Mathematical modelling of direct current corona discharge problems in air

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***Abstract* The problems of direct current corona discharge in air are modelled. For numerical solution is used higher-order finite element method together with space adaptivity algorithm. The methodology is illustrated by a typical example whose results are discussed.**

***Keywords* Corona, numerical analysis, finite element methods, time varying problems**

I. Introduction

Corona discharge is an electrical discharge, which is brought by the ionization of particles in electrical fields. Corona occurs in places, where is hight gradient of electrical potential.

Applications for corona discharge have existed for many years, dating to the first electrostatic precipitator. Since then, corona has been used in several applications.

II. Mathematical model

Dynamic of electron, positive ion and negative ion in air is possible describe by partial differential equations (PDEs), which are derive from Boltzmann's equation. This equations is possible write down for each particle type

$$\frac{\partial n_\mathrm{e}}{\partial t} + \mathrm{div}\ (-n_\mathrm{e}\boldsymbol{w}_\mathrm{e} - D_\mathrm{e}\cdot \mathrm{grad}\ n_\mathrm{e}) = R_\mathrm{e}\,, \qquad (1)$$

$$\frac{\partial n_\mathrm{p}}{\partial t} + \mathrm{div}\ (n_\mathrm{p}\boldsymbol{w}_\mathrm{p} - D_\mathrm{p}\cdot \mathrm{grad}\ n_\mathrm{p}) = R_\mathrm{p}\,, \qquad (2)$$

$$\frac{\partial n_\mathrm{n}}{\partial t} + \mathrm{div}\ (-n_\mathrm{n}\boldsymbol{w}_\mathrm{n} - D_\mathrm{n}\cdot \mathrm{grad}\ n_\mathrm{n}) = R_\mathrm{n}\,, \qquad (3)$$

where n_e, n_p and n_n denotes the number of density of the electrons (e), positive (p) and negative (n) ions (n $(-)$), t (s) time, $\boldsymbol{w}_\mathrm{e}$ (), $\boldsymbol{w}_\mathrm{n}$ and $\boldsymbol{w}_\mathrm{p}$ vectors of drift velocity ($\boldsymbol{w}$ $(\mathrm{m}^2\cdot\mathrm{v}^{-1}\cdot\mathrm{s}^{-1})$) and D_e, D_p and D_n diffusion coefficients (D $(-)$). The sources R_e, R_p and R_n on the right-hand sides of equations describes the rate of particles production due to the chemical reactions.

The rate of electron production includes electron impact ionization, electron attachment, electron-ion recombination, background ionization $R_0 = 10^7\ \mathrm{m}^{-3}\cdot\mathrm{s}^{-1}$ (ionization by previous discharges or by other processes such as cosmic rays or natural radioactivity) and also photoionization R_ph.

$$Re = \alpha n_\mathrm{e}|\boldsymbol{w}_\mathrm{e}| - \eta n_\mathrm{e}|\boldsymbol{w}_\mathrm{e}| - \beta_\mathrm{ep}n_\mathrm{e}n_\mathrm{p} + R_0 + R_\mathrm{ph}\,, \qquad (4)$$

where α (m^{-1}) stands for Townsend's ionization coefficient (the number of ionization events on the length which passed by electron in the electric field), η (m^{-1}) attachment coefficient (the number of attachment events on the length passed by electron in electrical field), β_ep $(\mathrm{m}^3\cdot\mathrm{s}^{-1})$ recombination coefficient (the number of recombination events of electrons with positive ions in volume per time).

The rate of ions production includes electron impact ionization, electron-ion recombination, ion-ion recombination, background ionization and photoionization for positive ions

$$Rp = \alpha n_\mathrm{e}|\boldsymbol{w}_\mathrm{e}| - \beta_\mathrm{ep}n_\mathrm{e}n_\mathrm{p} - \beta_\mathrm{pn}n_\mathrm{p}n_\mathrm{n} + R_0 + R_\mathrm{ph}\,, \qquad (5)$$

and electron attachment and ion-ion recombination for negative ions

$$Re = \eta n_\mathrm{e}|\boldsymbol{w}_\mathrm{e}| - \beta_\mathrm{pn}n_\mathrm{p}n_\mathrm{n}\,. \qquad (6)$$

Described equations are coupled with Poisson's equation

$$\mathrm{div}\ (\varepsilon_\mathrm{r}\varepsilon_0\mathrm{grad}\ \varphi) = -q(n_\mathrm{p} - n_\mathrm{e} - n_\mathrm{n})\,, \qquad (7)$$

where φ $(\mathrm{V}\cdot\mathrm{m}^1)$ is electric potential, ε_r $(-)$ relative permittivity, $\varepsilon_0 = 8.854\cdot10^{-12}\ \mathrm{F}\cdot\mathrm{m}^{-1}$ permittivity of vacuum and $q = 1.602\cdot10^{19}\ \mathrm{C}\cdot\mathrm{cm}^{-3}$ is elementary charge.

III. Numerical solution

For numerical solution of formulated mathematical model is used higher-order finite element method with space adaptivity algorithm (hp-FEM). The hp-FEM is a modern version of the finite element method which combines finite elements of variable size (h) and polynomial degree (p) in order to obtain fast exponential convergence.

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