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Frank Plumpton Ramsey: The Economic Phenomenon Who Died Prematurely

Abstract
Although F. P. Ramsey (1903–1930) died aged nearly 27, he managed to publish a few pioneering works in mathematics, logic and philosophy. But this article deals with his work in the field of general economic theory only. We can identify four topics in his three economic contributions. By all means these four Ramsey´s topics were ahead of his time, and have influenced the economics decades later. Ramsey´s first article contributes to the expected utility theory, in other words the decision problem under uncertainty. The second one contributes to the taxes theory and monetary policy theory. In the third one he built a new and unique methodological approach to economic modelling, which is the aim of this article. So called Ramsey´s model lies traditionally within the field of economic growth but under some modifications also within the field of public finance, supply-side economics and new classical macroeconomics. Ramsey´s model is the main ingredient of contemporary analysis of short- and long-run effects of macroeconomic stabilization policy.

Key words: economic growth, dynamic optimization, Keynesian economics, Lucas Critique, methodology of positive economics, neoclassical economics, Ramsey Frank Plumpton, utility, utilitarianism

1. Introduction

Frank Plumton Ramsey was born on Feb. 22, 1903 in Cambridge. He was the eldest child of two brothers and two sisters of Arthur Stanley Ramsey (1867-1964), a mathematician, vice-master of Magdalene College from 1915 until 1934, and Agnes Mary Wilson (1875–1927), a graduate of Oxford. His brother Michael Ramsey became the 100th Archbishop of Canterbury, which is the highest post in the Church of England, from 1961 to 1974. Frank Plumpton spent nearly all his life in Cambridge, where he developed his academic career. He studied mathematics at Winchester College and later at Trinity College, Cambridge, where he graduated in 1923 (bachelor degree). In autumn 1924 he became a fellow of King´s College, Cambridge, and lecturer in mathematics, although he did not study there previously. This was thanks to the support of John Maynard Keynes because from 1921 Ramsey attended his seminary called Political Economy Club (Keynes´ Club) and they became friends. In September 1925 he married Lettice Barker. The wedding took place in a Register Office because Frank Plumton was a "militant atheist", but quite tolerant to his brother Michael. The marriage produced two daughters. In 1926 he became the University Lecturer and Director of Studies in Mathematics at King´s College. On Jan. 19, 1930 Frank Plumpton Ramsey died of jaundice, which was a consequence of an operation – he suffered from chronic liver problems (Duarte 2009).

Through his short life, he did an amount of extraordinary works in mathematics, logic, philosophy and economics. He was strongly influenced by and also influencing his famous university colleagues, such as Bertrand Russell, George Edward Moore, Ludwig Wittgenstein and, last but not least, John Maynard Keynes. In the field of mathematics he improved Russell and Whitehead´s reduction of mathematics to logic, the well-known Ramsey´s theory that describes the conditions under which some order appears – how many elements must be in a structure to hold a particular property. He also compiled the first quantitative choice theory, which explains how personal beliefs (e.g. something could happen – maybe it will rain) and desires (to reach something – not to be wet) determine our decisions (to take an umbrella with us). In this work Ramsey criticised Keynes´s rejection of subjectivity of probability (Keynes 1921) and suggested a consistent theory of decision-making under uncertainty, which became a base of modern theories of John von Neumann and Oskar Morgenstern from 1944. In the field of philosophy Ramsey was strongly influenced by Wittgenstein. There are stories that Ramsey learned German by himself in a week to translate Wittgenstein´s first draft of *Tractatus Logico Philosophicus* in 1922, but the fact is that Ramsey had had German lessons at Winchester College for at least one year, so German was not unknown to him. Ramsey and Keynes´s effort to bring Wittgenstein back in Cambridge was successful in 1929, when he became a Ph.D. student (Keynes was the person who arranged a Trinity College grant for him) and Ramsey became his supervisor and Russell and Moore examiners (Mellor 1998).

Besides Keynes and Arthur Cecil Pigou (who held the chair in the Political Economy Club after Alfred Marshall´s retirement), he, as a member of the Club, also met Roy Forbes Harrod, Joan Robinson and Piero Sraffa, who were the well-known economists of the 20th century. However, the work of Ramsey was completely different from their works. We focus on the Ramsey´s most valuable contribution, i.e. his economic growth theory, the so-called Ramsey´s model.

The basic idea of the Ramsey´s model is the maximization of the nation´s utility. The utility of the nation depends on the amount of consumption and the amount of work. We will show how the nation can maximize its utility by setting the interest rate. We will also analyse the nation´s choice between consumption and investments. The Ramsey´s model is based on an infinite time horizon so the nation can maximize its utility over time.

1.1. The economic growth theory before Frank Plumpton Ramsey

The economic growth theory has not been in the centre of interest since the constitution of the neoclassical economics. The economic growth has been seen as an uninteresting topic. A neoclassical economist mainly focused on the theory of the partial and general economic equilibrium. This equilibrium was considered an ideal state. The equilibrium was also supposed to assure the optimal rate of the economic growth.

But the economic growth was a very important topic before the classical and neoclassical economics. This theory was widely researched by mercantilists. Mercantilism was quite an inconsistent group of ideas. Mercantilists´ books were often written as a list of advice to the monarch. Mercantilists offered solutions to many common issues dealt with by the monarch from the 16th to the late-18th centuries. They built no general economic theory.

The monarchs often had a problem with the growth of the nation´s wealth or the nation´s welfare. Mercantilists were convinced that the main source of the economic growth was the accumulation of gold by means of international business. They advised to the monarchs to maximize the nation´s export and minimize the import.[1] The increased amount of gold of the nations was supposed to be the source of the economic growth (Holman 2005, 15–19).

But David Hume argued that the increase of the amount of gold in the country caused an increase of prices. Then the more expensive goods are not able to compete with the others. The second problem was that the imported goods were less ex-

1) We can call this creating a current account surplus. Of course these terms were defined hundreds of years after the Mercantilist´s era.

pensive and more attractive. So this policy was not sustainabl (Holman 2005, 35–37). Adam Smith argued that the free trade eventually made all actors better off (Smith 2001).

Finally Mercantilism was replaced by the classical economics.[2]

There is another frequently mentioned economic growth theory, despite its relatively small practical impact. It is the population growth theory by the classic Thomas Malthus.[3] [4] Malthus wrote that there was a population limit given by the limited amount of land. His theory was based on the law of diminishing marginal returns.[5] According to Malthus the amount of food grows slower than the size of the population. The Malthus´ thoughts were recovered by The Club of Rome. This global think tank was founded in 1968 (Samuelson and Nordhaus, 2007) The Club of Rome opened a discussion about the limits of the global growth according to the pollution, the consumption of non-renewable natural resources etc.[6]

2. Methodology and the assumptions of the model

The methodology of the model was revolutionary but it can sound weird to a non-economist.

The world of the Ramsey´s model contains only one universal consumption good. These goods represent consumption of all goods in the economy. This is quite a big abstraction but we must realize that the main purpose of the model was ~~a~~ modelling the economic growth. This task is different from modelling a partial market equilibrium or creating supply or demand for one good. A macroeconomic analysis cannot contain these specific microeconomic problems because the models would be too complicated.

As Friedman wrote in his Methodology of positive economic,[7] a hypothesis should not be tested by its assumptions. Friedman´s main arguments are two. First of all, there is no scale to measure how much these assumptions are similar to the economic reality. So we cannot make an objective comparison of two or more models based on their assumptions. Friedman´s second argument is that an important hypothesis should "*explain much by little*".[8] Another Friedman´s argument is that getting the exact data from the reality could be too complicated or too expensive, so no sophisticated model could be easily used for economic forecasting[9] (Friedman 1997, 7–9).

Ramsey assumed that the utility of the entire population can be measured by a single function. Another important aspect is that we do not discount later enjoyments in comparison with the earlier ones. It means that the current members´ consumption is valued as a consumption of their children, their grandchildren and each successive generation. Ramsey himself wrote that this is "*a practice which is ethically indefensible and arises merely from the weakness of this imagination*". Ramsey also proposed a modification of his model by adding a discount rate to the utility function. It means that further utility is discounted, and so the utility of today is the more preferred alternative to the same utility in the future. This means that the subjects of the economy are impatient or aware of their mortality. This modification was added to every newer version of the Ramsey´s model. This modification is also important for finding the solution of the model as we will see later.

2) Unfortunately Mercantilism caused several economic disasters. For example Spain´s economy was damaged by the accumulation of gold from Latin America (Holman 2005, 15–19).

3) The main importance of this theory is probably an illustration of common mistakes of the early philosophers and political economists (like Marx). They deeply underestimated the growth of the labour productivity caused by the raising amount of the capital per capita. Marx believed that the industrial revolution would cause massive unemployment. It actually created new working opportunities and increased the standards of living rapidly (Frank and Bernanke, 2005).

4) POLÍVKA, Martin: Malthus Thomas Robert, fek.zcu.cz/kalendarium/EKONOM/Malthus_t_r.pdf, 17. 11. 2012.

5) Malthus made several extensions to his theory. The law of diminishing marginal returns was added as one of these extensions. So Malthus used it to support his theory and the theory was not originally derived from this law.

6) TURNER, Graham: A Comparsion of the Limits to Growth with thirty Years of Reality, www.csiro.au/files/files/plje.pdf, 18. 11. 2012.

7) Friedman wrote his essay in 1953 but we can still use it to advocate the Ramsey´s approach. Many economists used models with quite unreal assumptions before and after Ramsey. Friedman wrote that a hypothesis could not be refused only because its assumptions are not real.

8) Friedman exactly wrote, that "a hypothesis is important if it ´explains´ much by little, that is, if it abstracts the common and crucial elements from the mass of complex and detailed circumstances surrounding the phenomena to be explained and permits valid predictions on the basis of them alone. To be important, therefore, a hypothesis must be descriptively false in its assumptions; it takes account of, and accounts for, none of the many other attendant circumstances, since its very success shows them to be irrelevant for the phenomena to be explained."

9) Friedman proposed that economic models should be compared by accuracy of their forecasts.

The major controversy of the model is the problem of setting the interest rate and the consumption-investment ratio. This can be done in two ways. The first way is by voluntary actions of the members of the society. The main problem is that these members cannot do these actions because they don´t know these economic parameters and they don´t know how to make the proper decisions.[10] The second way is by the action of a central planner. The central planner would have to know the utility function of the society and also would have to have purely altruistic motivation. His only goal would have to be the maximization of the nation´s happiness.

Ramsey also assumed that a number of members of the nation do not change. Because the Solow´s model contained the possibility of the constant rate of the population growth, this was also added to the Ramsey´s model. We will see that this enhancement does not markedly change the solution or the conclusions of the Ramsey´s model.

Another important topic in economics is making the subjects´ expectations. Ramsey assumed that the subjects are rational.

Ramsey also assumed that the community will always be governed by the same motives. No generation will selfishly consume savings. All generations have the same utility function and all generations maximize the utility in an infinite time horizon (Ramsey 1928, 543–546).

3. Description of the model

Now we will explain a simplified version of the Ramsey´s model. One on the major problems of the Ramsey´s model was too a complicated mathematic method. Because the dynamic optimization with the Hamiltonians is a complicated mathematic method, we will explain the basic ideas of the Ramsey´s model on a simplified version. This simplified version was published by J. Black (1962, 360–366). The simplified version of the Ramsey´s model does not change any basic idea of the model.

The main difference between these versions is a stronger assumption about the production and the utility functions.[11] We will restrict our analysis to the utility functions and the production functions with constant elasticity of substitutions. We will also assume the marginal productivities to be constant.

Let´s start with the definition of the basic variables. Let r stand for the rate of the return on savings. The rate of the return on savings is constant and equal to the marginal productivity of capital. Let ρ be the rate of time-discounting, and let u_t denote the marginal utility of consumption at time t. We will describe the development of the economy from time o to infinity. Let u_0 be the value of the utility in time 0. Now we will use an important thought. Let´s consider that the utility is higher in time t than in time 0. Then the society could get a higher utility by decreasing the consumption in time t and by increasing consumption in time 0. The decline of the utility at time t would be smaller than the increase at time o because of the decreasing marginal utility of consumption. But if the nation gives up at time o, it allows the consumption at time t to increase by e^{rt} units. The giving up of the consumption increase of the capital of the nation and the marginal productivity of the capital is expressed by r. So we can say that $u_t = u_0 \cdot e^{-(r-\rho)t}$ for every $t > 0$. The $e^{-\rho t}$ is the time-discounting of the future utility and the e^{rt} is the increase of the future utility because of the higher amount of the capital goods (or the higher investment of the nation).

Let´s denote x to be the consumption of the nation. We can define the function of the consumption in time by $f(x_0, t)$, where t is time and x_0 is the consumption at time o. We can define the time-path of consumption by the value of consumption over time. This function connects the total utility and total consumption. We can find out the present value of the future consumption. This present value is given by the integral

$$\int_0^\infty f(x_0, t) \cdot e^{-rt} dt$$

We can also find the value of the present capital stock needed to finance such a consumption path.

The convergence of the integral is a very important problem of the solution. It depends on the variables g a r. Let´s define g as the growth rate of consumption. If $g = r$, than the nation has the same value of consumption in any t. This is so because moving a single unit of consumption of goods to the future has two effects. It decreases the utility of the consumption of the unit by e^{-gt} and also increases the possibility of further consumption by e^{rt}. But $e^{-gt} \cdot e^{rt} = 1$ so the effects eliminate each other. So the present value of $f(x_0, t)$ is $\int_0^\infty x_0\, dt$. This integral does not converge. If $g > r$ then $\int_0^\infty f(x_0, t) \cdot e^{-rt} dt$ does not converge either. If $g < r$ the integral $\int_0^\infty f(x_0, t) \cdot e^{-rt} dt$ converges. So if $g < r$ we can calculate the highest x_0 which can be afforded. Let us call this maximum value consumption the optimum consumption level x_0^*. The optimum savings ratio will depend on the shape of the utility function and of the extent of time-discounting.

We also need to make stronger assumptions about the utility function. Let´s denote U as the total utility, $u = \frac{dU}{dx}$ the marginal utility and $u' = \frac{du}{dx}$the derivative of marginal utility. We will assume a marginal utility function with constant elasticity $u = b \cdot x^{-\alpha}$ where α is a parameter of this function. For $\alpha = 1$ the utility function $U = b \cdot \ln(x)$ so $\lim_{x\to\infty} U = \lim_{x\to\infty} b \cdot \ln(\mathrm{x}) = \infty$. It´s easy to see we need to

10) We will discuss this problem later.

11) But these assumptions are quite common in microeconomics.

$\frac{1}{u} \cdot \frac{du}{dt} = (r - \rho)$. Because $\frac{du}{dt} = \frac{du}{dx} \cdot \frac{dx}{dt}$ and $\frac{u'}{u} = -\frac{1}{x}$ so $\frac{1}{x} \cdot \frac{dx}{dt} = (r - \rho)$ which is satisfied by $x_t = x_0 \cdot e^{(r-\rho)t}$. The present value is

$$\int_0^\infty x_0 \cdot e^{-\rho t} dt = x_0 \int_0^\infty e^{-\rho t} dt = x_0 \left[-\frac{e^{-\rho t}}{\rho} \right]_{t=0}^\infty = \frac{x_0}{\rho}$$

Now we will make decision about x_0. We know that $\frac{x_0}{\rho}$ is the present value of consumption over time. The present income is y_0 and the present value of income y_0 is given by $\frac{y_0}{r}$. So if the present value of consumption equals the present value of income $\frac{x_0}{\rho} = \frac{y_0}{r}$ than we maximize the x_0. The present income y_0 is produced by capital stock K. Because r is the productivity of the capital, then $y_0 = r \cdot K$. So $\frac{x_0}{\rho} = \frac{r \cdot K}{r}$ and $x_0 = \rho \cdot K$. The value of the consumption is $x_0 = \frac{\rho}{r} \cdot y_0$ and $\frac{r-\rho}{r} \cdot y_0$ is saved. If $\alpha = 1$ we need discounting $\rho > 0$ otherwise x_0 would be o.

Now let´s assume that $\alpha > 1$. It is easy to see that the utility function can be defined as

$$U = \mathrm{Z} - \frac{b}{\alpha - 1} \cdot x^{-(\alpha-1)}$$

We assume that the utility function U has the finite value for every x: $\lim_{x\to\infty} \mathrm{Z} - \frac{b}{\alpha-1} \cdot x^{-(\alpha-1)} = Z$. This value Z is a bliss point and we will discuss it later.

We still require $\frac{1}{u} \cdot \frac{du}{dt} = (r - \rho)$. By means of substitution we find $\frac{1}{x} \cdot \frac{dx}{dt} = \frac{(r-\rho)}{\alpha}$. This is satisfied by $x_t = x_0 \cdot e^{\left(\frac{r-\rho}{\alpha}\right)t}$. The present capitalised value of this is

$$\int_0^\infty x_t \cdot e^{-rt} dt = \int_0^\infty x_0 \cdot e^{\left(\frac{\rho+(\alpha-1)r}{\alpha}\right)t} \, dt = x_0 \cdot \frac{\alpha}{\rho + (\alpha - 1)r}$$

Again, the present value of the capital is $K = \frac{y_0}{r}$ and the consumption at time o is $x_0 = y_0 \cdot \frac{\rho+(\alpha-1)r}{ra}$. So the saving ratio will be $\frac{r-\rho}{r}$.

The second problem is the amount of work of the nation. We will assume the constant marginal productivity of labour. We also assume no technological growth. But there is the growing amount of capital K in the economy. If one unit of work is done, then e^{rt} fewer units of work need to be done in t years of time. The marginal disutility of labour should fall over time at a rate which will prevent the nation to gain by shifting work in time.

Again, we must choose an initial amount of work L_0. A rational nation would choose the initial amount of work according to the initial level of consumption. The L_0 is the initial condition of the model and it determines the amount of work for every t. We will assume that the marginal disutility of work increases with the amount done, and so, the higher initial consumption level x_0 chosen, the lower the initial level of work L_0 will be rational to choose. The amount of work at every t must also be chosen according to the amount of consumption. The optimum combination will be the highest time-path of consumption which can be permanently sustained together with the associated time-path of work.

We will calculate with the marginal disutility of labour in the form $v = z \cdot L^c$, where c is the parameter of the function, $c > 0$. Then $v' = \frac{dv}{dL} = z \cdot c \cdot L^{c-1}$. The equilibrium condition is that $\frac{1}{v}\frac{dv}{dt} = -r$, so $\frac{v'}{v}\frac{dL}{dt} = -r$ and $\frac{1}{L}\frac{dL}{dt} = -\frac{r}{c}$. It is easy to see that this is satisfied by $L_t = L_0 \cdot e^{-\left(\frac{r}{c}\right)t}$. The present value of the income stream is

$$\int_0^\infty L_0 \cdot w \cdot e^{-\left(\frac{r}{c}\right)t} \cdot e^{-rt} dt = L_0 \cdot w \cdot \frac{c}{r(1+c)}$$

where w is the real wage-rate. The real wage rate links the problem of the amount of the labour with the problem with the amount of consumption because the real wage rate is defined as the marginal utility of work.

4. The main problems of the model

Ramsey´s article had been forgotten for over thirty years. We can see this from Figure 1. There is a number of citations of the Ramsey´s article since 1931 according to the Web of Science. There was almost no citation of the article between 1930 and 1960. The popularity of the article grew rapidly 1970s and again after 2000 as can be found in the figure no. 1 – amount of per year quotation in Web of Science.

4.1. The existence of the bliss point

The problem of the bliss point is both philosophical and mathematical. The bliss point is nirvana or the point of maximal utility. Ramsey defined the bliss point as *"the maximum obtainable rate of enjoyment or utility"* (Ramsey 1928, 545). The bliss point can also be described as a point of maximum happiness. It is obvious that the bliss point is not purely an economic concept. The search for the bliss point has existed in the world literature since the ancient history[12] (Sedláček 2009, 35–37).

The bliss point is also important for the existence of the solution. If there is a bliss point in the utility function then the utility function U tends toward the bliss level as $t \to \infty$. The integral of the difference between the bliss point and U converges (Chakravarty 1962, 178–182). The other possible formulation is $max \int_0^\infty U(c_t) \cdot e^{-\delta t} dt$. This formulation became more popular in the later versions of the model (Britto 1973, 1358). But there is a need for discounting the utility because of the convergence of the

12) Sedláček described and criticized the principles of the modern economy. It was a deficiency of Sedláček´s book that he disregarded the Ramsey´s model in his analysis. DESCUBES, Irena: Recenze publikace Tomáše Sedláčka Ekonomie dobra a zla, http://www.ekonomikaamanagement.cz/cz/clanek-recenze-publikace-tomase-sedlacka-ekonomie-dobra-a-zla.html, 18.11.2012.

integral. Without discounting $e^{-\delta t}$ the integral from 0 to ∞ would not converge. The integral of the undiscounted utility would converge for the finite time period (i.e. for the integral with bounds from 0 to $T, T \in \mathbb{R}$) (Chakravarty 1962, 178–182).

A steady-state can be generally defined as a situation in which the various quantities grow at a constant rate (Barro and Sala-i-Martin 1995, 17). The population of the Ramsey´s model gets to the steady-state after reaching the bliss point. The population can generally get to the steady-state at the finite time or it can converge to the steady-state asymptotically (Ramsey 1928, 545). Generally after reaching the steady-state there should be no change in the subject´s behaviour.

4.2. Sophisticated mathemathics in the model

The first problem of Ramsey´s article is a very sophisticated mathematic theory. Ramsey was not an economist but he was a mathematician and a philosopher. The mainstream economic theory used much less mathematics in 1920s than today. Most economists did not have enough knowledge to understand the model (Duarte 2009, 173). John Maynard Keynes who was widely aware of Ramsey´s work wrote: "*The article is terribly difficult reading for an economist, but it is not difficult to appreciate how scientific and aesthetic qualities are combined in it together*" (Keynes 1933). But we can see that Keynes was aware of the importance of this methodology for the economic theory. He also described the article as "*one of the most remarkable contributions to mathematical economics ever made, both in respect of the intrinsic importance and difficulty of its subject, the power and elegance of the technical methods employed, and the clear purity of illumination with which the writer´s mind is felt by the reader to play about its subject*" (Keynes 1930).

4.3. The economic situation after the publication of the model

Another problem of the model was the economic situation at the beginning of 1930s. The Great Depression, the greatest economic crisis of the 20th century, started in 1929. It was only one year after the publication of Ramsey´s article. The economic theory faced a great problem. Neoclassic economics failed in both explaining the causes of the crises and finding a way to solve them.

The Great Depression was both a political and economical problem. Politicians actually had to start new economic policies without supporting the economic theory. For example The New Deal started in 1933. Keynes published his opus magnum *The General Theory of Employment, Interest and Money* in 1936. This book was not just a reaction to the recent economic development. He built an entirely new economic paradigm which was called Keynesian economics. Keynes saw almost no importance in the long term economic development. He wrote that the government and monetary authorities should focus on the short run policies. Their main goal should be the stabilization of the short run development of the economy. He actually wrote "*The long run is a misleading guide to current affairs. In the long run we are all dead.*"

Keynes actually offered a way to solve the short fluctuations of the economy. He wrote that the government should increase the government spending at the time of depression. The central bank should also boost up the economy by printing money and lowering interest rates (Holman 2005, 370; Sojka 1991).

We can see that there is a huge difference between Keynes´ and Ramsey´s economic theories. Ramsey was concerned with the infinite time horizon. It is actually the largest antagonism to the Keynesian economics we can imagine.

4.4. The problem of measurability of the utility

Another problem of Ramsey´s article was the measurability of the utility. This is maybe one of the most criticized concepts in economics.[13] We should distinguish the measurability of the utility in microeconomics and macroeconomics.

First we will discuss the problem in microeconomics. We can use a simple example of a consumer who consumes only two goods: goods X and goods Y. If the utility of the consumer is measurable then it has to be possible to measure the utility of the consumption of any amount of X and Y. This theory is called cardinal theory of utility. We can call any combination of X and Y a consumption bundle. For example we will assume that the utility function of the consumer is $U = 3 \cdot X + 2 \cdot Y$. The utility of the consumption bundle with$10\ X$ and $5\ Y$ is 40. The utility of consumption $14\ X$ and $8\ Y$is twice as big as the utility of the previous case. The simplest argument against the measurability of the utility is that there is no way how to measure this. The consumer himself is not able to tell if the utility in the second example is twice as big as in the first example or 1.947 times bigger or 2.123 bigger. And even if he could, this utility function can change any time.

Microeconomics solves this using an indifference curve. The indifference curve is a graph showing a set of all consumption bundles that are indifferent to each other. Consumer is also able to decide which of the two bundles is more attractive for him or if he is indifferent to them. Modern microeconomics is built on this concept which is called the ordinal theory of utility (Varian 1995, 34–70).

Now we can see the advance of fragmentation our analysis on two parts. The indifference curves are sufficient for microeconomics because they allow us to compare the utility of one consumer. They do not allow us to compare the utility of two or of two million consumers. But the aggregate utility function assumes this possibility.

13) An example of such a critique can be found at (Rothbard 2005, 6–20).

For example if person A works for 8 hours and consumes 10 items of goods and person B works for 9 hours and consumes 11 items of goods then we can calculate that the utility of this society is 12. And if person B works for only 8 hours and consumes 11 items of goods then the utility of the society grows to 12.5. This is very hard to accept but it is necessary to optimize the society´s utility.

The concept of the measurability of the utility is connected with utilitarianism. Ramsey´s aggregate utility function may remind of John Rawls´ concept (Mankiw 1999, 417–435). But this similarity is quite inaccurate.[14] Ramsey´s utility function is the function of the aggregate utility of consumption and the aggregate disutility of labour. It means that the nation´s utility can be increased only by raising the nation´s consumption or by decreasing the nation´s labour time. This means that there is no way to increase the nation´s utility by redistribution of the national income which is definitely not a utilitarian statement. So Ramsey´s economy contains two production factors: labour and capital goods. We also assume universal capital goods.[15] This allows the nation to choose simply how much of the production will be consumed and how much of the production will be used as the capital goods. The amount of the created capital goods is the investment of the nation. On the other hand, Ramsey assumed that the aggregate utility function of the nation existed, and so some economists were confused and claimed that Ramsey was a utilitarian.

Ramsey´s model was also a deterministic model. There is no random variable, parameter or function in the model. The conditions of the nation´s development do not change forever. It is obvious that the economic situation in 1930s or 1940s was far away from this concept.

5. The economic growth theory before the resurrection of the Ramsey´s model

5.1. The dynamic version of the Keynesian theory – The Harrod´s model

The most important article about the growth theory was *An Essay in Dynamic Theory* by Roy Forbes Harrod published is 1939. This growth model was very different from the Ramsey´s model. It was a dynamic version of the main Keynesian thoughts. Harrod worked with an unstable development of economy. The main reason of this instability is the difference between the expected demand for product eY_t at time t and the real demand for product Y_{t-1} at time $t-1$. The investment I_t is determined by this difference. He also assumed a *fixed ratio* between the amount of the capital and the amount of the product C. The investment at time t is given by

$$I_t = (eY_t - Y_{t-1}) \cdot C.$$

The output at time t depends on the investment at time t and the savings ratio of the economy s and it is given by $Y_t = \frac{1}{s} \cdot I_t$.

Harrod explained the instability of the development of the economy by wrong expectations of the investors. It is easy to see that the ratio of expected demand to actual demand is

$$\frac{Y_t}{eY_t} = \frac{C}{s} \cdot \frac{eY_t - Y_{t-1}}{eY_t}.$$

We can call $\frac{eY_t - Y_{t-1}}{eY_t}$ the expected rate of growth. We can see that if $\frac{eY_t - Y_{t-1}}{eY_t} = \frac{s}{C}$ then the expected rate of growth equals the actual rate of growth.

The main problem is that the investors do not correct their expectations. If $eY_t > \frac{s}{C}$.then the investors think that they underestimated the rate of growth, and so they will raise their expectations. By analogy they will lower their expectations if $eY_t < \frac{s}{C}$. So the growth rate of the economy is very unstable (Čihák and Holub 12–13).

So Harrod distinguished three rates of growth: the actual (ex post rate), the natural and the warranted ones. There is full employment of labour and capital at the natural rate. The warranted rate is the rate in which the intended savings and intended investments are equal (Asimakopulos and Weldon 1965, 54–56).

Ramsey expected a perfectly stable development of the economy. There was no difference between the expected and actual growth of the economy. All subjects were rational and knew the future development of the economy. The importance of the Harrod´s model is felt after the formulation of the rational expectation hypothesis. According to this hypothesis it is impossible for the subjects to repeat their mistakes indefinitely.

14) We will explain this inaccuracy because it was one of the reasons of the early rejections of Ramsey´s article.

15) This assumption is more similar to the economics of that period. It was used even in the neoclassical microeconomics. This assumption does not necessarily mean that there is only universal capital goods in the economy. It means that the firms use the most effective production goods.

5.2. The basic neoclassic growth model

The basics of the neoclassical growth theory are contained in a model published by Robert Merton Sollow and Trevor Swan in 1956.[16] We will see that despite the date of the publication, the Sollow´s model is less advanced than the Ramsey´s model.

The model uses a neoclassical production function. Solow admired Harrod´s work but he saw that in the fixed proportion of the factors of production there is too much rigidity in the long-run development (Solow 1956, 55–56).

The production function allows substitution between the labour and the capital. The Solow´s model allows an exogenous technologic growth. The growth increases the productivity of labour,[17] so we will multiply the amount of labour L by A which represents the technology growth. We will assume constant economies of the scale and diminishing marginal return.

We will explain the Solow´s model using the Cobb-Douglas production function defined as

$$F(K,L) = K^{\alpha} \cdot (AL)^{1-\alpha}.$$

It is easy to see that the Cobb-Douglas production function fits our assumptions. Also $\lim_{K\to\infty} \frac{\partial F}{\partial K} = 0$ and $\lim_{L\to\infty} \frac{\partial F}{\partial L} = 0$. We will see that these conditions make the economy possible to converge (Čihák and Holub 2000, 16–28).

We will also assume that the rate of growth of the population n, the rate of growth of the technology g and the rate of depreciation of the capital δ are constant. We can write this formally as $\dot{L}(t) = n \cdot L(t)$ and $\dot{A}(t) = g \cdot A(t)$. The amount of consumption is given by the Keynesian consumption function. This function has a constant saving rate s and a constant consumption rate defined as $(1 - s)$. The amount of savings is always given by $s \cdot Y_t$ and it is equal to the amount of gross investment. The net investment is an actual change of capital and it equals

$$\frac{\partial K}{\partial t} = \dot{K} = s \cdot Y(t) - \delta \cdot K(t).$$

We will define $k = \frac{K}{AL}$ as capital per unit of effective work. The effective work grows because of both the technology and population growth. Now we must express the change of k in time which is a fundamental formula of the Solow´s model:

$$\begin{aligned}\frac{\partial k}{\partial t} = \dot{k}(t) &= \frac{\partial k(t)}{\partial K(t)} \cdot \dot{K}(t) + \frac{\partial k(t)}{\partial K(t)} \dot{L}(t) + \frac{\partial k(t)}{\partial A(t)} \dot{A}(t) \\ &= \frac{\dot{K}}{A(t) \cdot L(t)} \\ &- \frac{K(t)}{[A(t) \cdot L(t)]^2} \left[A(t) \cdot \dot{L}(t) + \dot{A}(t) \cdot L(t)\right] \\ &= \frac{\dot{K}}{A(t) \cdot L(t)} - \frac{K(t)}{A(t) \cdot L(t)} \cdot \frac{\dot{L}(t)}{L(t)} \\ &- \frac{K(t)}{A(t) \cdot L(t)} \frac{\dot{A}(t)}{A(t)}\end{aligned}$$

This can be written as

$$\dot{k}(t) = s \cdot f\big(k(t)\big) - (n + g + \delta) \cdot k(t)$$

The capital per effective labour is increased by investment and decreased by the growth of population, the technology growth and the depreciation. The definition of the steady-state of the Solow´s model is the state with the constant amount of the capital per effective labour. It can be written as

$$s \cdot f(k^*) - (n + g + \delta) \cdot k^*$$

where we used k^* for the constant amount of capital per effective labour.

This equation has significant implications for the government policy. The government is able to maximize the nation´s consumption at the steady state by changing the rate of savings s. We know that the product equals investment plus consumption for any t including the steady-state. So we can express consumption at steady-state c^* as

$$c^* = f(k^*) - \ s \cdot f(k^*) = f(k^*) - (n + g + \delta) \cdot k^*$$

and we can calculate the effect of the rate of savings on consumption at steady-state

$$\frac{\partial c^*}{\partial s} = [f'(k^*) - (n + g + \delta)] \cdot \frac{\partial k^*}{\partial s}$$

It can be proved that the raising of s leads to the raising of k^*. So the derivation $\frac{\partial k^*}{\partial s}$ is always bigger than 0. So the effect of rising of s depends on the contrast of the marginal product of capital $f'(k^*)$ and sum $n + g + \delta$. We can explain this. The investment needed for the rising of k is $n + g + \delta$ and the increase of the product by the rising of k is $f'(k^*)$. So if there is too much capital per effective labour, it is too expensive to stay at the steady-state and this leads to a low level of consumption. The sum $n + g + \delta$ is constant and $f'(k^*)$ declines while k^* is raising . So for the low value of k^* the nation can increase its consumption at the steady-state by raising s and for the high value of k^* a nation can increase its consumption at the steady state-state by lowering s. So the optimal rate of the net investment equals $n + g$. Because the real interest rate equals the net return from investment then the real interest rate should equal $n + g$ (Čihák and Holub 2000, 16–26).

16) Solow and Swan invented the model independently. The model is commonly named as Solow´s and then as the Solow–Swan´s model because Solow published a group of other articles about this topic.

17) This is called Harrod´s-neutral technologic growth.

We saw that the Solow´s model solves the problem of the optimal nation´s investment as the Ramsey´s model. But the Solow´s model uses the simple Keynesian consumption function instead of the nation´s utility function. Solow also assumed that the size of labour force equals the size of the population. The population in the Ramsey´s model optimizes its amount of ~~the~~ free time as well as the amount of the consumption.

5.3. The future development of the Ramsey´s model

Economics made several modifications of the Ramsey´s model.

Cass and Samuelson made a modification with the finite time horizon. They proved that the optimal path would be near a certain steady-state path for a large fraction of the time (Britto 1973, 1358).

Another extension was an explicit definition of the open economy model. This means that two (or more) economies are connected with international business. Both economies are solving the optimization problem. There are a lot of possible versions of this modification. We can make a model of two equally sized economies or a model of one big economy and one small economy. Then the big economy has much greater influence on the small economy (Barro and Sala-I-Martin 1995, 97–108).

The Ramsey´s model is also important for the supply side economics. The best known proposition of the supply side economics is the Laffer curve. Actually the Laffer curve was not a new idea but it was mentioned by David Hume or Adam Smith (Wanniski 1978, 5–7). But the supply side economics formulated it more exactly and popularized it among economists, politicians or journalists. The Laffer curve implies that for high tax rates the rising of the rate can lower the government´s budget income. There are several reasons for it. People avoid paying taxes; avoiding is more attractive because people save more money than by avoiding at the low rate. People can also work less because the net income of their work gets lower (Holman 2005, 470–474). But Arthur Laffer and other economists used mainly verbal arguments to support it (cf. Wanniski 1978). But the Laffer curve can be derived from the Ramsey´s model. By adding the government and the fiscal policy the Ramsey´s model can show the effects of the taxes and the government spending on the consumer´s behaviour. Because of both consumption and free time in the utility function people can substitute the consumption by free time to gain greater utility. So the Laffer Curve can be derived by the methodology of economics (Čihák and Holub 2000, 64–74).

The Ramsey´s model used one item of consumption goods. There are new versions of the model with more consumption goods which can be substituted. Economists also use many ways of technological changes of a model. They make models based on technology growth given by the government investment to the research or given by the amount of the capital in the economy. The technological changes are modpossible modification of his model. He supposed that everyone would discount future utility. Then in a state of equilibrium there will be no savings so $\frac{dx}{dt} = \frac{dc}{dt} = 0$ and $\frac{\partial f}{\partial c} = \rho$ (Ramsey 1928, 555–559).

The basic overlapping generation model was published by Peter Diamond in 1965. This model was an extension of the Ramsey´s model. Diamond added a two stage man´s life: a productive and a post-productive age. The models based on this idea are often used to pension systems modelling and forecasting.

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Fürst Ernst Rüdiger Starhemberg als Antidemokrat und Kämpfer für die österreichische Unabhängigkeit gegen den Nationalsozialismus

Abstract
The article addresses two basic interpretations of Ernst Rüdiger Starhemberg, who was the leader of the Austrian Heimwehr movement and member of the cabinets of Engelbert Dollfuss and Kurt von Schuschnigg. There is no controversy concerning Starhembergs primary political stance as antidemocratic. His Heimwehr with a radical anti-Marxist position and bearing all the signs of a fascist movement had been steadily pushing for a change of the political system. And the authoritarian course of the Dollfuss' government that Heimwehr helped establish culminated in a civil war. The second thesis, Starhemberg as a fighter against the Nazis, is more problematic. In spite of a partly common ideological means, mainly the anti-Marxism and anti-Liberalism, Starhemberg clearly advocated the idea of an independent "Austrianism" as a "better" and untarnished Germanic identity. Starhembergs Heimwehr decisively helped the Austrian government to successfully withstand the onslaught from Austrian Nazis and the pressure coming from the Third Reich in the critical time between spring 1933 and summer 1934.

Key words: Austria, Politics, Ideology, Dollfuss, Nationalism

1. Einleitung

Fürst Ernst Rüdiger Starhemberg (geb. 1899–gest. 1956) war österreichischer Soldat, Politiker und Heimwehrführer. Er nahm am ersten Weltkrieg in der k. und k. Armee teil.