Using Z-transform for solution of steady state and transient state in single-phase voltage inverter system

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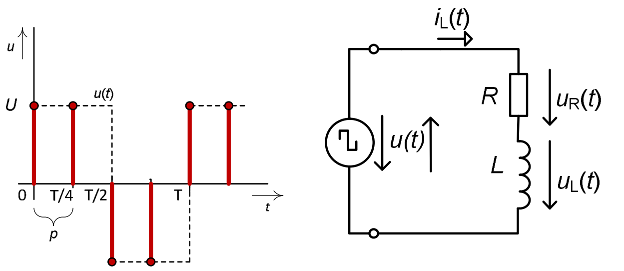
Abstract **The novel approach of analysis and modelling of power single-phase voltage source inverter with resistive - inductive load is introduced in the paper. The mathematical description of this system is using state space model with Z-transform method and series. Created impulse switching functions are used as exciting functions of this state space model. It makes possible to obtain the state variables in steady state and transient state in time domain using simulation model.**

Keywords **Z-transform, state space model, series, impulse switching function.**

I. Introduction

There are many methods of power converters analyses. Classical analytic methods, Laplace transform or Fourier analysis is suitable mainly for steady state operation [1], [2]. Transient analysis uses dynamic state space modelling or Z-transform method. One of the fastest methods is with uses impulse switching functions (ISF). For power converters systems ISF are strongly non-harmonic. Then it deals with power series of time pulses. Fig. 1 shows possible impulse switching function of considered single phase voltage inverter system and equivalent scheme of this system with resistive – inductive load.



a) b)

Fig.1. Impulse switching function of single phase voltage inverter system a) and equivalent scheme of this system with resistive – inductive load b)

II. Impulse switching function using Z-transform

Using basic definition of Z-transform - and taking into account Z-images of constant- and alternating series [3] we can write

$$U(z)=U\frac{z^2+z}{z^2+1}, \tag{1}$$

where $U(z)$ is output voltage in z-plain and roots of polynomial of denominator are

$$z_{1,2}=\pm\mathrm{j}=\pm(-1)^{\frac{1}{2}}=e^{\pm\frac{1}{2}} \tag{2}$$

placed on boundary of stability in unit circle [3], [4].

Applying inverse Z-transform for converter output phase voltages in Z-domain we can create impulse switching functions. For inverse Z-transform $U(z)\leftrightarrow\{u_n\}$ one can use the residua theorem [4], [5]

$$\sum_{i=1}^{N}\underset{z=z_i}{res}\,U(z)z^{n-1}=\sum_{i=1}^{N}\lim_{z=z_i}(z-z_i)U(z)z^{n-1}, \tag{3}$$

where $n = 0, 1, 2, \ldots, N$ is number of poles.
Then

$$U(z)\leftrightarrow\{u_n\}=\sum_{i=1}^{N}\lim_{z=z_i}(z-z_i)U\frac{z+1}{(z+1)(z-1)}z^n. \tag{4}$$

After adapting

$$u_n=u\left(n\frac{T}{4}\right)=U\sqrt{2}\sin\left(n\frac{\pi}{2}+\frac{\pi}{4}\right). \tag{5}$$

Taking discrete state-space model for single-phase converter output current as state-variable considering 1st order load (resistive-inductive)

$$x_{n+1}=\mathbf{F}_{T/4}x_n+\mathbf{G}_{T/4}\left\{u\left(n\frac{T}{4}\right)\right\}, \tag{5}$$

where $\mathbf{F}_{T/4}, \mathbf{G}_{T/4}$ are fundamental and transition matrices of system parameters. After their findings, we can determine the load current too.

III. Conclusion

Using direct and inverse Z-transform and series it is possible to derive functions as discrete voltage sequences from output voltage of power converters. Further the determination of state variables waveforms will be made.

IV. Acknowledgements

This work has been supported from project No. APVV–0138–10 and R&D operational program No. OPVaV-2008/2.1/01-SORO, ITMS 26220120046 funded by European regional development fund (ERDF).

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