Finding multiple DC operating points of MOS circuits fabricated in submicrometer technology

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Extended abstract

***Abstract*— The paper is focused on the analysis of circuits containing MOS transistors fabricated in submicrometer technology, having multiple DC operating points. The transistors are characterized by intricate models BSIM 3 and BSIM 4. To find the operating points an algorithm is proposed, based on the homotopy concept and the simplicial method. The algorithm is capable of finding multiple DC operating points but it does not guarantee finding all of them. For illustration a numerical example is given.**

Keywords—analog circuits, faulty diagnosis, multiple soft faulty, restart homotopy

DC analysis of MOS circuits fabricated in nanometer technology is a difficult task due to very intricate model of an MOS transistor described by several hundred nonlinear equations. Consequently, the methods for finding DC operating points are uncertain. This is why the programs based on SPICE simulator offer sets of different methods rather than one universal method. The problem is much more difficult and time consuming if a circuit possesses multiple DC operating points. As a result the methods which guarantee finding all the operating points are capable of analysing only small size circuits. Recently several papers have been published which are oriented on finding multiple operating points, but do not guarantee determining all of them [1]-[5]. Unfortunately, most of these methods exploit low level models of the MOS transistors. This paper offers a method for finding multiple DC operating points using intricate models of MOS transistors.

Let us consider a DC circuit containing MOS transistors, fabricated in submicrometer technology, driven by voltage and current sources. The transistors will be characterized by models BSIM 3 and BSIM 4 which are specified by a set of several hundred nonlinear equations. Let us connect between terminals of the devices and the ground auxiliary voltage sources $e_1,\ldots,e_n$ as shown in Fig. 1. The currents flowing through these sources will be labeled $i_1,\ldots,i_n$. These currents can be considered as nonlinear functions of voltages $e_1,\ldots,e_n$, i.e.:

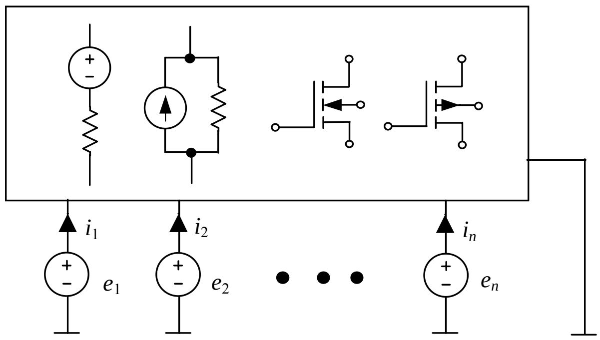

Fig. 1. MOS circuit with additional voltage sources

$$\boldsymbol{i} = \boldsymbol{f}(\boldsymbol{e}) \ , \tag{1}$$

where $\boldsymbol{i} = [i_1 \cdots i_n]^{\mathrm{T}}$, $\boldsymbol{e} = [e_1 \cdots e_n]^{\mathrm{T}}$. Unfortunately, the nonlinear function $\boldsymbol{f}(\boldsymbol{e})$ is not given in explicit analytical form, making the analysis more complicated. The values of $e_1,\ldots,e_n$ for which all the currents $i_1,\ldots,i_n$ are equal to zero determine a DC operating point of the circuit. In such a case the equation

$$\boldsymbol{f}(\boldsymbol{e}) = \boldsymbol{0} \ , \tag{2}$$

holds. Since the circuit has multiple DC operating points we search for several sets $\{e_1,\ldots,e_n\}$ which meet equation (2). To find them we use the Newton homotopy method [6], which set up the equation with an additional variable α, called a homotopy parameter

$$\boldsymbol{h}(\boldsymbol{e},\alpha) = \boldsymbol{f}(\boldsymbol{e}) - (1-\alpha)\boldsymbol{f}\left(\boldsymbol{e}^{(0)}\right) = \boldsymbol{0} \ , \tag{3}$$

where $\boldsymbol{e}^{(0)} = \left[e_1^{(0)} \cdots e_n^{(0)}\right]^{\mathrm{T}}$ is an arbitrary point and $\boldsymbol{h}(\boldsymbol{e},\alpha) = [h_1(\boldsymbol{e},\alpha) \cdots h_n(\boldsymbol{e},\alpha)]^{\mathrm{T}}$. At $\alpha = 0$ equation (3) reduces to $\boldsymbol{f}(\boldsymbol{e}) - \boldsymbol{f}\left(\boldsymbol{e}^{(0)}\right) = \boldsymbol{0}$ having the solution $\boldsymbol{e}^{(0)}$,

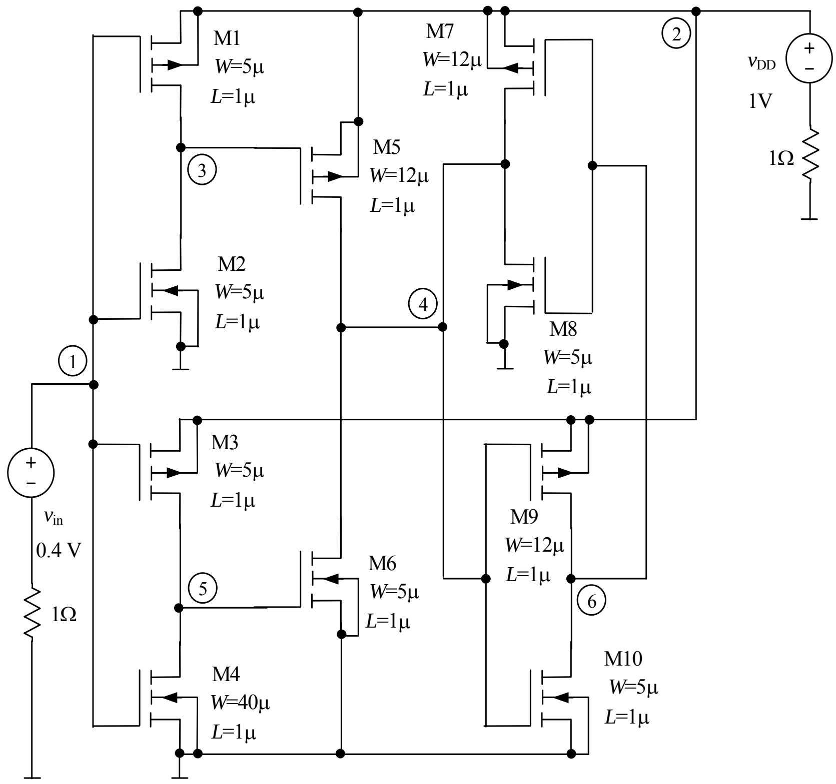

Fig. 2. An exemplary Schmitt-trigger CMOS circuit

whereas at $\alpha = 1$ it becomes the original equation (2). As α varies a homotopy path is traced and each intersection of the path with $\alpha = 1$ plane is a DC operating point. To trace the homotopy path we use the simplicial method described in reference [7] and successfully applied to fault diagnosis of analog circuits in paper [6]. The method enables us to overcome the mentioned above difficulty resulting in the lack of explicit analytical form of the function $\boldsymbol{f}(\boldsymbol{e})$. At any step of the simplicial–homotopy approach the circuit is analysed, for actual values of the voltages $e_1, \ldots, e_n$ using the SPICE simulator. The developed procedure has been implemented in MATLAB, whereas the circuit analyses are carried out using WinSpice (SPICE3F4) and both environments have been joined together. For illustration a numerical example is given.

Numerical example

Let us consider the circuit shown in Fig. 2. The MOS transistors are characterized by the BSIM 4 model. The homotopy path generated by the proposed method intersects the $\alpha = 1$ plane at three points leading to three DC operating points:

$$\boldsymbol{e}_1^* = \begin{bmatrix} 0.4000 \\ 0.9998 \\ 0.9744 \\ 0.9998 \\ 0.0195 \\ 0.0000 \end{bmatrix}, \quad \boldsymbol{e}_2^* = \begin{bmatrix} 0.4000 \\ 0.9998 \\ 0.9744 \\ 0.0000 \\ 0.1953 \\ 0.9998 \end{bmatrix}, \quad \boldsymbol{e}_3^* = \begin{bmatrix} 0.4000 \\ 0.9995 \\ 0.9741 \\ 0.4896 \\ 0.0195 \\ 0.4896 \end{bmatrix},$$

where the components of the above vectors are voltages $e_1, \ldots, e_6$ in volts.

REFERENCES

[1] G.S. Gajani, A. Brambilla, A. Premoli, "Numerical determination of possible multiple DC solutions on nonlinear circuits", *IEEE Trans. Cir. Syst.-I*, vol. 55, pp. 1074-1083, 2008.

[2] L.B. Goldgeisser, M.M. Green, "A method for automatically finding multiple operating points in nonlinear circuits", *IEEE Trans. Cir. Syst.-I*, vol. 52, pp. 776-784, 2005.

[3] A. Reibiger, W. Mathis, T. Nähring, Lj. Trajković, "Mathematical foundations of the TC-method for computing multiple DC-operating points", *Int. J. Appl. Electromagnet. Mech*., vol. 17, pp. 169-191, 2003.

[4] M. Tadeusiewicz, A. Kuczyński, "A very fast method for the DC analysis of BJT circuits", *Circuits Syst. Signal Process.,* vol. 32, pp. 433-451, 2013.

[5] M. Tadeusiewicz, S. Hałgas, "A method for finding multiple DC operating points of short channel CMOS circuits", *Circuits, Systems, and Signal Processing*, DOI: 10.1007/s00034-013-9582-7, 2013.

[6] M. Tadeusiewicz, S. Hałgas, „Global and local paramtertic diagnosis of analog short-channel CMOS circuits using homotopy-simplicial algorithm", *International Journal of Circuit Theory and Applications*, Published online in Wiley Online Library (wileyonlinelibrary.com), DOI: 10.1002/cta.1904, 2013.

[7] W.I. Zangwill, C.B. Garcia, *Pathways to solutions, fixed points, and equilibria*, (Prentice-Hall Series in Computational Mathematics, N.J., 1981.