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Application of the Monte Carlo method for investigation of dynamical parameters of rotors supported by magnetorheological squeeze film damping devices

J. Zapoměl[a,*], P. Ferfecki[a], J. Kozánek[a]

[a] *Department of Dynamics and Vibrations, Institute of Thermomechanics, Department of Mechanics, Dolejškova 1402/5, 182 00 Praha 8, Czech Republic*

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Abstract

A flexible suspension with damping devices is an efficient technological tool for reducing forces transmitted between the rotor and its frame. To achieve optimum performance of the damping elements, their damping effect must be adaptable to the current operating conditions. In practical rotordynamic applications this is offered by magnetorheological squeeze film dampers. Some of parameters, which determine behaviour of rotors, may have uncertain character. Then a probabilistic approach is needed for analysis of such systems. In this paper there is investigated the vibration amplitude of a rigid rotor damped by two magnetorheological squeeze film dampers and magnitude of the force transmitted to the stationary part during the steady state operating regime. The uncertain parameters of the studied system are the rotor unbalance and speed of its rotation. The Monte Carlo method was employed for this analysis.

Keywords: uncertain parameters of rigid rotors, magnetorheological dampers, force transmission, Monte Carlo method

1. Introduction

Unbalance of rotating machines is the main source of increase of time variable forces transmitted to the rotor frame. A frequently used technological solution which makes it possible to reduce their magnitude is represented by application of a flexible suspension with added damping elements. Both the theory and practical experience show that to achieve optimum performance of the damping devices their damping effect of must be controllable. This is offered by semiactive magnetorheological squeeze film dampers.

Values of some geometric, operational or technological parameters of rotating machines may be uncertain or their magnitudes may slightly vary during the operating regime. Then the approaches based on stochastic principles should be utilized for their investigation. The worst scenario method [3], the theory of fuzzy sets and interval mathematics [1, 4, 6, 11], the probability methods [2] and variational procedures belong to them. The worst scenario approach is based on searching for the worst combination of the input uncertain parameters. Its drawback is that it does not take into account probability of occurrence of such a case. The theory, confirmed by practical experience, shows that the fuzzy set approach overestimates influence of uncertain parameters on behaviour of mechanical systems. Therefore, some correction procedure is needed to minimize this undesirable effect. The probabilistic method of the Monte Carlo type requires

*Corresponding author. Tel.: +420 597 323 267, e-mail: zapomel@it.cas.cz.

to perform a large number of repeated computational simulations for randomly generated values of uncertain quantities. Application of this method seems to be suitable for investigation of practical problems as it is not complicated and may bring reliable and accurate predictions.

If there is specified an allowed value of some parameter (e.g. the vibrations amplitude, magnitude of the transmitted force, state of stress, etc.), then the Monte Carlo method enables to determine the system reliability ψ

$$\psi = 1 - \frac{N_{DIS}}{N_{SAT} + N_{DIS}}, \tag{1}$$

where N_{SAT}, N_{DIS} are the numbers of simulations when the required condition is satisfied and is not satisfied respectively.

The force transmitted to the rotor casing during the steady state operating regime depends on unbalance and on speed of the rotor rotation. In this paper, values of both these parameters are considered to be uncertain and their influence on the force transmission between the rotating and stationary parts and on amplitude of the rotor vibration is the subject of investigations.

2. The investigated rotor system

The rotor of the studied rotating machine (Fig. 1) is rigid and consists of a shaft and of one disc. Two magnetorheological squeeze film dampers are used to mount the rotor with the bearing housings flexibly coupled with the rotor frame. The whole system can be considered as symmetric relative to the disc middle plane perpendicular to the shaft axis.

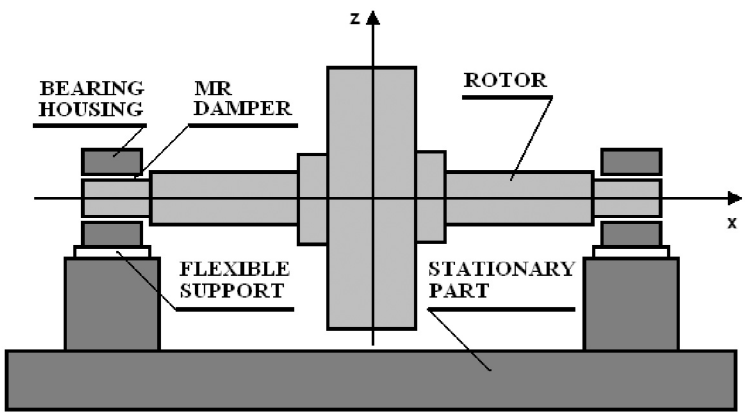

Fig. 1. The investigated rotor system

The rotor turns at constant angular speed, is loaded by its weight and excited by the disc unbalance. The squirrel springs are prestressed to eliminate their deflection (relative to the bearing housings) caused by the rotor weight.

The principal parts of each magnetorheological damper (Fig. 2) are two rings between which there is a layer of magnetorheological fluid. The outer ring is fixed to the damper's body. The inner ring is coupled with the shaft by a rolling element bearing and with the stationary part by a squirrel spring. The damping effect is produced by squeezing the lubricating layer produced by the rotor lateral vibrations. The damper is equipped with an electric coil generating magnetic flux passing through the layer of the magnetorheological liquid. As resistance against its flow depends on magnetic induction, the change of the current can be used to control the damping force.

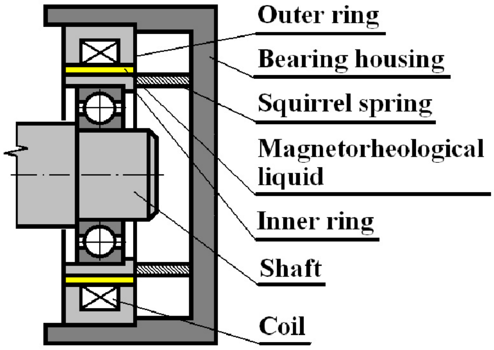

Fig. 2. Magnetorheological damper

The mathematical model of the squeeze film magnetorheological damper is based on assumptions of the classical theory of lubrication except those for the lubricant. As the magnetorheological oils are liquids with the yielding shear stress, the lubricant is represented by bilinear theoretical material whose properties depend on magnetic induction. In addition, it is assumed that the geometric and design parameters of the damper enable to consider it as short.

In the computational model, the rotor, the bearing housings and the frame are considered as absolutely rigid and the magnetorheological squeeze film dampers are represented by springs and force couplings.

Taking into account the system symmetry, the vibration of the rotating machine is governed by a set of four nonlinear equations of motion

$$m_R\ddot{y}_R + b_P\dot{y}_R + 2k_Dy_R - 2k_Dy_B = m_Re_T\omega^2\cos(\omega t + \psi_R) + 2F_{mry} + 2F_{psy}, \tag{2}$$

$$m_R\ddot{z}_R + b_P\dot{z}_R + 2k_Dz_R - 2k_Dz_B = m_Re_T\omega^2\sin(\omega t + \psi_R) + 2F_{mrz} + 2F_{psz} - m_Rg, \tag{3}$$

$$m_B\ddot{y}_B + b_B\dot{y}_B - k_Dy_R + (k_D + k_B)\,y_B = -F_{mry} - F_{psy}, \tag{4}$$

$$m_B\ddot{z}_B + b_B\dot{z}_B - k_Dz_R + (k_D + k_B)\,z_B = -F_{mrz} - F_{psz} - m_Bg, \tag{5}$$

where m_R, m_B are masses of the rotor and the bearing housing, b_P is the coefficient of the rotor external damping, k_D is the squirrel spring stiffnesses, k_B, b_B are the stiffness and damping coefficients of the bearing housing support, e_T is eccentricity of the rotor centre of gravity, g is the gravity acceleration, t is the time, y_R, z_R, y_B, z_B are displacements of the rotor and bearing housings centres in the horizontal and vertical directions, ω is angular speed of the rotor rotation, ψ_R is the phase shift, F_{mry}, F_{mrz}, F_{psy}, F_{psz} are the y and z components of the magnetorheological damping and prestress forces and (.), (. .) denote the first and second derivatives with respect to time.

To obtain the steady state solution of the equations of motion, a trigonometric collocation method was applied.

3. Determination of the magnetorheological damping forces

Magnetorheological fluids belong to the class of liquids with yielding shear stress. This implies that in the area where the shear stress between two neighbouring layers is less than critical one a core is established. In this region the magnetorheological oils behave almost like solid bodies.

Further attention is focused only on dampers whose geometric and design parameters make it possible to treat them as short. The thickness of the oil film depends on positions of the centres of the damper rings relative to the damper body [5, 7]

$$h = c - e_H \cos(\varphi - \gamma), \tag{6}$$

where h denotes the thickness of the film of magnetorheological oil, c is the width of the gap between the inner and outer rings of the damper, e_H is the rotor journal eccentricity, φ is the circumferential coordinate and γ denotes the position angle of the line of centres (Fig. 3), X, Y, and Z read for the local coordinates describing positions in the magnetorheological oil in the circumferential X, radial Y and axial Z (perpendicular to X and Y) directions respectively. Axes y and z define directions of the fixed frame of reference in which the rotor vibration is investigated.

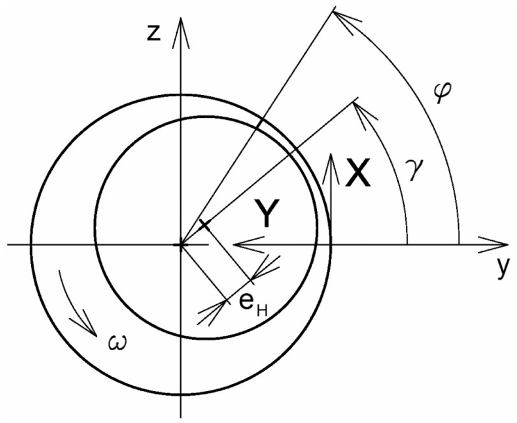

Fig. 3. Coordinate systems of the damper

Character of the flow in the lubricating film is depicted in Fig. 4. The pressure distribution in the layer of the full lubricating film is governed by the Reynolds equation adapted for bilinear material. Its derivation starts from the equation of continuity, the equation of equilibrium of the infinitesimal element specified in the lubricating layer and from the constitutive relationship of the magnetorheological oil [10].

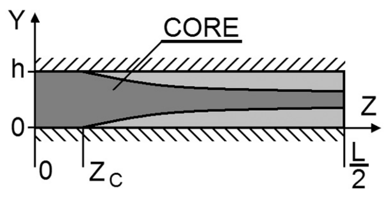

Fig. 4. The core formation in the lubricating film

After performing some manipulations the resulting relations for the pressure distribution p in the oil film read

$$p = \frac{6\eta_C \dot{h}}{h^3}\left(Z^2 - Z_C^2\right) + p_C \qquad \text{for} \qquad 0 \le Z < Z_C,\ Z_C < \frac{L}{2}, \tag{7}$$

$$a_3 p'^3 + a_2 p'^2 + a_1 p' + a_0 = 0 \qquad \text{for} \qquad Z_C \le Z \le \frac{L}{2}, \tag{8}$$

where

$$a_3 = 0.5\, h^3 \eta_C, \tag{9}$$

$$a_2 = -3h^2 \left(\tau_C - \tau_y\right) \eta_C - \left(12\dot{h}Z + C\right) \eta\, \eta_C, \tag{10}$$

$$a_1 = -6h\tau_C^2 \eta_C, \tag{11}$$

$$a_0 = 4\tau_C^3 \eta_C - 12\tau_y \tau_C^2 \eta_C - 8\tau_C^3 \eta, \tag{12}$$

or if only the core is formed in the gap

$$p = -\frac{6\eta_C \dot{h}}{h^3} \left(\frac{L^2}{4} - Z^2\right) + p_A \qquad \text{for} \qquad 0 \le Z < \frac{L}{2},\ \frac{L}{2} \le Z_C. \tag{13}$$

The derived relations for the pressure distribution (7), (8) and (13) are valid only for the dampers symmetric to their middle plane perpendicular to the shaft axis. τ_y is the yielding shear stress, τ_C is the shear stress at the core border, η is the dynamic viscosity of the oil if no magnetic field is applied, η_C is viscosity of the oil in the core, Z is the axial coordinate, Z_C is the axial coordinate of the location where the core border touches the lower and upper walls of the gap and C in (10) is the integration constant. Its value is determined from the boundary condition expressing that for $Z = Z_C$ the pressure gradient is $p' = p'_C$. L is the axial length of the damper and p_A denotes pressure in the ambient space (atmospheric pressure).

The relation between the shear stresses τ_C and τ_y and the axial coordinate Z_C are given by the following expressions [9]

$$\tau_C = \frac{\eta_C}{\eta_C - \eta}\, \tau_y, \tag{14}$$

$$Z_C = -\frac{\tau_C h^2}{6\eta_C \dot{h}}. \tag{15}$$

Relation (8) represents a cubic algebraic equation for unknown value of the pressure gradient in the axial direction p'. The root which has the physical meaning must satisfy two conditions:

- is real (not complex),
- $p' < -\frac{2\tau_C}{h}$.

When the pressure gradient p' is known, the pressure profile in the oil film is consequently calculated by the integration

$$p = \int_{\frac{L}{2}}^{Z} p' \,\mathrm{d}Z + p_A. \tag{16}$$

Relations (7), (8) and (13) are valid only for the case when $0 \le Z$ and $\dot{h} < 0$.

In cavitated areas (in areas where the thickness of the oil film rises with time) pressure of the medium is assumed to be constant and equal to the pressure in the ambient space. The nonlinear damping forces are calculated by integration of the pressure distribution in the oil film (taking into account the different pressure distributions in non-cavitated and cavitated areas) around the circumference and along the length of the damper

$$F_{mry} = -R \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_0^{2\pi} p_d \cos\varphi \,\mathrm{d}\varphi \,\mathrm{d}Z, \tag{17}$$

$$F_{mrz} = -R \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_0^{2\pi} p_d \sin\varphi \,\mathrm{d}\varphi \,\mathrm{d}Z, \tag{18}$$

where p_d is the pressure distribution in the damper clearance, R is the radius of the damper inner ring and φ is the circumferential coordinate defining positions in the oil film.

Results of theoretical analyses and measurements done by researchers and producers of magnetorheological liquids show that the dependence of the yielding shear stress on magnetic induction can be approximated by a power function

$$\tau_y = k_y B^{n_y}, \tag{19}$$

where k_y and n_y are material constants of the magnetorheological oil (n_y is often equal to 2) and B is magnetic induction.

In the simplest design case when the inner and outer rings of the damper can be considered as a divided core of an electromagnet the relation for the yielding shear stress can be expressed [8]

$$\tau_y = k_C \left(\frac{I}{h}\right)^{n_y}, \tag{20}$$

where I is the electric current and k_C denotes the design coefficient whose value depends on the number of the coil turns, on material parameters of the damper body and on arrangement of its individual parts.

4. Results of the computational simulations

The technological parameters of the investigated rotor system are: mass of the rotor 450 kg, mass of the bearing housing 100 kg, the rotor external damping coefficient 50 N · s/m, the squirrel spring stiffness 2.0 MN/m, the stiffness and damping coefficient of the bearing housing support 50.0 MN/m, 1 000 N · s/m.

The task was to find out if the vibration amplitude and the maximum force transmitted to the rotor frame in the vertical direction were less than the allowed values 100 μm, 3 850 N respectively at the nominal rotor running speed of 170 rad/s.

The unbalance and the rotor angular velocity are uncertain parameters in the investigated case. Therefore, they are given by their mean values and estimated deviations 0.036 ± 0.009 kg·m and 170 ± 6.0 rad/s, respectively. The Monte Carlo method was adapted to perform the analysis. The unbalance probability function is assumed to be uniform because it can take any value from some interval given by accuracy of the balancing procedure. As the speed of the rotor rotation is set by a controller which forces the rotor to achieve the nominal angular velocity, the speed probability function is assumed to have a normal distribution.

In Figs. 5 and 6 there are drawn the frequency response characteristics and dependence of amplitude of the variable component of the force transmitted to the rotor frame in the vertical direction calculated for mean values of the unbalance, nominal angular velocity and magnitudes of the applied current of 0.0, 0.5, 1.0, 1.5 and 2.0 A. The results show that to suppress the vibration amplitude below the allowed value the current must be at least 1.5 A. The orbits of the rotor centre and of the centres of the bearing housings are drawn in Fig. 7 and the corresponding time histories of the force transmitted to the rotor frame in the horizontal and vertical directions can be seen in Fig. 8. Both these results are related to the nominal speed of the rotor rotation.

The results show that the requirements put on performance of the rotating machine without taking into account the system uncertainties are met.

To study influence of the uncertain parameters on behaviour of the investigated rotor, the Monte Carlo method was employed and 200 000 computational simulations were carried out. The probability functions of the rotor unbalance and speed of its rotation are drawn in Figs. 9 and 10.

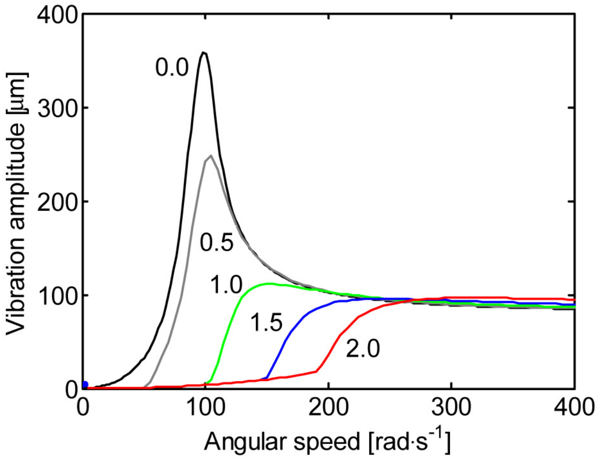

Fig. 5. Frequency response characteristic

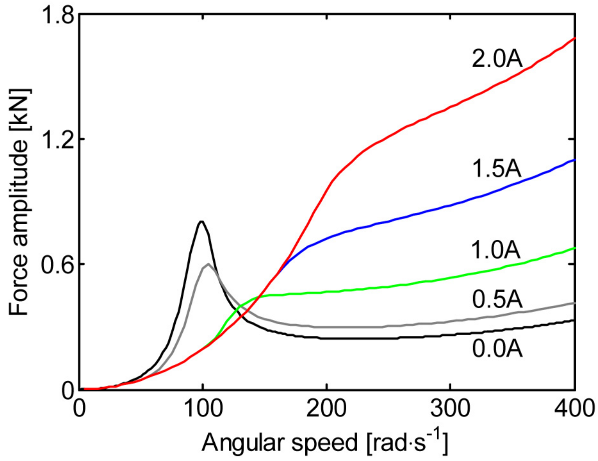

Fig. 6. Force-frequency relationship

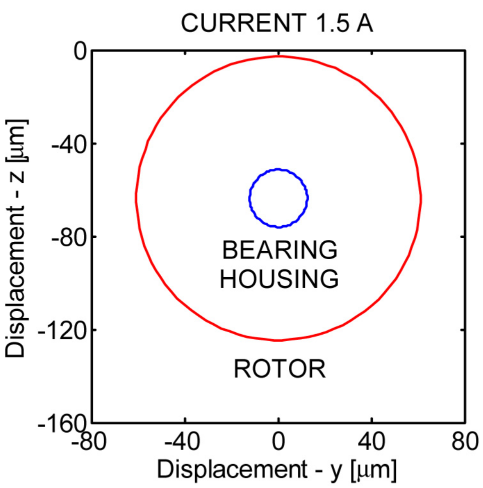

Fig. 7. Rotor and bearing housing centre orbits

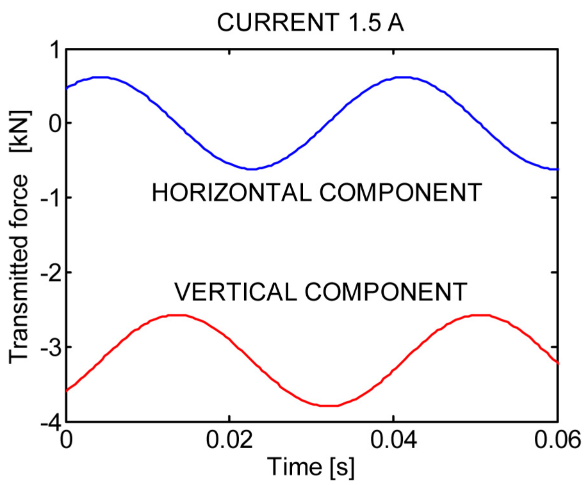

Fig. 8. Transmitted force components (nominal speed)

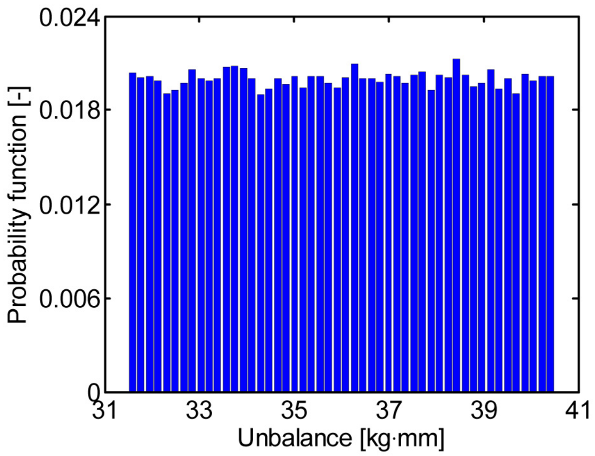

Fig. 9. Probability function – unbalance

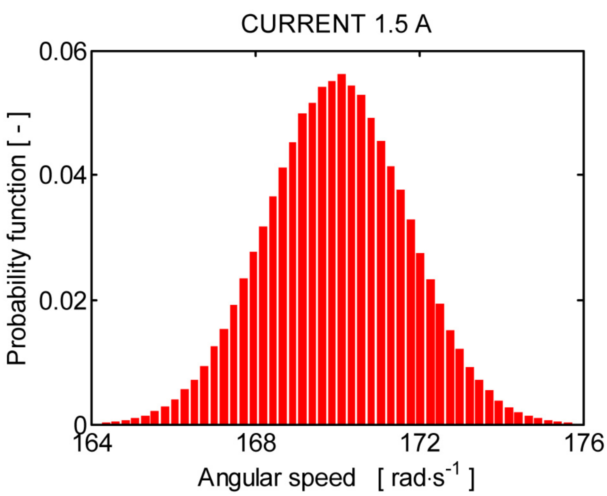

Fig. 10. Probability function – angular speed

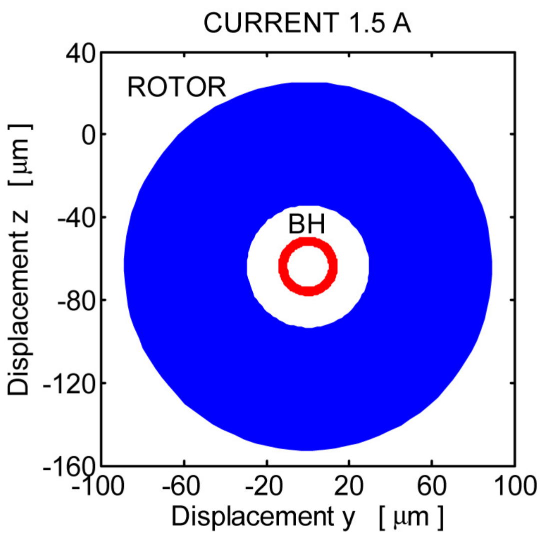

Fig. 11. Rotor orbit (BH-bearing housing)

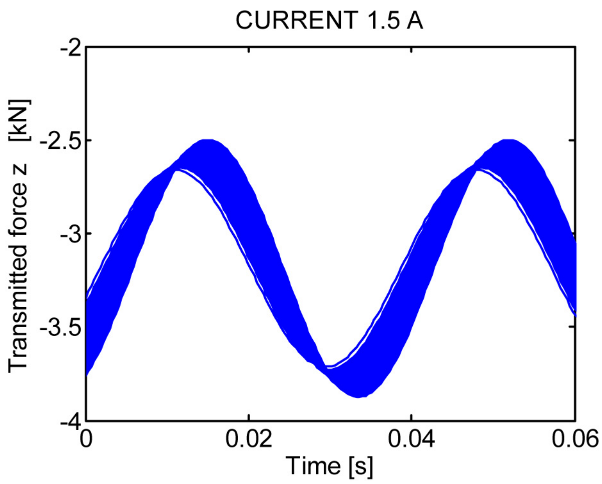

Fig. 12. Maximal transmitted force in the vertical direction

The effect of the uncertain parameters on size of the orbits of the rotor and of the bearing housing centres and the time history of the force transmitted to the rotor frame in the vertical direction are depicted in Figs. 11 and 12. The probability functions of the resulting vibration amplitude and of the maximum transmitted force are drawn in Figs. 13 and 14 which correspond to the estimated values of these quantities of 61 ± 15 μm and $-3\,802 \pm 38$ N respectively. The negative sign of the force denotes that it acts in the direction from the bearing housing to the frame.

Results of the performed simulations confirm that amplitude of the rotor vibrations does not exceed the allowed value (Figs. 11 and 13). The probability function of the maximum force transmitted to the rotor frame in the vertical direction depicted in Fig. 14 shows that the rotating machine turning at the angular speed of 170 rad/s works with reliability of 95 %. Both these results are related to the applied current of 1.5 A.

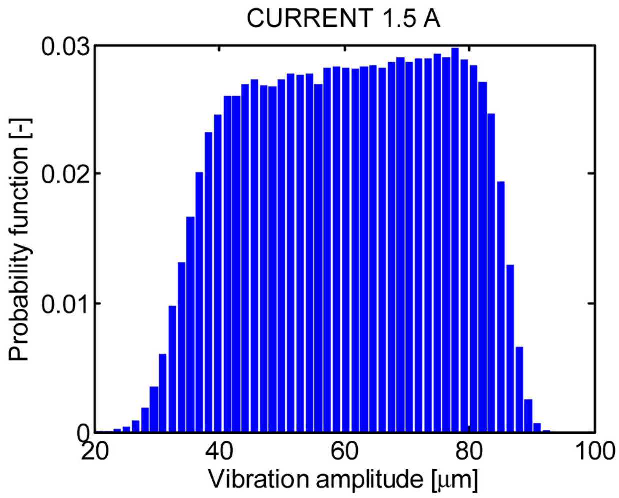

Fig. 13. Probability function – vibration amplitude

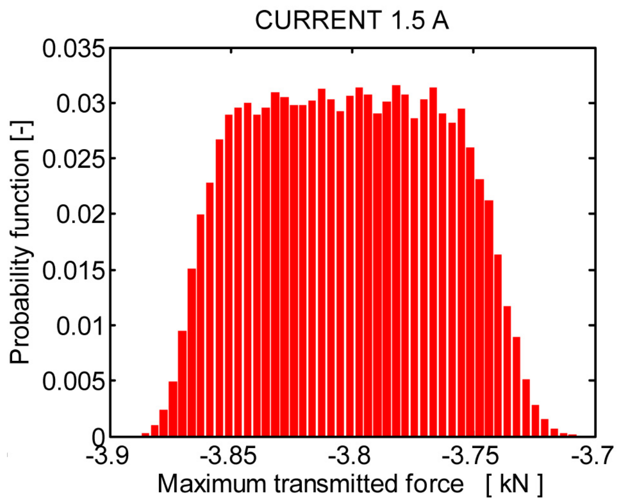

Fig. 14. Probability function – transmitted force

5. Conclusions

The probabilistic Monte Carlo method is a strong tool for dynamical investigation of mechanical systems whose behaviour is effected by geometric, design or operational parameters of uncertain values. The results are expressed in the form of interval numbers and characterized by probabilistic quantities. It follows from the theory, confirmed by experience, that to obtain the reliable results a large number of computational simulations must be performed which makes this method considerably time consuming. Advantage of the Monte Carlo method is that the number of simulations needed for achieving the required accuracy of the results does not depend on the number of uncertain system parametres. Some its modifications (e.g. stratified sampling, importance sampling, antithetic variates) based on reducing the variance of uncertain quantities make it possible to decrease the number of simulations which speeds up the calculations and rises efficiency of the procedure.

Acknowledgements

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