

# A Crowd Simulation Using Individual-Knowledge-Merge based Path Construction and Smoothed Particle Hydrodynamics

Weerawat Tantisiriwat  
Dept. of Computer Engineering  
Faculty of Engineering  
Chulalongkorn University  
Bangkok 10330, THAILAND.  
weerawat.tan@gmail.com

Arisara Sumleeon  
Dept. of Computer Engineering  
Faculty of Engineering  
Chulalongkorn University  
Bangkok 10330, THAILAND.  
arizx@hotmail.com

Pizzanu Kanongchaiyos  
Dept. of Computer Engineering  
Faculty of Engineering  
Chulalongkorn University  
Bangkok 10330, THAILAND.  
pizzanu@cp.eng.chula.ac.th

## ABSTRACT

Previous researches on crowd simulation are often based on pre-computed path planning to reduce computational time; however, the pattern of the crowd locomotion is usually unrealistic and the simulation itself is also inflexible. This paper presents a novel technique to model an interactive time crowd simulation by using automatic path construction and Smoothed Particle Hydrodynamics (SPH) in order that each individual can automatically search for the destination without path pre-computation. Two alternating stages approaches are used in our simulation. Firstly, the environment and crowd are set up, and crowd is then moved in accordance with SPH in order to achieve smooth locomotion without fixed pattern. Each agent from a group of crowd can create a map that contains the knowledge of local environment that it gains from perceptions. The local path can be extended and shared corresponding to the new environment or knowledge sharing with other agents. An automatic path determination is done in the next stage by using the potential field. The result from path selection is then used to set the direction of external force in SPH model. By using our method, natural crowd locomotion under a variety of conditions, such as forming and separating lane, obstacles avoidance and escape from unknown area can be represented in interactive time.

## Keywords

Crowd simulation, Particle methods, fluid dynamics, Smoothed Particle Hydrodynamics

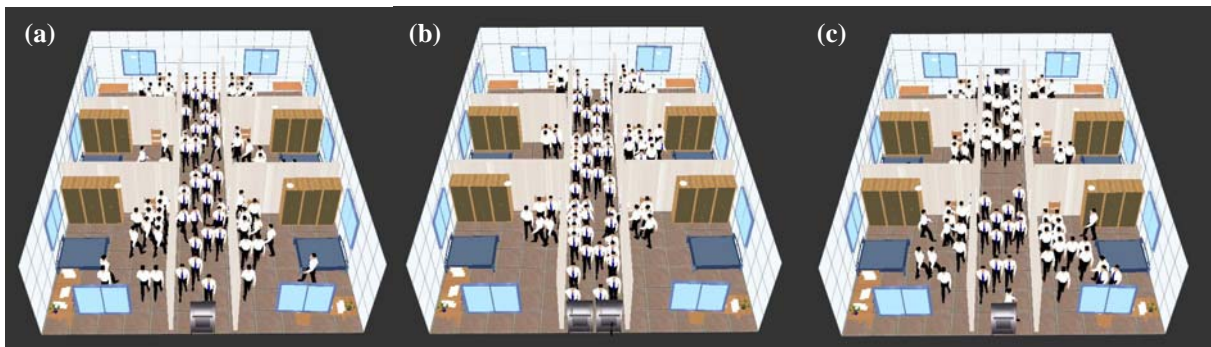
## 1. INTRODUCTION

With the advance in computer technology, it is not surprising that sometimes we can not figure out between what is real and what is computer-generated. Crowd simulation is one of the most popular among computer graphics researchers. Visual realism of crowd simulation involves more than model for decision and evaluation of environment for automatic interactivity of individual since it also has to avoid collision while moving in local direction and matching velocity of neighboring individual. It is interesting and yet not easy to model a crowd simulation that has natural crowd locomotion. This research proposes a crowd

simulation that can realistically model path searching behavior of crowd when it is in unknown place by using automatic path construction, which is modeled by imitating decision behavior of human. Local knowledge of each individual is going to be evaluated in order to find path. Crowd locomotion can be described using fluid dynamics which can naturally simulate crowd motion.

Many researches have proposed various techniques based on pre-computed path for guiding each individual to the destination, for example, Probabilistic Roadmap which uses the construction of all possible path in the environment and A\* which uses heuristic function for searching path to the destination. In addition, potential field is another method that is used to guide crowd locomotion toward the destination. Despite the fact that path planning can make crowd motion smoother, the decision behavior is not modeled naturally since each individual has knowledge of overall path in environment. Imagine if someone can escape from the unknown place where he has never been to, this is not possible since human gain knowledge from

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**Figure 1. Path selection of crowd simulation (a) one destination, (b) two closed destination, (c) two separated destination**

experience, which is different from person to person, and then that knowledge is used to select path. As a result, the assumption of global knowledge about path in the environment presented in previous works is not correctly model human decision process.

Agent-based approach is widely used to model crowd. It suits well with human behavior model since thought, decision and locomotion of each individual can be freely determined. However, it is time-consuming when applied to crowd simulation. For this reason, fluid dynamics is used to describe crowd locomotion. This method can model visual plausibly large crowd simulation. Without using agent in modeling, crowd can only move to its assigned direction and the model have no decision behavior.

For this reason, we propose a crowd simulation that coupling fluid dynamics with Agent-based approach, which is used in modeling human decision, path searching and path construction from local knowledge to create characteristic crowd locomotion. We can animate crowd simulation that has more visual realism in path determination and each individual's decision. Our model can simulate smooth crowd locomotion under various environments and conditions.

This paper is organized as follows: in Section 2 some related works are presented, in Section 3 we describe the path construction approach. Locomotion behavior is proposed in Section 4. The implementation is described in Section 5. Section 6 presents obtained result, while Section 7 is conclusions and future works.

## 2. RELATED WORKS

Since 1987, Reynolds has proposed Boids-model that described behavior of unit in the group by using specific local rule for assigning each unit behavior [Rey87b]. It was developed thereafter to model automatic interactivity [Rey99c] which was based on the idea that unit can move toward its assigned destination while handling collision avoidance from surrounding units and obstacles in the environment and locating near its neighboring units to form group behavior. This work has been developed in many

works to increase the realism in various aspects. Social potential field [Rie95a] describes the interaction force between each unit of group. It can be repulsive when each unit reaches the limited distance. However, it was not model crowd realistically. Braun has proposed crowd simulation that model individual behavior [Bra03b]. Each individual behavior is modeled by using various parameters. This research based on Herbing model [Hel00a] that suits with crowd simulation in emergency. It was improved to increase more realism by [Lat04a] which has proposed emergency avoidance force and force that try to help the victims. Wei has proposed Autonomous Pedestrians [Wei05a] that focus on modeling interactive behavior of crowd to the environment in its eyes sight. Hughes has proposed a crowd simulation using fluid dynamics to describe crowd locomotion [Hug03b]. Crowd is viewed as continuous dense particle. This work can animate impressive crowd locomotion that is frequently seen. Recently, Treuille has developed 3D crowd simulation based on previously stated work [Tre06a]. Massive amount of crowd can be simulated in interactive time by calculating equations of fluid motion using Eulerian scheme. This work can generate smooth crowd locomotion but the detailed behavior of each individual is lacked and the computation time is based on control volume.

However, the model of crowd behavior is not enough for realistic crowd simulation. Pre-computation of path is one of the factors that make crowd animation looks more plausibly. Various techniques have widely been used. One of them is path construction using grid table in A\* [Rus94d]. Using this method can easily bring unit to the destination; however, the resulting animation is not plausible especially in vehicles motion around the corner. Probabilistic Roadmap (PRM) [San02a] uses random placing of points onto the environment and connecting these points using lines to obtain the main connected path. Bayazit [Bay04a] is coupling PRM with flocking for flocking simulation. However, the efficiency of the constructed path depends on the number of

randomly-placed points. Using potential field to construct the path of crowd is one alternative that can generate smooth crowd motion. Its principle lets crowd moves toward the direction that has less potential value which can specify the factors that affect the potential value as Treuille has presented in continuum crowds [Tre06a].

Although pre-computed path makes crowd locomotion look smoother, it is still lack of a natural decision behavior since each individual is assumed to have same level of knowledge. It is not possible since human gain knowledge from experiences and uses these experiences to select different paths. This can be seen in the work of Murakami has proposed which stated about using agents to bring the victims to the exit [Mur02b]. Crowd has unordered movements that are resulted from individual perception in its area. Agents can choose the corrected path to the exit using less time while other people who have to randomly search for the exit itself or they can escape faster when they were told by agents.

We propose the crowd simulation using Individual-Knowledge-Merge based Path Construction and Smoothed Particle Hydrodynamics which integrates decision model using individual knowledge and can be shared and merged to create global path without the need of pre-computation of path. As a result, crowd can have more natural locomotion than previous methods. SPH is pure Lagrangian method; it means that we can explicitly specify the rules to individual. In our system, we model crowd volume as a set of particles which each of these represents each individual composing crowd.

### 3. PATH CONSTRUCTION

In this section, we will discuss about the path construction of crowd so that it can move to any positions in the environment. Path is constructed based on local knowledge of each individual, which is different from individual to individual and results from each individual's experience. It means that if one individual have global knowledge, it can construct the path direct to the destination without randomly move through the environment. We will describe the design of knowledge construction imitated from human perception by classifying it as perception from vision and from communication. We then described about knowledge analysis to construct a path of each individual. The model of optimized path decision which is derived from various factor and knowledge sharing of each individual that is used to construct global path of the environment is described in last two subsections.

## 3.1 Perception

Perception is the starting point in development of human abilities. The most obvious and most used is vision and communication and these abilities are used in daily lives. In this research, we use them to generate the perception of new environment and receive the news from communication. This section describes about the adaptation of human perception, which composes of perception from vision and communication to model a crowd simulation.

### 3.1.1 Vision

Vision is one of the most important ability to perception. Since human can learn many things from vision, a crowd simulation should not neglect modeling of vision. This research models the vision of each individual by setting two rules in order to increase realism to our crowd simulation. These rules are limiting vision of each individual when the object or scene is out of sight or when seeing through opaque material. In the technical term, the agents perceive the environmental data by shooting the limited sensor to detect the data as shown in Figure 2.



Figure 2. Vision

### 3.1.2 Communication

This research sets this kind of perception by setting the limited distance to communicate since communication is occurred in finite distance that each person can hear each other. The information can be exchanged immediately when two of individuals reach this limited distance as shown in Figure 3.

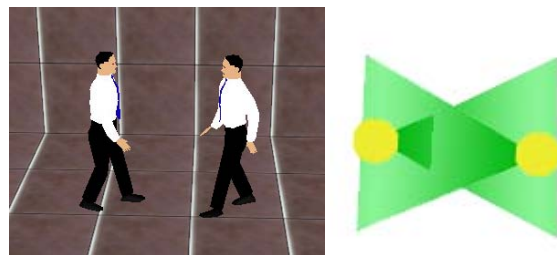


Figure 3. Communication

## 3.2 Cognition

Cognition develops from perception and it can be either large or little amount depending from experiences the one gained from their past experiences of vision and communication. Environment recognition is one of the most used skills in our lives. This research uses this idea to model recognizable crowd in order that each

individual is closer to real human. The recognition of each individual result either from seeing in limited distance as stated in section 3.1.1 or from communicating with other individuals as in 3.1.2. We assign instant recognition of environment to each individual perception. Crowd can remember immediately and its memory is not deleted until the end of the simulation. Moreover, an individual can recognize more environments while moving in the environment. It means that if it can perceive the whole environment then it can also recognize that environment. Knowledge gain from recognition that is used in path construction is called map-knowledge. This map-knowledge is classified into two groups: walkable area and unwalkable area as shown in Figure 4. These data make each individual know the area that can connect to other unknown areas.

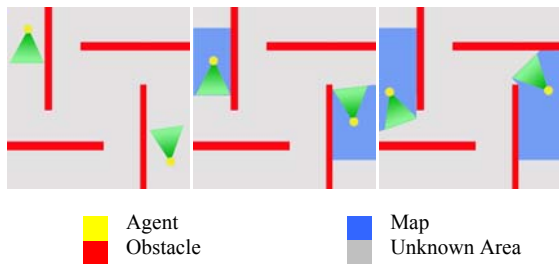


Figure 4. Map construction

### 3.3 Decision

Decision is an important part that affects realism of a behavior model. Path selection is one of the most frequent seen in our daily lives. This section presents the decision model for optimum path selection that yields the most convincing crowd locomotion toward the destination. In the comparison of each possible path can be evaluated from using cost in walking into that path, which composes of the distance, crowd density and the inconvenience of moving. Each actor is weighted by an adjustable parameter and can be written as follows:

$$C = \alpha S_{pi} + \beta \rho_{pi} + \gamma g_{pi}$$

- where  $C$  is the cost in walking into that path  
 $pi$  is the path from present position toward the destination  $i$   
 $S$  is the distance  
 $\rho$  is the crowd density within eyesight  
 $g$  is the inconvenience of moving from the height of the ground at the eye level  
 $\alpha, \beta, \gamma$  is the weight of  $S, \rho, g$  respectively

We use potential field as a method to construct the path of crowd. More details of the algorithm can be found in [Tsa05b]. We start by assign potential at the destination zero value and gradually increase the value when the distance is further. We assume that each individual construct its path from its local knowledge, not calculating from global environment. Each individual can search for its optimum path from using only its knowledge. Since we do not use pre-computed path, we can have two methods of path construction. These are path construction from cover and uncover knowledge, which can be described as follows:

#### 3.3.1 Cover Knowledge

If an individual has global knowledge toward the destination, it means that it know at least one path to the destination. For this reason, we can generate potential field from the destination to the position of that individual directly by constructing from its knowledge-based as described earlier.

#### 3.3.2 Uncover Knowledge

For an individual that has no global knowledge toward the destination, which is called the main destination in this section, it has to observe possible paths that may lead to the main destination. It has to move in the unknown area to observe new environment. The connection area between known an unknown area is assigned to be the minor destination that can lead to the main destination by calculating potential field as described earlier. However, an individual can have more than one connection area, so we evaluate potential field from the connection area of every point by choosing the minimum value of evaluated cost from every connection area of the environment at the end of the time step of the simulation. As a result, an individual can perceive the connection area and the optimum path to choose as shown in Figure 5.

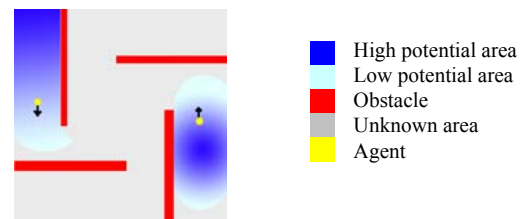


Figure 5. The local potential density map

### 3.4 Locomotion

Crowd locomotion results from path selection in previous process. Normally, a model of an individual motion is only assigning its path toward the destination that does not collide with obstacles and other individuals. However, when it comes to crowd simulation, this is not enough since flocking of massive crowd yield characteristic pattern of

locomotion. Although individuals within the same group share the same destination, it does not mean that each individual has to move toward the destination in the same pattern. Uncertain patterns are occurred since each individual has its own decision that is characterized in the real life crowd. This research models a crowd simulation by using Smoothed Particle Hydrodynamics. Crowd volume can be viewed as particle systems that move in accordance with the rule of fluid dynamics along the direction determined by potential field. It can animate smooth crowd locomotion under various conditions, for example forming a lane when passing narrow area and separating when passing wide area without collision occurred. The rule of smooth crowded locomotion is described in the next section.

### 3.5 Knowledge Sharing

Knowledge sharing is one of common behaviors that can be seen every day. One of these that involves with this research is knowledge sharing from communication as seen in asking for path of tourists. This research models knowledge sharing as describe earlier in section 3.1.2. Knowledge is new environment that it does not know before. Knowledge sharing makes an individual constructing a path to new environment without having seen it earlier by itself. An individual can gain much knowledge if it receives knowledge from other individuals from different areas, and then it can construct the optimized path to the destination faster.

## 4. LOCOMOTION BEHAVIOR

Locomotion behavior is based on Smoothed Particle Hydrodynamics (SPH) which is an interpolation method that approximates the value of a continuous field quantity and its derivative by using discrete sample points. SPH has been widely used for astrophysical problems and fluid dynamics, and has recently gained its popularity among computer graphics researchers. In SPH, the scalar quantity  $A$  at the position  $\mathbf{r}$  is approximated by a summation interpolant.

$$A_s(\vec{r}) = \sum_{j \in N} A_j \frac{m_j}{\rho_j} W(\vec{r}_i - \vec{r}_j, h) \quad (1)$$

Where  $A_j$  is the value of a field quantity at particle  $j$ .  $m_j, \rho_j$  are the mass and density of particle  $j$ .  $N$  is a set of the neighboring particle within the smoothing length,  $h$ .  $W(\vec{r}, h)$  is the weighting function referred as smoothing kernel in SPH. The smoothing length sets  $W=0$  for  $|\vec{r} - \vec{r}_j| > h$  in order to limit the interaction within finite radius. In addition, this smoothing should be even and normalized in order to

correctly represent physical quantities and ensure second order accuracy interpolation. In this work, we used the special-proposed kernels as in Gross et al. [Mul03a] to achieve real time result.

The gradient and laplacian of the summation interpolant can be written as;

$$\nabla A_s(\mathbf{r}) = \sum_{j \in N} m_j \frac{A_j}{\rho_j} \nabla W(\vec{r} - \vec{r}_j, h) \quad (2)$$

$$\nabla^2 A_s(\mathbf{r}) = \sum_{j \in N} m_j \frac{A_j}{\rho_j} \nabla^2 W(\vec{r} - \vec{r}_j, h) \quad (3)$$

Our crowd motion is described by using the equations of fluid flow, which are two equations: Navier-Stokes and continuity. The first equation states that the momentum of the flow is always conserved and any changes in fluid velocity result from self-convection, pressure gradient, internal resistance and the external forces that act upon the fluid. However, the convective term can be omitted in Lagrangian description. Navier-Stokes equation can be written in term of the substantial derivative as follow,

$$\frac{D\vec{v}}{Dt} = -\frac{1}{\rho} \nabla p + \mu \nabla^2 \vec{v} + \vec{f} \quad (4)$$

Where  $\vec{v}, \rho, P, \mu$  are the fluid velocity, density, pressure and the kinematic viscosity, respectively.  $\vec{f}$  is the external body forces in which the gravitational force and user-interaction force are included.

From equation 4, the right hand side implies that there are three forces that responsible for the motion of a particle; pressure force, viscous force and external body force. The sum of these forces is used to calculate the acceleration of particle  $i$

$$\vec{a}_i = \frac{d\vec{v}_i}{dt} = \frac{\sum \vec{f}_i}{\rho_i} \quad (5)$$

$$\sum \vec{f}_i = \vec{f}_i^{pressure} + \vec{f}_i^{visc} + \vec{f}_i^{ext} \quad (6)$$

Where

$$\vec{f}_i^{pressure} = -\sum_{j \in N} m_j \frac{p_i + p_j}{2\rho_j} \nabla W(\vec{r}_i - \vec{r}_j, h) \quad (7),$$

$$\text{and } \vec{f}_i^{visc} = \mu \sum_{j \in N} m_j \frac{\vec{v}_j - \vec{v}_i}{\rho_j} \nabla^2 W(\vec{r}_i - \vec{r}_j, h) \quad (8)$$

Another equation is continuity equation or mass conservation. It states that the rate of density change

of an infinitesimal fluid element equal the total amount of mass per volume entering and leaving the volume occupied by the element. Thus, mass is always conserved during flow.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{v}) = 0 \quad (9)$$

Since the number of particles and their individual mass are constant, the mass conservation can be omitted completely. As a result, we need to evaluate only Navier-Stokes equation to obtain the velocity and the position of each particle.

In our work, we use the Navier-stokes equation to set the rule of our crowd locomotion. The two leftmost terms on the right side of the equation are defined internal interaction within group. These terms are used to achieve smooth movement and handle collision. The external forces which are used to set the direction of crowd are derived in the second stage of the simulation. The amount of force applied is set to be constant, while the direction is resulted from the decision.

## 5. IMPLEMENTATION

Our crowd simulation based on behavior model and SPH which can generate smooth crowd locomotion in interactive time without using pre-computed path. Although it can create a smooth flow of crowd in various situations, our model suits best with the case of crowd escape from unknown environment. The process of crowd simulation is started when the individuals are assigned the random position. The system will repeats the vision creation, map creation, path creation and movement creation processes until the crowd reaches its destination while the animation is shown in Display process as shown in Figure 6. Each of them can be described in details as:

### 5.1 Perception and Recognition

The agent is received knowledge from its perception by shooting the limited sensor to detect the environmental data. The agent of each group can collect the knowledge from individuals in the same group and then merges the knowledge to keep in its map. The map composes of a known environment, connection area and obstacles. The information is then used in decision process to select the path. Their maps can be merged when two individuals have

meeting within the interactive distance as described in section 3.1.2. An agent can also use the new knowledge same as using the knowledge which is gained from its perception.

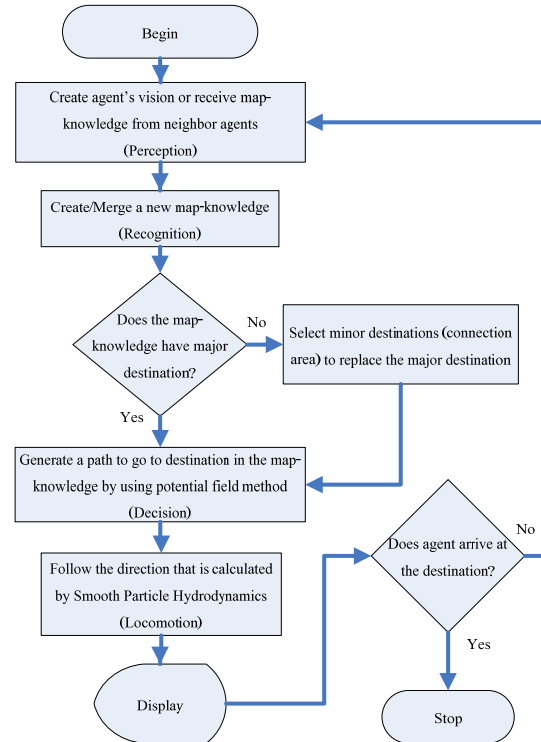


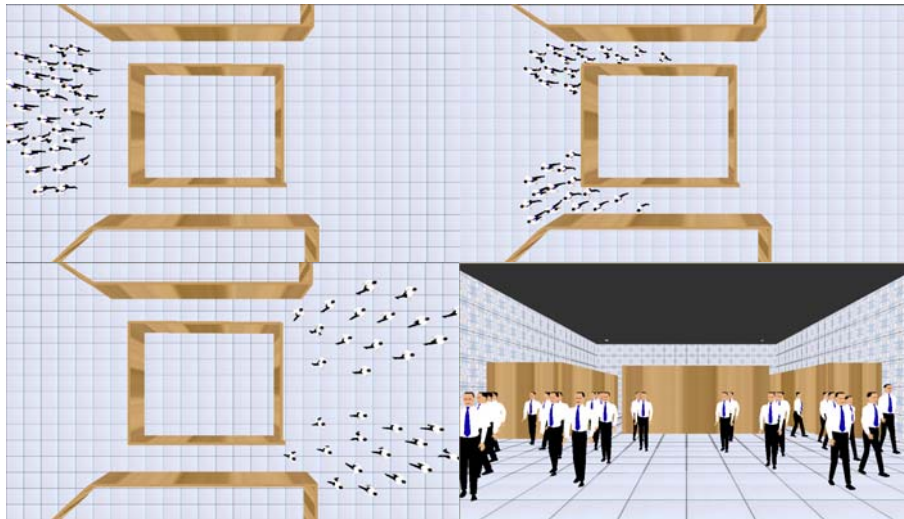
Figure 6. Simulation flow diagram

### 5.2 Decision

Path can be constructed by the agent after creating the map. An agent from each group can select the path toward the destination, using its map. In the case that the map covers the main destination, potential is generated from the destination along the map in the agents' knowledge as described in Section 3.3.1. In another case, potential is generated as described in Section 3.3.2. This generated potential is used to guide the individuals to select the direction, and this selected direction is used as the driving force in locomotion process.

Results	Approaches		
	Proposed Approach	Fluid-based	Agent-based
Non pre-computed global path	✓		
Automatic goal seeking	✓	✓	✓
Making independent decisions	✓		✓
Interactive rate of large crowd simulation	✓	✓	
Natural phenomena of crowd locomotion	✓	✓	

Table 1. Experimental results



**Figure 7. Smooth flow under variety of conditions such as forming lane in narrow area or separating in wide area**

### 5.3 Locomotion

Crowd locomotion can be created after the direction of each group is known. The locomotion can be described by using SPH which is stated in section 4. Crowd volume seems to be collection of fluid particles which move in accordance with the external driving force that act on them. This force is set to have a constant value to keep the walking speed of each individual, while the direction of the force is different since it is based on the potential field approach. When an individual travels to new environment, its knowledge can always be increased.

## 6. EXPERIMENTAL RESULTS

In our experimental results; three types, Agent-based, Fluid-based and Individual-Knowledge-Merge based (IKM), of simulation were compared. The experiments demonstrated how to find a path of the crowd simulation in the unknown complex environment with the same initial conditions. Individuals were archived in random position and tried to the goal.

First, Agent approach based on Autonomous Pedestrians research could fast to seek the goal because it used pre-computed path to guide all individuals. This approach could not generate the loosed way behavior which was seen in the real world. The system could demonstrate independent path selection by self-individual decision but could not demonstrate a natural crowd behavior when the individuals formed a large group.

Next, Fluid approach based on continuum crowds research could seek the goal same as first approach and it could demonstrate a natural large crowd phenomena in real time but could not specify each individual decision behavior.

Finally, IKM approach could seek the goal but did not use pre-computed global path. The individual had a difference direction from its vision to find the destination which consisted of both correct and incorrect path in the beginning. Then, it had the most correct path in the next time when it communicated with each others. Moreover, it could demonstrate natural large crowd phenomena in interactive rate as shown in Figure 7.

The results shown that IKM approach could be used with various situations which were flexible more than pure Agent-based or Fluid-based as shown in Table 1.

## 7. CONCLUSIONS AND FUTURE WORKS

This paper proposes a crowd simulation that coupling fluid dynamics with Agent-based approach, which is used in modeling human decision, path searching and path construction from local knowledge to create a characteristic crowd locomotion. We can animate a crowd simulation that has more visual realism in a path determination and each individual's decision. Our model can simulate smooth crowd locomotion under various environments and conditions. Each individual can recognize the difference knowledge based on their past experiences of perception, it can only generate own appropriate path to find the destination naturally. Moreover, the knowledge of each individual can be merged when they have communication. Individual uses a knowledge sharing to construct a path to new environment without having seen it earlier by itself, because the new knowledge is new environment that it does not know before. An individual can gain much knowledge, if it receives knowledge from other individuals from different areas, and then it can construct the

optimized path to the destination faster. We group the individuals that have the same knowledge and destination for computational time reduction by specifying all individual to use agent's computational results.

In the future work, we can improve the behavioral model for simulate the more realistic human behavior. This paper assumes that the shared knowledge can be used by receiver immediately which reduces a little of realistic recognition behavior of human, because the human must spends the time to learn the knowledge in the real world. We can propose an approach to address this problem by setting the time of perceptions to recognize the environment which is translated to the local knowledge in next time. Moreover, the system can be increased or improved the factors that have influence with decision behavior in several situations.

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