Visibility Complexity of Animation in Flatland

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ABSTRACT

In this paper we address the problem of measuring the visibility complexity of scene animation in flatland. In our previous work we proposed a complexity measure which quantifies the information transfer in a static scene. Here we introduce two measures to capture the complexity of movement. The first approach measures the dissimilarity between successive frames of an animation using the variation of information exchange between each pair of patches. The second one is the euclidean distance between form factor distributions. We present preliminary results which show that both approaches attain a satisfactory and very similar animation classification.

Keywords: animation, complexity, euclidean distance, information theory, visibility

1 Introduction

Complexity is an active research area, and in the two last decades many complexity measures have been proposed from different fields [1, 9]. But what is complexity? In the majority of cases complexity is related to difficulty: "The meaning of this quantity should be very close to certain measures of difficulty concerning the object or the system in question: the difficulty of constructing an object, the difficulty of describing a system, the difficulty of reaching a goal, the difficulty of performing a task, and so on" [11]. In the particular case of a 3D scene, the complexity measure that we have proposed in our previous work [7, 8] is scene mutual information, which can be interpreted as the difficulty of computing accurately the visibility and radiosity in a scene. Scene mutual information, which is an information theory measure, quantifies the information transfer in a scene, and also the correlation or dependence among all their points or patches. Note that our approach to complexity is different from the ones based on integral geometry results [4] and reachability graph [12].

Here we apply mutual information and euclidean distance for studying the visibility complexity in dynamic environments, such as the ones considered in [2, 6]. Although we only deal with the flatland case, the results obtained can straightforwardly be extended to 3D scenes. Some of the most important applications we envisage for animation complexity are cost prediction for visibility and radiosity recomputations and the development of meshing strategies to obtain an accurate discretization. The study of 2D animation complexity has also potential applications in fields such as robot motion and architectural design.

In this paper we measure the animation complexity of several 2D scenes, and we analyze its behaviour in relation to position, quantity, and size of the moving objects.

The organisation of this paper is as follows: In section 2 we present the framework for studying animation visibility complexity in flatland. After considering different alternatives, we define in section 3 two animation complexity measures. In section 4 we compute the complexity of different sequences of frames and analyze the main reasons for the growth in complexity.

2 Framework

The most basic information theory definitions [3, 5] applied to 3D scene visibility were presented in [7]. In this section, mutual information is adapted to flatland by only changing the area of each patch with its length and the total area with the total length (see [14] for details). Flatland visibility and form factors are studied in [10, 13].

Thus, *discrete scene visibility mutual information* is defined by

$$I_s = \sum_{i=1}^{n_p} \sum_{j=1}^{n_p} \frac{L_i F_{ij}}{L_T} \log \frac{F_{ij} L_T}{L_j}$$

where n_p is the number of patches (2D segments), F_{ij} is the form factor between the patches i and j, L_i is the length of patch i and L_T is the total length of the scene (the sum of segment lengths). Discrete mutual information can be interpreted as the average information transfer in a scene. Moreover, it can be expressed as the Kullback-Leibler "distance" [3, 5], or discrimination, between the scene probability distribution $\{\frac{L_i F_{ij}}{L_T}\}$ and its independence distribution $\{\frac{L_i L_j}{L_T^2}\}$ [7].

On the other hand, *continuous scene visibility mutual information* is defined by

$$I_s^c = \iint_{x,y \in \mathcal{L}} \frac{F(x,y)}{L_T} \log(L_T F(x,y)) dx dy$$

where $\mathcal{L}$ is the set of segments that forms the environment, x and y are points on segments of the environment and $F(x,y)$ is the differential form factor between x and y. This integral can be solved by Monte Carlo integration. Similarly to [7], the computation can be done efficiently by casting global lines uniformly distributed upon segments. Thus, continuous mutual information can be approximated by

$$\begin{aligned} I_s^c &\simeq \frac{1}{N} \sum_{k=1}^{N} \log(L_T F(x_k, y_k)) \\ &= \frac{1}{N} \sum_{k=1}^{N} \log(\frac{L_T cos\theta_{x_k} cos\theta_{y_k}}{2d(x_k, y_k)}) \end{aligned}$$

where θ_{x_k} and θ_{y_k} are the angles which the normals at x_k and y_k form with the segment joining x_k and y_k, $d(x_k, y_k)$ is the distance between x_k and y_k, N is the total number of pairs of points considered, which is equal to the total number of intersections divided by two, and the value of $F(x,y)$ is $\frac{cos\theta_x cos\theta_y}{2d(x,y)}$ for mutually visible points and zero otherwise.

In [7, 8] continuous scene visibility mutual information has been proposed as an absolute measure of the complexity of scene visibility and discrete mutual information as a complexity measure of discretised scene visibility. We have also shown that when a patch is refined into m subpatches discrete mutual information increases or remains the same, and continuous mutual information of a scene is the least upper bound to discrete mutual information:

$I_s \leq I_s^c$. We also established two proposals which show a close relationship between complexity and discretization: (i) the greater the complexity the more difficult it is to get a discretization which expresses with precision the visibility or radiosity of a scene and (ii) among different discretizations of a scene the best is the one with the highest discrete mutual information. Thus, while continuous mutual information expresses how difficult it is to discretise a scene to compute accurately the visibility, discrete mutual information gives us a measure of how well it has been discretised.

We also introduced [15] a measure that quantifies the visibility complexity of a scene region. Given a point x in a region, we can compute the complexity at this point by casting random lines from it in all directions. The *complexity at point* x is expressed by

$$C_p \simeq \frac{1}{N}\sum_{k=1}^{N}\log(\frac{L_T cos\theta_{y_k}}{2d(x,y_k)}) \qquad (1)$$

where N is the number of lines cast, θ_{y_k} is the angle which the normal at y_k forms with the segment joining x and y_k, and $d(x, y_k)$ is the distance between x and y_k. It can be interpreted that in a more complex region it will be more costly to insert an object than in a less complex one. From the complexity of each point we can obtain a complexity map of a region [15].

3 Animation visibility complexity measures

A scene with moving objects is a dynamic system. The relationship between their points changes at each step, and consequently its complexity. Since each movement is modelled as a collection of small movements, the animation complexity will be given by the sum of the complexities of each step, and obviously the bigger the number of frames, the higher the animation complexity. This complexity (or dissimilarity between two frames) is a measure of the degree of recomputation required.

An animation complexity measure has to capture the variation of interactions between all the points or patches of a scene. With this aim we will analyze four possible dissimilarity measures: the first two will be rejected and the other two will show good behaviour. In order to compare two frames, a restriction is imposed: the discretization should not be changed. And, obviously, the finer the discretization the finer the measures.

3.1 Difference between complexities

As I_s^c represents the complexity of a frame, we could try to define the animation complexity between two successive frames as the absolute value of the difference between the respective continuous mutual information I_s^c. But the difference between complexities does not express the cost of movement. For example, it is easy to imagine a scene in which the movement of an object does not change the complexity and, despite this, the transformation can have a high cost. This subtraction of complexities does not contain dynamic information. In fact, I_s or I_s^c express a global property of a system but there is a loss of information with respect to the diversity of the relationships between the pairs of points or patches of a scene. In conclusion, this proposal is not appropiate.

3.2 Kullback-Leibler distance

In the context of information theory, the most used discrimination measure between probability distributions is the

Kullback-Leibler distance. The relative entropy or Kullback-Leibler distance between two probability distributions $p = \{p_i\}$ and $q = \{q_i\}$, which are defined over the same set of states $S = \{1, \ldots, n\}$, is defined by $D_{KL} = \sum_{i=1}^{n} p_i \log \frac{p_i}{q_i}$ and measures the inefficiency of assuming that the distribution is q when the true distribution is p. In our case, it should be given by

$$D_{KL} = \sum_{i=1}^{n_p} \sum_{j=1}^{n_p} \frac{L'_i F'_{ij}}{L_T} \log \frac{L'_i F'_{ij}}{L_i F_{ij}}$$

where $p = \{\frac{L_i F_{ij}}{L_T}\}$ and $q = \{\frac{L'_i F'_{ij}}{L_T}\}$ are the probability distributions of two successive frames. It is easy to see that some probabilities can be zero, those corresponding to pairs of non-visible patches which, in another frame, can become visible to each other, and then $p_i \log \frac{p_i}{0} = \infty$. In consequence, this measure fails in the majority of cases.

3.3 Animation complexity

As we have seen, discrete scene visibility mutual information is given by

$$I_s = \sum_{i=1}^{n_p} \sum_{j=1}^{n_p} \frac{L_i F_{ij}}{L_T} \log \frac{F_{ij} L_T}{L_j}$$

Moreover, we can consider that the terms

$$I_{ij} = \frac{L_i F_{ij}}{L_T} \log \frac{F_{ij} L_T}{L_j}$$

form part of a symmetric mutual information matrix ($I_{ij} = I_{ji}$), where each term represents the exchange (or transfer) of information between the patches i and j. We observe that negative values appear when $F_{ij} < \frac{L_j}{L_T}$. This situation reflects a very low interaction between the two patches involved.

In the context of information theory, we propose an animation complexity measure C_a that quantifies the variation of interactions between all the patches for n successive frames (labeled from 0 to $n-1$). This measure is defined by the sum of the complexities C_a^k of each animation step:

$$\begin{aligned} C_a &= \sum_{k=1}^{n-1} C_a^k \\ &= \sum_{k=1}^{n-1} \sqrt{\sum_{i=1}^{n_p} \sum_{j=1}^{n_p} (I_{ij}^{k-1} - I_{ij}^k)^2} \end{aligned}$$

where C_a^k is the complexity of a movement between the frames $k-1$ and k and I_{ij}^k is the exchange of information between the patches i and j in the frame k.

We have proposed root squared differences against absolute value differences because of their much higher robustness.

3.4 Euclidean distance

Finally, a non-information-theoretic measure, the euclidean distance D_e, is defined by the sum of the euclidean distances D_e^k between the probability distributions of successive frames:

$$\begin{aligned} D_e &= \sum_{k=1}^{n-1} D_e^k \\ &= \sum_{k=1}^{n-1} \sqrt{\sum_{i=1}^{n_p} \sum_{j=1}^{n_p} \left(\frac{L_i^{k-1} F_{ij}^{k-1}}{L_T} - \frac{L_i^k F_{ij}^k}{L_T} \right)^2} \\ &= \sum_{k=1}^{n-1} \sqrt{\sum_{i=1}^{n_p} \left(\frac{L_i}{L_T} \right)^2 \sum_{j=1}^{n_p} \left(F_{ij}^{k-1} - F_{ij}^k \right)^2} \end{aligned}$$

where D_e^k is the euclidean distance between the frames $k-1$ and k, $\{\frac{L_i^k F_{ij}^k}{L_T}\}$ is the probability distribution of the frame k, and in the last equality we have considered $L_i^{k-1} = L_i^k = L_i$ because the discretization of all the frames is the same.

As we will see in the next section, this measure exhibits a very similar behaviour to C_a, and thus could be considered as a cheaper computational alternative to this one.

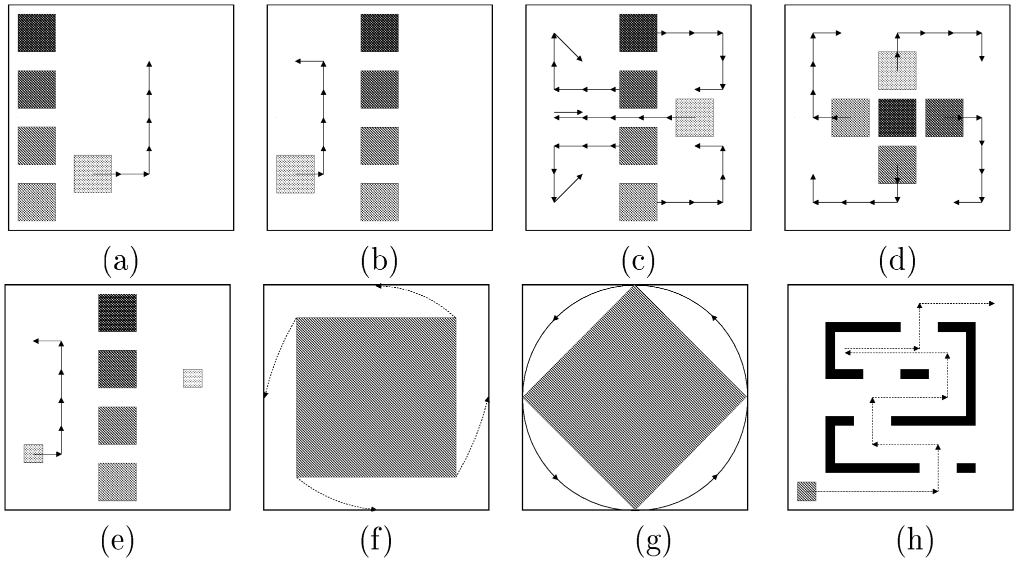


Figure 1: Scenes with a vectorial representation of movement.

scene	*frames*	n_p	I_s^c *range*	C_a	D_e
a	7	176	[3.224413, 3.309799]	0.126312	0.030174
b	7	176	[2.889719, 2.959785]	0.241348	0.048400
c	7	176	[2.774633, 3.196687]	0.748059	0.129423
d	7	176	[2.738304, 3.298303]	0.710385	0.120634
e	7	176	[2.843272, 2.877578]	0.132887	0.032049
f	10	200	[3.218023, 3.484319]	0.500879	0.085347
g	2	200	3.476344	0.139460	0.027501
h	75	234	[3.483784, 3.539923]	2.227256	0.372783

Table 1: I_s^c range, C_a, and D_e for the scenes in figure 1, where n_p is the number of patches and 10^5 global lines were used to obtain these values. All the patches of all the scenes have the same length.

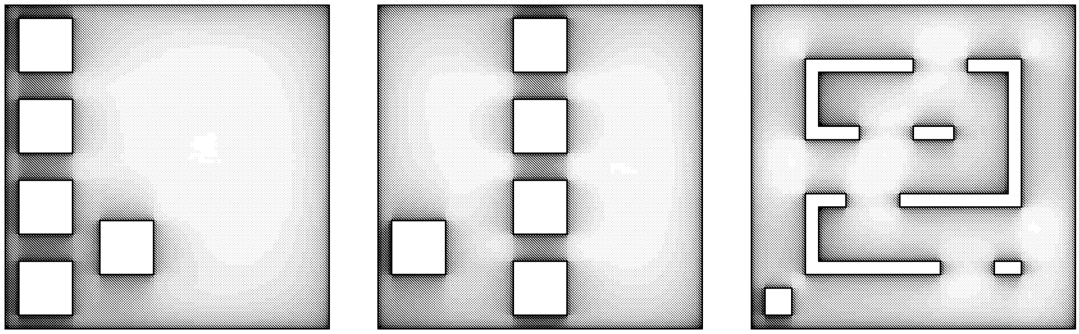
4 Results and discussion

In order to illustrate the feasibility of animation complexity measure, we compute C_a and D_e for eight sequences of frames (figure 1) whose values are contained in table 1.

For each sequence of frames, 10^5 global lines have been cast to obtain an approximated Monte Carlo solution for the form factors [16], by counting the number of intersections between pairs of segments which are visible. The first two sequences (figures 1(a, b)) show a moving square following two different paths. Animation in figure 1(b) is more complex than in figure 1(a) because the movement is produced in a more complex region (between the wall and four objects). This can be seen in figures 2(a, b) where we show the complexity maps [15], computed with formula 1, corresponding to both sequences (figures 1(a, b)). It is interesting to remark that in the sequence from figure 1(b) the scene complexity I_s^c is lower than in the other one, where the four objects are placed in the middle of the scene.

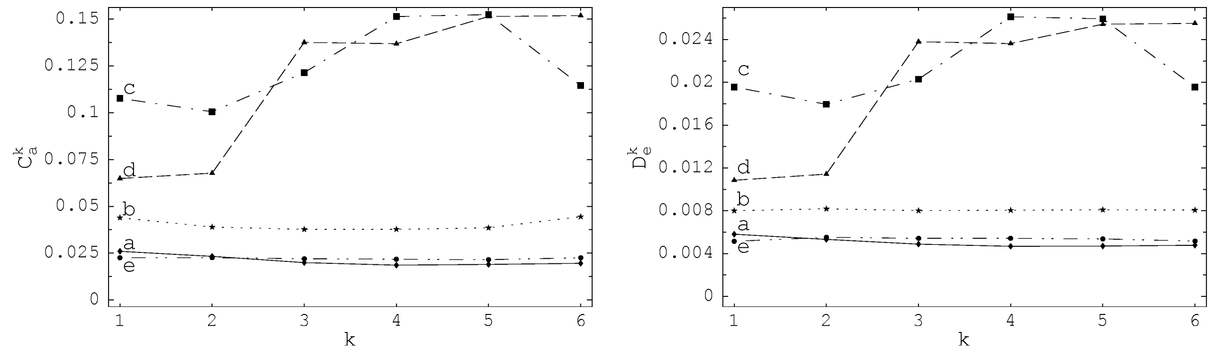
In figures 1(c,d) all the objects are moved simultaneously. As we could expect, the animation complexity increases outstandingly.

In figure 1(e), the decrease of the size of



(a) [2.697, 6.880] (b) [3.041, 6.694] (c) [3.836, 7.312]

Figure 2: Complexity maps (increasing from white to black) and ranges of the complexity of a region corresponding to figures 1(a, b, h) respectively. The computation has been carried out casting 10^3 lines from each cell of a 96×96 grid [15].



(a) (b)

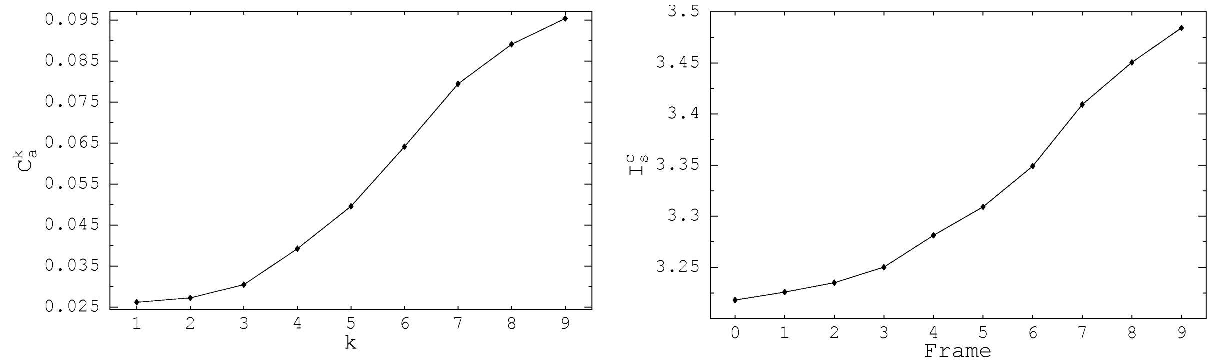
Figure 3: Evolution of C_a^k (a) and D_e^k (b) for figures 1(a) to 1(e).

the moving square implies a decrease in animation complexity. Figure 3 collects together the first five sequences (figures 1(a) to 1(e)) and shows the animation complexity and the euclidean distance of each step. An almost identical behaviour can be observed from both graphs. In our future work the reasons for this behaviour will be analyzed.

In figure 1(f), an interior square rotates (5 degrees on each step) in a square enclosure from a position with parallel sides to a position where the vertexs of the interior square almost touch the enclosure. In this case, the animation complexity increases on each step (figure 4(a)), similarly to the scene complexity I_s^c (figure 4(b)). Figure 1(g) simply represents a rotation of 90 degrees. In this case, the animation complexity is high because the variation of the relationship between the patches has been important.

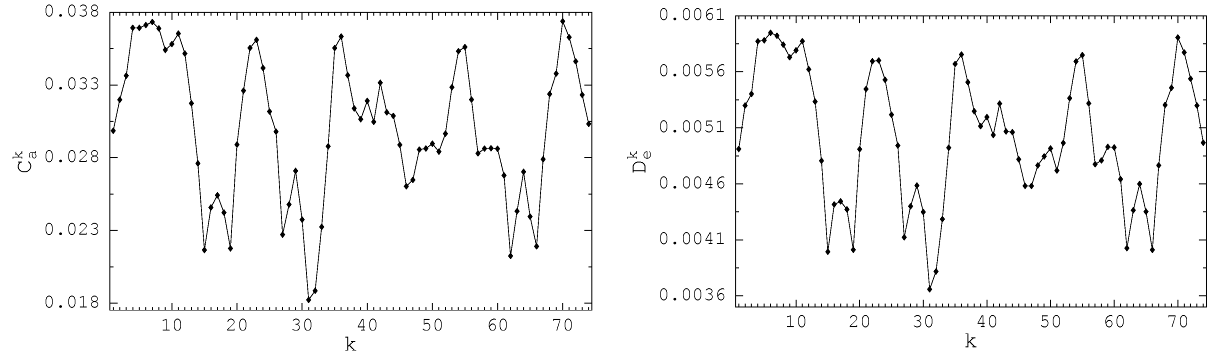
Finally, in the labyrinth scene (figure 1(h)) the high complexity is due to the big number of frames. In figure 2(c) we show the corresponding complexity map and in figure 5 we observe again a very similar evolution of C_a and D_e. From all these experiments, we conclude:

- Both measures, C_a and D_e, capture well the complexity of the animation, exhibiting very similar behaviour.
- The animation will be more complex if it is produced in more complex regions of a scene.
- The increase in moving objects in-



(a) (b)

Figure 4: Evolution of C_a^k (a) and I_s^c (b) for figure 1(f).



(a) (b)

Figure 5: Evolution of C_a^k (a) and D_e^k (b) for figure 1(h).

creases the animation complexity.

- The bigger the moving objects, the higher the animation complexity.

Future work will be addressed to extending these measures to radiosity and to studying the relationship between C_a and D_e from both theoretical and computational points of view.

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