Nailing and Pinning: Adding Constraints to Inverse Kinematics

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ABSTRACT

Inverse kinematics is commonly applied to compute the resulting movement of an avatar for a prescribed target pose. The motion path computed by inverse kinematics, however, often differs from the expected or desired result due to an underconstrained parameter space of the degrees-of-freedom of all joints. In such cases, it is necessary to introduce additional constraints, for instance by locking a joint's position and / or rotation. We present a method to fix a joint in terms of position and explain how to incorporate these constraints into the inverse kinematics solution.

Keywords: animation, inverse kinematics, positional / rotational constraints

1 INTRODUCTION

Many Computer Graphics applications, such as virtual environments, computer games, and interactive stories, feature animated characters, for instance humans and animals. To create animated sequences, the animator should be able to position and move all parts of the character. In many animation systems, a *skeleton* (or *articulated figure*) of the character is used to specify positions and motion. Such skeletons consist of rigid links (denoted as segments in H-Anim [HAWG] terminology) connected by joints. Usually, the articulated figure has a hierarchical structure, where each joint has its own coordinate system and is positioned relatively to the coordinate system of its parent. An articulated figure can often be divided into kinematic chains (limbs) where each chain has one end that is free to move, called the end-effector. To obtain a specific configuration of the articulated figure, each joint needs to be set to the correct rotation angle to obtain the required position. Specifying the configuration of the figure by rotating each joint one by one down the hierarchy is denoted as *forward kinematics* (FK). In contrast, when the position and orientation of a specific end-effector is given, the rotation angles of all joints further up the hierarchy can be computed using *inverse kinematics* (IK).

We present a method to pin a joint to a position in space and explain how to incorporate these constraints into the inverse kinematics solution.

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2 RELATED WORK

The Resolved motion-rate method introduced by Whitney [Whi69], is one of the methods that are frequently used to solve the IK problem. Many extentions have been proposed, such as the pseudo-inverse method [MK85], the Jacobian transpose method [Wel93] and the selectively damped least squares method [BK03]. Two approaches making use of the resolved motion-method are the weighting strategy and the task-priority approach. With the weighting strategy, such as in [BMW87], when tasks get into conflict, the algorithm will distribute the residual error among the tasks according to their weight. Therefore no task is exactly satisfied unless one task's weight is higly dominant with respect to other weights. With the task-priority approach, conflicts are dealt with directly at differential level. When all goals cannot be satisfied simultaneously, the task with the highest priority reaches its goal while the residual error of the other tasks are minimized [BB98]. We use the task-priority approach.

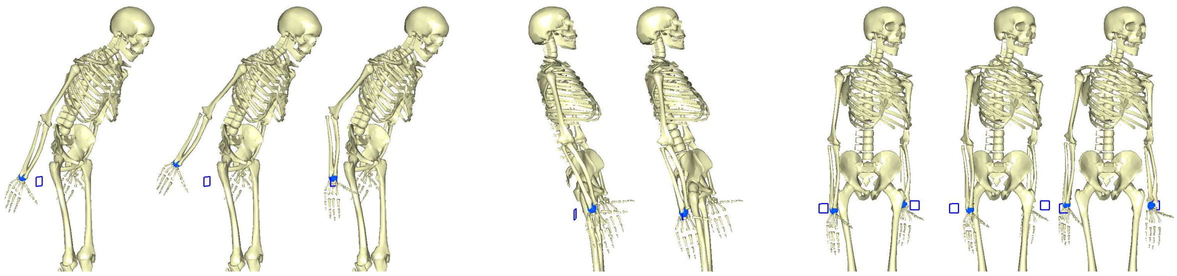

Figure 1: Constraints for keeping wrist pinned when spine is rotated. Left three images: leaning forwards moving pinned wrist; the shoulder first moves the pinned wrist away from the pinned position; the elbow moves the pinned wrist back to its pinned position. Middle two images: Moving the pinned wrist back to its position when leaning backwards. Right three images: twisting moving wrist from pinned position; the shoulder first moves the pinned wrist away from the pinned position; the shoulder moves the pinned wrist back to its pinned position

Constraints Rotational constraints, where the joint's rotation about an axis is restricted within joint angle limits, are addressed [MF98] [BT97]. Badler *et al.* [BMW87] addressed positional constraints by describing how to position an articulated figure with a weighting strategy. However, joint angle limits and rotational constraints are not considered.

3 OUR APPROACH

Inverse Kinematics Method

In our system we use the task-priority algorithm with damped least-squares. We implemented the recursive algorithm, including linear equality and inequality constraints that are satisfied after each iteration step, as discussed in [BB98].

Dealing with Positional Constraints

To efficiently check whether a joint caused a pinned joint to move from its position, each joint J_i is assigned a chain K_i $(i = 1, \ldots, n)$ that is used in the pinning algorithm (see Table 1). A chain K_i is a part of the hierarchy of the articulated figure that contains the joint J_i. For a hierarchy similar to the H-Anim specification [HAWG], the chain K_i usually starts at the joint J_i and proceeds down the hierarchy to the leave nodes. For instance, the chain corresponding to the right shoulder joint would start at the shoulder, and proceed via elbow and wrist to all finger joints of the right hand. When checking whether a joint J_i caused another pinned joint to move, only joints in the chain K_i are tested. This provides the flexibility to allow a joint to be pinned for rotations by a specified chain, but not pinned for rotations by the other joints outside the chain.

The pseudo-code of our algorithm for handling pinned joints is listed in Table 1.

4 ADDITIONAL CONSTRAINTS

In this section, we discuss special constraints that can be used for the arm of a human-like articulated figure when positional constraints are added to the IK problem.

```
for each simulation step
  changed = FALSE
  compute q̇ , q
  ∀ joints i = 1,...,n
    θ_curr = current rotation of J_i
    θ_new = θ_curr + q̇_i
    ∀ joints j in chain K_i
      if (J_j is pinned)
        pos_curr = current position of J_j
        set rotation of J_i to θ_new
        pos_new = current position of J_j
        if (pos_curr ≠ pos_new)
          ∀ joints p = i+1,...,n
            φ_curr = current rotation of joint J_p
            φ_new = φ_curr + q̇_p
            set rotation of J_p to φ_new
        pos_new = current position of J_j
        if (pos_curr ≠ pos_new)
          set rotation of J_i to θ_curr
          changed = TRUE
        ∀ joints t = i+1,...,n
        set rotation of J_t to q_t
      set rotation of J_i to θ_curr
    if (NOT changed)
      set rotation of J_i to θ_new
```

Table 1: Our algorithm.

Reachable Space

In many applications where a reaching task is applied, it should be tested whether the goal is within the reachable space of the hand, to determine whether the spine should remain fixed or should be allowed to rotate in order to obtain a natural pose. When the shoulder position is fixed, the reachable space can be roughly approximated by a half-sphere [Zha96]. However, when there is a positional or rotational constraint set for the elbow, the method discussed in [Zha96] has to be extended to deal with these cases.

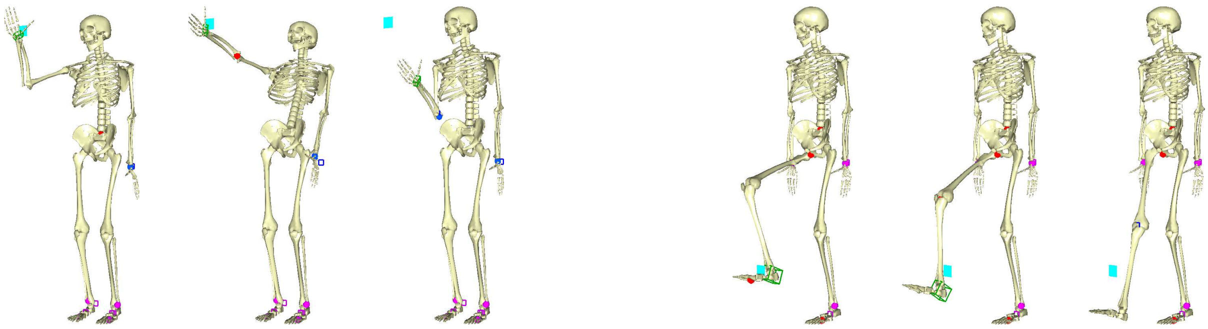

Figure 2: Different types of constraints affect the motion towards a target position. Left three images: the goal (indicated by the cyan square) is reached if no constraints are imposed on the right arm; the skeleton compensates for a rotational constraint of the right elbow by leaning backwards; the goal cannot be reached if the right elbow is pinned in space (left to right). Right three images: the right ankle reaches the goal without any constraints; a fixed rotation of the right knee prevents the ankle from fully reaching the goal; a pinned right knee makes it impossible for the ankle to get close to the goal (left to right).

If the elbow is not pinned in terms of position, the origin of the sphere is the shoulder position, the radius of the sphere is the arm length and the x-, y- and z-axis of the sphere is approximately the shoulder joint base frame. When the elbow is pinned in terms of position, the origin of the sphere is the elbow position, the radius of the sphere is the lower arm length and the x-, y- and z-axis of the sphere is approximately the elbow joint base frame.

To determine if the goal position is within the reachable space of the hand, the goal position is transformed into spherical coordinates.

Let θ define the azimuthal angle in the xz-plane and ϕ the polar angle from the y-axis. Let r be the distance from the goal position to the origin (radius). Let ℓ_L define the length of the lowerarm, ℓ_U the length of the upperarm and d the distance between the hand and the shoulder. Then,

$$\begin{aligned} r &= \sqrt{x^2+y^2+z^2}\,, \\ \theta &= \tan^{-1}\frac{x}{z}\,, \\ \phi &= \cos^{-1}\frac{y}{r} \end{aligned}$$

with

$$-\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}\,, \tag{1}$$

$$-\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}\,, \tag{2}$$

$$0 \leq r \leq \ell_U + \ell_L\,. \tag{3}$$

If the elbow is pinned to a position in space, Equation (3) should be changed to:

$$0 \leq r \leq \ell_L\,. \tag{4}$$

If the elbow has a rotational constraint, and the arm is outstreched, Equation (3) should be changed to:

$$r = \ell_U + \ell_L\,. \tag{5}$$

If the elbow has a rotational constraint, and the arm is not outstreched, Equation (3) should be changed to:

$$r = d. \tag{6}$$

θ and ϕ should satisfy Equations (1) and (2), respectively, and r should satisfy either one of Equation (3), (4), (5) or (6), according to the constraints set for the elbow. If these equations are satisfied, the goal position is within the reachable space of the hand and the rotation of the vertebrae is fixed. Otherwise the goal position is not within the reachable space of the hand and the rotation of the vertebrae cannot be fixed.

Spine Rotation

Bending forwards or backwards Assume the wrist is pinned and the spine is rotated, bending the character forward. In Figure 3 let ℓ_L be the distance between the shoulder and wrist, $\mathbf{x}$ the distance between the current and pinned wrist position and $\mathbf{d}$ the distance between the shoulder and pinned wrist position. Angle B can then be calculated making use of the *Law of Cosines*:

$$\begin{aligned} x^2 &= \ell_L^2 + d^2 - 2\ell_L d\cos B \\ \Rightarrow\quad B &= \cos^{-1}\left(\frac{\ell_L^2+d^2-x^2}{2\ell_L d}\right)\,. \end{aligned} \tag{7}$$

The shoulder's rotation around the x-axis is set in such a way that it moves the wrist away from the pinned position. Now the rotation of the elbow that is necessary to move the wrist back to its pinned position, is calculated. In Figure 3 let ℓ_L be the length of the lowerarm, $\mathbf{x}$ the distance between the current and pinned wrist position and $\mathbf{d}$ the distance between the elbow and pinned wrist position. Angle B can now be calculated as discussed above.When the spine rotates, bending the character backwards, only the rotation for the shoulder should be calculated. This is illustrated in Figure 1.

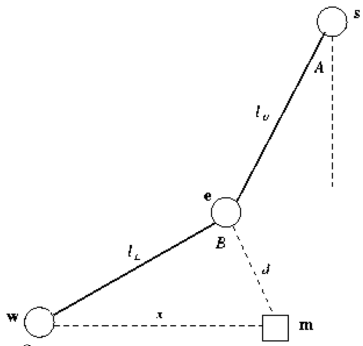

Figure 3: Triangle that is formed between the shoulder, elbow, and wrist.

Twisting Assume the wrist is pinned and the spine is rotated, resulting in a twist. Then, in Figure 3 let ℓ_L be the distance between the shoulder and wrist, $\mathbf{x}$ the distance between the current and pinned wrist position and $\mathbf{d}$ the distance between the shoulder and pinned wrist position. The rotation of the shoulder is calculated as discussed above. The rotation of the shoulder around the y-axis is set such that the wrist is moved towards its pinned position. Then the rotation of the shoulder around the x-axis is calculated in the same way, moving the wrist towards its pinned position. This is illustrated in Figure 1.

5 RESULTS

Rotating spine When the spine is rotated and the wrist is pinned, the rotation will move the wrist from its pinned position, as can be seen from the first pose from each group in Figure 1. Then by adding the constraints discussed in Section 4, the wrist is moved back to its pinned position.

Reachable space On the left of Figure 2 the task is to move the right wrist upwards towards the goal position. When there are no constraints on the arm, the goal position is within the reachable space (discussed in Section 4) of the hand and therefore the vertebrae do not rotate. When the elbow is fixed, the goal position is not within the reachable space of the hand (the distance to the goal position is not equal to the length of the arm) and therefore the rotation of the vertebrae are not fixed. To compensate for the rotational constraint of the elbow, the character leans backwards to reach the goal. When the elbow is pinned to a position in space, the goal position is further away than the length of the lowerarm and therefore not within the reachable space of the hand. In an attempt to get as close as possible to the goal position, the skeleton turns towards the goal, but the goal cannot be reached.

The full version of the paper with a detailed discussion, as well as a comparison between our method and the weighting strategy, can be found at: http://www.mpi-sb.mpg.de/resources/VirtualHumans/publ/wscg2005.pdf

6 FUTURE WORK

In future we want to apply the algorithm to motions such as walking and jumping, e.g. walking up or down stairs, where one foot needs to stay at a position while stepping downwards or upwards. Another possible application could be a ballet dancer that needs to keep the hand on the bar while doing the ballet movements and where in many movements one foot has to stay pinned at a certain position. We also want to extend the algorithm to include the possibility to pin a joint to a relative position in space, e.g. that the hands stay on the back of the dancing partner while they are dancing, i.e. moving in space.

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