$ are delivered by the FOB during the digitization of the object. $V_1$ was obtained while the FOB delivered $F_1$ and $V_2$ while the FOB delivered $F_2$, where

$$F_1 = \begin{pmatrix} -0.979 & 0.056 & 0.198 & 455.42 \\ 0.196 & -0.026 & 0.98 & -121.41 \\ 0.06 & 0.998 & 0.014 & -120.90 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and

$$F_2 = \begin{pmatrix} -0.415 & 0.031 & 0.909 & 431.29 \\ 0.909 & -0.04 & 0.415 & -75.44 \\ 0.049 & 0.999 & -0.012 & -121.16 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The transformation $N$ from $V_2$ to $V_1$, but now measured in the FOB frame, is $N = F_1 (F_2)^{-1}$. We find

$$N = \begin{pmatrix} 0.5873 & -0.8092 & 0.0054 & 141.7320 \\ 0.8098 & 0.5868 & -0.0277 & -429.76 \\ 0.0191 & 0.0209 & 0.9992 & -6.4853 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Now $M$ and $N$ are similar matrices, which means that they have the same eigenvalues (not necessarily the same eigenvectors). The most important quantity that $M$ and $N$ have in common is the amount of rotation $\theta$, defined as

$$\theta = \mathrm{acos}((w_{11} + w_{22} + w_{33} - 1) / 2),$$

where $w_{ii}$ refers to matrix diagonal element (*i, i*). We find for $N$ and $M$:

$$\theta_M = 54.37°,\ \theta_N = 54.08°,$$

indeed very similar values. The interpretation is that the total amount of rotation does not change when measured relative to different coordinate systems. Both the scanned object and FOB's sensor have rotated some 54 degrees be it measured from different frames. According to Poincaré, each rotation matrix also defines the axis about which the rotation occurs as follows:

$$a = \frac{1}{2\sin\theta} \begin{pmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \\ 0 \end{pmatrix}.$$

For $M$ and $N$ we find

$$a_M = \begin{pmatrix} 0.0234 \\ -0.7608 \\ -0.6201 \\ 0 \end{pmatrix} \text{ and } a_N = \begin{pmatrix} 0.0350 \\ 0.0085 \\ 1.0004 \\ 0 \end{pmatrix}.$$

The vectors $a_M$ and $a_N$ have unit length approximately. We observe that $N$ is nearly a rotation about the *z*-axis as measured in the coordinate system of the FOB, whereas $M$ specifies a rotation about an axis roughly half-way between the negative $y$- and $z$-directions, as measured in the scanner's frame.This is in accordance with the setup displayed in Fig. 1.

Now we can verify that the axes of rotation are themselves invariant under the rotation, so we expect that $M a_M = a_M$ and $N a_N = a_N$. Numerically we find:

$$Ma_M = \begin{pmatrix} 0.03502 \\ -0.76453 \\ -0.61495 \\ 0 \end{pmatrix} \text{ and } Na_N = \begin{pmatrix} 0.019112 \\ 0.005662 \\ 1.0005 \\ 0 \end{pmatrix},$$

which is confirming.

In Figure 3 we noticed that the rotation was clockwise as viewed in the picture, thus counterclockwise about the axis $a_M$. So, indeed, the rotation of $\theta_M = 54.37°$ has the correct (positive) sign.

Until now we only have information about the direction of the rotation axes, not their locations. To find the location of the rotation axis as measured in frame S, we need to find points $p_a$ (not vectors) on the line $l_M$ of rotation. This calls for computing the eigenvectors of $M$, as we will do later. The rotation part of $M$ rotates $p_a$ by angle $\theta_M$ about the axis through the origin of the scanner with direction $a_M$. By this rotation the point is displaced by the component of vector $-v_M = (-820.36, 236.7886, -325.2566)^T$ perpendicular to the rotation axis, where $v_M$ is the translation applied after this rotation, which is the 4th column of matrix $M$. By applying the rotation and the translation in sequence, point $p_a$ remains invariant, as expected for a point on $l_M$. The displacement due to rotation, $|v_M|$ equals to $2r_M |\sin(\theta_M)|$, where $r_M$ is the distance between the line $l_M$ and the scanner's origin. By definition $v_M$ should be perpendicular to $a_M$ (which can be verified by noting that the inner product $v_M \cdot a_M$ vanishes, using the data from $M$). Let us define $c_M$ as the midpoint of the line of back translation. We still don't know the location of this line. We know that its length and direction are specified by $v_M$. We also know that the vector $c_M$ is perpendicular to both $a_M$ and to $v_M$, or in vector notation

$$c_M = \frac{a_M \times v_M}{|a_M \times v_M|} |c_M|,$$

where $|c_M| = 1/2\ |v_M|\ /\ \mathrm{tg}(1/2\ \theta_M)$, from simple planimetry. The point $d_M$ on the physical line $l_M$ of rotation closest to the origin of the scanner is

$$d_M = c_M + 1/2\ v_M$$

and the point $d_M - v_M$ is the result of rotating $d_M$ due to $M$. The line of rotation relative to the scanner is thus given by

$$l_M = d_M + \lambda a_M,\ \lambda \in \mathbb{R}. \qquad (3)$$

Similarly, the line of rotation $l_N$ as observed in the FOB's reference frame is calculated from the data in $N$ as

$$l_N = d_N + \lambda a_N,\ \lambda \in \mathbb{R}. \qquad (4)$$

Numerically we find

$$l_M = \begin{pmatrix} -19.5 \\ 630.5 \\ -776.0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 0.024 \\ -0.761 \\ -0.620 \\ 0 \end{pmatrix}, \qquad l_N = \begin{pmatrix} 491.6 \\ -76.1 \\ -19.0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 0.035 \\ 0.0085 \\ 1.0004 \\ 0 \end{pmatrix}.$$

We recall that $l_M$ and $l_N$ represent the same physical axis of rotation, measured in the coordinate systems of $S$ and $T$, respectively. We therefore know the relative placements of $S$ and $T$ up to a shift along the axis of rotation. Also the orientation of $T$ around that axis is undetermined. If we repeat the experiment, where the object is rotated about a different axis (*e.g.* roughly perpendicular to $l_M$), then the placement ${}^{S}T$ can be resolved. A calibration procedure could theoretically be organized as follows:

1. Obtain a set of pairs of lines ($l_M$, $l_N$) where the object displacements should not all have a rotation component about the same axis directions.
2. Create point set $A$ as a finite collection of points in lines $l_M$ and, similarly, set $B$ containing points in lines $l_N$.
3. Apply a registration algorithm to the sets $A$ and $B$ to find the transformation $X$ such that the transformed set $XA$ matches $B$. Then $X$ is the relative placement ${}^{T}S$ as used in equation (1).

Obviously, the registration procedure in step 3 is not quite commonly applied since there is no prior knowledge about the correspondence between the points in $A$ and $B$, except for their being on a specific line in space.

Another approach to obtain $X$ is based on the equality of the eigenvalues of $M$ and $N$. If $H$ and $G$ denote the matrices of eigenvectors of $M$ and $N$, respectively and $\Lambda$ the diagonal eigenvalue matrix (which is equal for $M$ and $N$), then it holds that $M=G\Lambda G^{-1}$ and $N=H\Lambda H^{-1}$ and from equation (1):

$XH\,\Lambda\, H^{-1}X^{-1} = G\Lambda G^{-1}$,

which holds if $X=GH^{-1}$. $X$ is then interpreted as the frame of eigenvectors of $M$ relative to the frame of eigenvectors of $N$. However, since two of the

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eigenvalues (of $M$ as well as of $N$) are equal (to unity), this quantity is not unique. The two eigenvectors with eigenvalue 1 represent the direction of rotation and a point on the line of rotation, where the latter indeed has a non-zero 4th component, as is needed for the homogeneous coordinates of a 3D point. Every linear combination of this point and this direction vector remain invariant under $M$ (resp $N$), and thus collectively define the line of rotation $l_M$ and $l_N$.

# 4. METHOD TO DETERMINE $^ST$

In the previous section we considered the measurement of a single rotation axis relative to frame $S$ and relative to frame $T$, where these lines are denoted $^SL$ and $^Tl$, respectively. We can construct frames $^SL$ and $^TL$ from the data defining the lines in equations (3) and (4), which we rewrite as

$$^Sl = {}^Sd + \lambda\, {}^Sa$$

$$^Tl = {}^Td + \lambda\, {}^Ta.$$

Then we define frame $^TL$ as

$$^TL = \begin{pmatrix} \hat{x} & \hat{y} & ^Ta & ^Td \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

where $\hat{y} = {}^Ta \times \hat{q}$, and $\hat{x} = \hat{y} \times {}^Ta$, where $\hat{q}$ is any unit vector not parallel to $^Ta$. Recall that $^Ta$ is a unit length vector. $^TL$ is a frame with origin $^Td$ and with $z$-direction pointing in the direction $^Ta$. When applying the similar construction method to the line $^Sl$ we obtain the frame $^SL$. The matrix $Y = {}^SL\,({}^TL)^{-1}$ has the property that

$$Y\,{}^TL = {}^SL\,({}^TL)^{-1}\,{}^TL = {}^SL$$

$$\text{and } Y\,{}^Tl = {}^Sl,$$

a property that $^ST$ should have too.

However, it can be seen that for any $^TL'$ obtained after shifting $^TL$ over its own $z$-axis and/or after rotation about its own $z$-direction it holds that $Y\,{}^TL' = {}^SL$. Therefore, if we define

$$^TL\,(\delta,\gamma) = {}^TL\, D_z\,(\delta) R_z(\gamma),$$

where $D_z(\delta)\, R_z(\gamma) = \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 & 0 \\ \sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & \delta \\ 0 & 0 & 0 & 1 \end{pmatrix}$,

and we define $Y(\delta, \gamma) = {}^SL\,({}^TL(\delta,\gamma))^{-1}$, then

$$Y(\delta,\gamma)\,{}^Tl = {}^Sl, \text{ for all } \delta \in \mathbb{R},\ \gamma \in [0, 2\pi].$$

This confirms that we can retrieve, from data $^Sl$ and $^Tl$, the matrix $^ST$ up to transformation $D(\delta)\,R_z(\gamma)$.

A method to find the appropriate $\delta$ and $\gamma$ is as follows. Obtain $n$ measurements ($^Sl_i$, $^Tl_i$), $i$=1,..., $n$, $n$>1. From these measurements we can derive $^Sa_i$, $^Sd_i$, $^Ta_i$ and $^Td_i$ and hence the frames $^SL_i$ and $^TL_i$ for $i$=1, ..., $n$, as described above. Then we search for the pair $(\delta', \gamma') \in \mathbb{R}\times[0, 2\pi]$ for which holds

$$Y_1\,(\delta', \gamma')\,{}^Ta_i = {}^Sa_i\,,\ 2 \le i \le n, \qquad (5)$$

where $Y_1((\delta', \gamma') = {}^SL_1\,({}^TL_1\,((\delta', \gamma')\ )^{-1}$ is the $Y$-matrix derived from one particular measurement ($i$=1 in this case). Here we take all possible transforms derived from $^SL_1$ and $^TL_1$ and test them on the correct transformation of the rotation directions $^Ta_2$, ..., $^Ta_n$.

The test expressed in equation (5) is extended by

$$Y_1\,(\delta', \gamma')\,{}^Td_i \in {}^Sl_i\,,\ 2 \le i \le n, \qquad (6)$$

since point $^Td_i$ is, by definition, contained in line $^Tl_i$ and therefore $X\,{}^Td_i$ should be contained in $^Sl_i$ as well for any $i$. We denote the distance of the point to the line by $|g|$. If equation (5) holds the unit vectors on the left- and right-hand side are equal. We denote their difference as $|f|$.

In Figures 4 and 5 the behavior of $|f|$ and $|g|$ as function of $(\delta, \gamma)$ are shown for a simple measurement with $n$=2. For the particular experiment (we found $\delta'$ = -811.7mm and $\gamma'$ = 239.1°, corresponding to

$$X = Y(\delta', \gamma') = \begin{pmatrix} 0.189 & -0.982 & 0.022 & -18.12 \\ -0.662 & -0.144 & -0.735 & 89.21 \\ 0.724 & 0.125 & -0.678 & -1176.54 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Initial numerical experiments indicate that the differences $|\theta_N - \theta_M|$ (defined in section 2) remain below 0.5 degrees and the differences $|u_N - u_M|$ remain below 0.5mm.

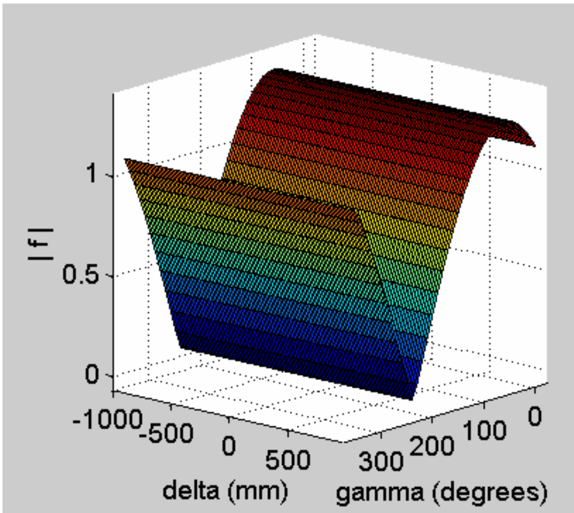


**Figure 4. Directional deviation as function of ($\delta$, $\gamma$).**

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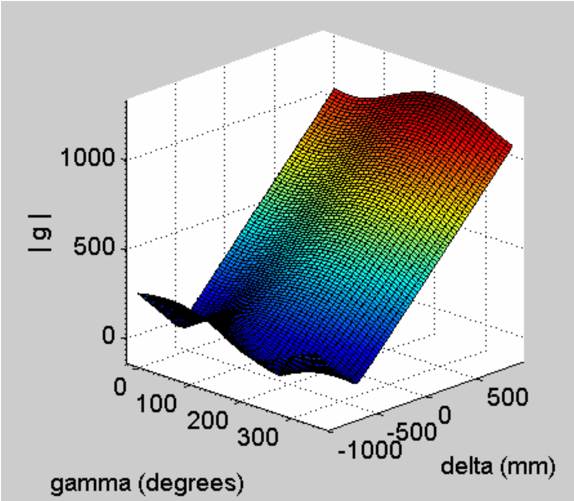


**Figure 5. Positional deviation as function of (δ, γ).**

## 5. SUPPORTED SCAN VIEW REGISTRATION

If we have determined the coordinate system $X = {}^{S}T$ of the sensor transmitter's location relative to the scanner's coordinate system we can define a procedure to apply the 6D sensor data in the scan view registration process.

Using the quantity $X = {}^{S}T$ we can compute the location of the sensor's local origin relative to the scanner as a function of time:

$${}^{S}p(t) = {}^{S}T\,{}^{T}p(t),$$

where ${}^{T}p(t) = (x(t), y(t), z(t))^{T}$ represents the position coordinates delivered by the sensor. Similarly, the sensor's orientation relative to $S$ as a function of $t$ can be computed. More importantly, if two scan views $V_j$ and $V_{j+1}$ need to be registered, the following procedure can be taken during digitization of an object:

1. Take a scan view at time $t=t_j$ resulting in a point set (or surface mesh) ${}^{S}V_j$.
2. Record the placement ${}^{T}F(t_j)$.
3. Displace the object into a new position and/or orientation, and take a scan view. Let us denote the time at which this scan view is taken as $t=t_{j+1}$. The resulting geometric set is ${}^{S}V_{j+1}$.
4. Record the placement ${}^{T}F(t_{j+1})$.
5. Supply the geometric sets ${}^{S}V_i$ and $A_{j+1}\ {}^{S}V_{j+1}$ to the scanner's registration software, where matrix $A_{j+1}$ is defined as

$$A_{j+1} = X\ {}^{T}F(t_j)\ ({}^{T}F(t_{j+1}))^{-1}\ X^{-1}.$$

Scan views ${}^{S}V_j$ and $A_{j+1}{}^{S}V_{j+1}$ will approximately match. Depending on the accuracy of the measured sensor placements and on the accuracy of similarity transformation $X$, registration of the two scan views can happen without user intervention.

The user can proceed by taking the next scan view at $t=t_{j+2}$, which will be pre-positioned using transformation $A_{j+2}$ as to automate the registration with the previous scans.

If the user decides to change the position of the sensor relative to the scanned object, for example because the sensor occludes a portion of the surface, then the following scan view at $t=t_{j+n}$ cannot be pre-positioned relative to the previous scans unless the position of the object remains unchanged between $t_{j+n-1}$ and $t_{j+n}$.

The calibration needs to be repeated only when the FOB's magnetic transmitter and the scanner device are moved relative to each other.

## 6. CONCLUSIONS

To make 3D scanning acceptable for inexperienced users, the registration procedure should be simplified. When data is recorded from a 6D sensor attached to the scanned object, the registration algorithm can be augmented. Since no placement data is available from the 3D scanner, but only transformations between scanned views, the calibration of scanner to sensor requires the similarity transform of pairs of transformations to be determined. This condition is different from those of calibration procedures typical for Augmented Reality devices. We verified numerically that based on a single transformation (obtained by applying the registration algorithm), the similarity transform can be obtained up to a translation along the axis of rotation and a rotation about this axis. We proposed a method to resolve the ambiguity by considering two or more registrations. Since two of the eigenvalues of the transformation matrices are equal, the direct solution of the similarity transform in terms of the eigenvectors is not possible with information from a single registration alone. We have found a method to compute the right similarity transform based on multiple measurements. A practical procedure to augment the scanning process has been described.

Work is in progress to determine the accuracy and error sources of the calibration process, as well as the added value of the method in practical applications. Devices different from the FOB will be experimented, since the FOB causes object occlusion and the connection cable presents a hindrance. Wireless sensors or tracking based on markers could be considered as alternatives. The authors would like to thank the reviewers for their helpful comments.

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