Improved Particle-based Ice Melting Simulation with SPH Air Model

Jakub Domaradzki
Institute of Computer Science
Warsaw University of Technology
ul. Nowowiejska 15/19
00-661 Warsaw, Poland
J.Domaradzki@stud.elka.pw.edu.pl

Tomasz Martyn
Institute of Computer Science
Warsaw University of Technology
ul. Nowowiejska 15/19
00-661 Warsaw, Poland
T. Martyn@ii.pw.edu.pl

ABSTRACT
This paper presents an improved method for simulating melting of ice. The melting process is implemented as a result of the heat transfer between ice objects and fluids (water and air). Both the solids and the fluids, including air, are modeled as a set of particles with specified temperatures, which can vary locally during simulation. The proposed new particle-based air model allows one to consider in simulation the influence of the natural air convection on the ice melting process. Moreover, the model makes it possible to melt the ice object in a controllable way by means of external heat sources. The motion of air and water, originally described by the Navier-Stokes equations for incompressible fluids, is computed using the Smoothed Particle Hydrodynamics (SPH) algorithm, which we modify to properly handle our particle-based air and its interactions with ice and water. Thanks to a GPU-based implementation, the proposed method allows us to run the simulation of ice melting at interactive speed on an average PC.

Keywords
SPH, ice melting, natural air convection, interactive techniques

1. INTRODUCTION
Simulations of natural phenomena are widely used both in science (physical simulation) and entertainment industry (special effects in movies and computer games). Therefore there is a need for simulation techniques resulting in physically correct and, at the same time, visually attractive outcomes. How to combine the two aspects within efficient and robust simulation is still an active research area in computer graphics. It seems that over the last few years, ice melting is one of the natural phenomena that caught the special attention of computer graphics community.

In this paper we focus on simulation of ice melting regarded as a result of the interactions between ice, melted water, and air. Although a number of efficient methods for simulation of melting ice have been proposed, the majority of them neglect the influence of the natural air convection [PPLT06] or significantly simplify it using heuristic functions [IUDN10]. On the other hand, the more accurate approaches that consider ice-air interactions in the melting process use computationally expensive techniques [FM07] and hence they are far from "interactive-time" methods.

In general, the widespread methods of ice melting simulation can be divided into two categories: the grid-based approaches and the particle-based approaches. The methods in the first category represent a modeled physical system as a 3D uniform grid of voxel cells. The cells remain static during simulation and store local physical quantities of the system, and calculations are done between neighboring cells [FM07]. The main issue with this approach is that it is difficult to handle details that are essential not only for the final visual appearance but also play important role in the melting process itself (e.g. droplets of melting water on the ice surface).

The methods from the second category rely on discretization of objects into particles. From computational point of view, particles are utilized in a similar way as cells. However, in opposite to cells, particles can move freely. As a consequence, usually with lesser storage and computational requirements, tiny aspects of phenomenon, such as the mentioned water droplets, can be involved in simulation [IUDN10].

2. OUR CONTRIBUTION

Our main goal was to develop a method for ice melting simulation which would consider the effects of the natural air convection in ice-air interactions and, at the same time, could be run at interactive speed. As a result of our work we propose in this paper a new method for ice melting simulation, in which ice (and possibly other solids, e.g. glass) as well as fluids (air and water) involved in the heat transfer, are represented by sets of particles. Such a representation seems to be a natural extension of the previous particle-based ice melting approaches and allows one to consider the fine-grained effects of the natural air convection in simulation. The movement of air particles caused by local changes in temperature and their interactions with particles representing other objects are computed with the aid of a variant of the Smoothed Particle Hydrodynamics (SPH) algorithm. As a consequence, our method combines advantages of the voxel-based and particle-based approaches in that it accurately computes air-ice interactions and, at the same time, considers fine-grained details such as droplets of melting water. Moreover, the proposed approach allows us to augment the ice melting simulation by the possibility of influencing the temperature and, thus, movement of air particles with the aid of external heat sources. We propose a new external heat source model, which acts similarly to hair dryer and allows one to melt ice object in a controllable way.

3. RELATED WORK

One of the early attempts to simulate ice melting was a method by Fujishiro and Aoki [FA01] in which morphology operations and form factors were utilized. The computation related to heat transfer and melting was based on voxels and didn't take into account generation of water due to the phase transition. To deal with this challenge, Carlson et al. [CMIT02] treated solids as high viscosity fluids and based their simulation on solving the Navier-Stokes equations. The phase transition was realized by influencing on the fluids viscosity with respect to temperature changes. Nevertheless, due to the low viscosity of water, the method cannot be used to simulate the flows of melted water. In turn, Matsumura et al. [MT05] used one of the grid-based techniques (MAC method) and simulated the melting of ice including the natural convection of surrounding air. However, due to the rough and static representation of objects with the grid of voxels, the simulation did not handle the fine-grained effects of the ice melting process (such as droplets of melting water). A different model of air founded on a voxel grid and utilizing a technique analogous to photon mapping for calculating thermal radiation was presented by Fujisawa et al. [FM07]. While the results of the method are quite satisfying, its computational cost is high and interactive simulations are impossible. In the context of the method presented in this paper, the most relevant method was presented by Iwasaki et al. [IUDN10]. The solution is based on particles and takes into account many aspects of the phenomenon, in particular visualization of tiny elements, such as water droplets. However, the influence of the air surrounding the ice object was simplified by applying a constant ambient temperature. The heat energy transferred to the ice surface was brought to a heuristic function which depends on an area exposed to air influence.

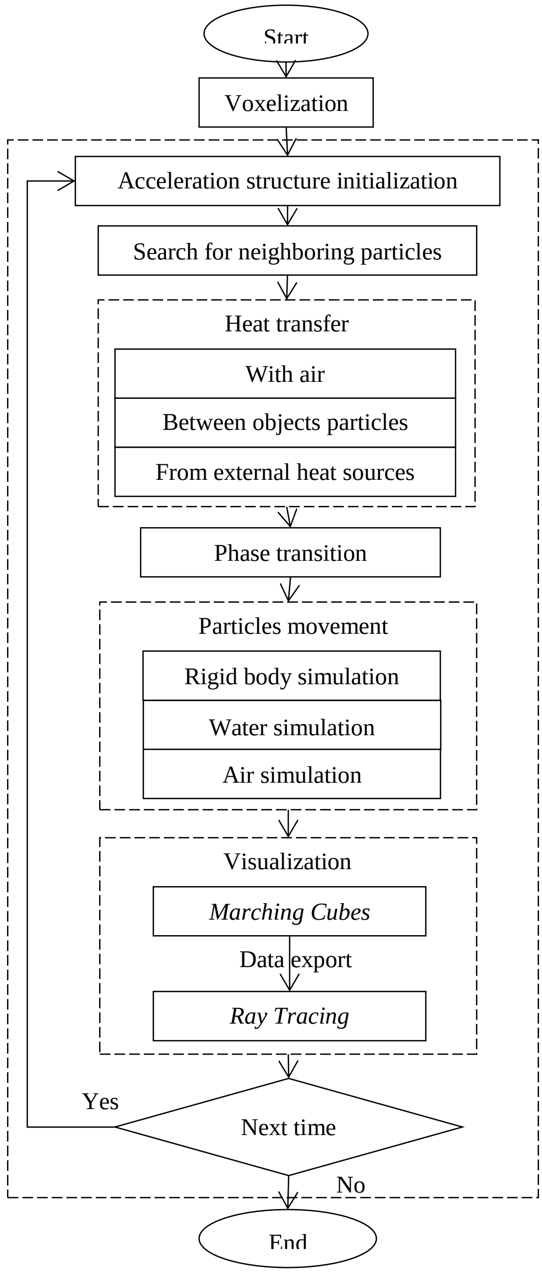

Figure 1. Flowchart of ice melting simulator

To our knowledge, there is no research on representing air with particles for the purpose of the ice-melting simulation. On the other hand, Müller et al.

[MSKG05] showed how to handle multiple fluids with different rest densities. One of the exemplary results of their method were air bubbles rising in the water. However, as shown in [SP08], miscible fluids with a density ratio larger than 10 cannot be realistically simulated with the standard SPH algorithm. Therefore they proposed a different density model, in which all neighbor particles are treated as if they belong to the same fluid. Although densities in the interface are computed correctly, but the structure created by particles cannot be broken to small volumes to simulate, for instance, water droplets. Recently some research on multifluid dynamics systems with an interesting method of density calculation has been presented in [OCD13].

4. OVERVIEW OF THE PROPOSED METHOD

An overview of our simulation method is presented in Fig. 1. There are distinguished operations that are repeated for every time step. From the standpoint of the simulation execution time, the most important part is the initialization of an acceleration structure. A commonly used uniform grid [G10] was chosen, due to preferable local region of interest. Subsequently, all computations are performed on particles on the basis of the data "carried" by neighboring particles of a given particle. Then, the heat transfer between particles is calculated. Depending on their temperatures, this may lead to the phase transition of some ice particles into water particles. Next, the movement of particles is computed, considering the motion of melted water and surrounding air, as well as the ice object collisions and its stability under gravitation. The final stage is visualization. The particles are treated as metaballs and surfaces of visible objects are reconstructed with the use of the marching cubes algorithm [LC87], and a realistic image of the current scene are obtained with ray tracing.

Attribute	Description
M	mass
r	position
u	velocity
ρ_0	rest density
ρ	current density
μ	viscosity coefficient
T	temperature
K	gas constant (stiffness)

Table 1. Particle attributes

5. PARTICLE-BASED ICE MELTING SIMULATION

In our method of ice melting simulation each physical object is modeled using a set of moveable particles. Each particle stores a collection of attributes (Table 1). Their values specify physical quantities of the volume represented by the particle and are updated at every simulation step.

5.1. Heat transfer

The heat is transferred between particles of all media: ice and its surroundings. The increase in temperature can be calculated using the equation [MSKG05]:

$$\frac{\partial T_i}{\partial t} = \alpha \sum_{j \in N_i} m_j \frac{(T_j - T_i)}{\rho_j} \nabla^2 W(r_{ij}, h_H), \quad (1)$$

where α is the thermal diffusion constant, t is the time, N_i is the set of particles whose distances are smaller than h_H from particle i, W is a smoothing kernel, and r_{ij} is the distance vector $r_i - r_j$.

Other forms of heat transfer are described in Sec. 6 and 7.

5.2. Water simulation

In order to simulate water, we used technique called Smoothed Particle Hydrodynamics (SPH) [MCG03], which solves the Navier-Stokes equations for incompressible fluids. The SPH is based on the assumption that we can distinct certain forces acting on water particles, namely the pressure force f_{press}, the viscosity force f_{vis}, and external forces.

The pressure force is trying to keep the fluid in the incompressible state and is described by the equation:

$$f_{press} = -\sum_{j \in N_i} \frac{p_i + p_j}{2} \frac{m_j}{\rho_j} \nabla W(r_{ij}, h_F), \quad (2)$$

where p_i is the current pressure for particle i, and is calculated as:

$$p = k(\rho - \rho_0). \quad (3)$$

The viscosity force is used as an internal friction between particles. It is computed using the equation:

$$f_{vis} = \frac{\mu}{\rho_j} \sum_{j \in N_i} u_{ji} m_j \nabla^2 W(r_{ij}, h_F), \quad (4)$$

where u_{ji} is the velocity difference.

The external forces include all other forces acting on fluid, such as gravity, buoyancy, and the interfacial tension which was used in [IUDN10] to create water droplets.

All the mentioned forces result in particle acceleration that is integrated using the Leap-Frog scheme.

The SPH algorithm is also used to simulate the motion of the air particles, but in a slightly different manner (Sec. 6).

5.3. Ice simulation as rigid body

An ice object is represented by a set of particles, and its movement depends on forces acting on those particles. These include the forces generated by water particles and the forces resulting from objects collisions (the repulsive, the frictional, and the damping force). To compute the force interaction between ice and water (e.g., to simulate ice floating on water) we treat an ice particle as if it were a water particle. The total force acting on the ice object is the sum of forces coming from all its particles [TSK07].

6. SPH AIR MODEL

In this section we present a new air model based on the SPH technique. Our model takes advantage of the results from [MSKG05] on simulating air bubbles and handling multiphase environment in SPH.

6.1. Air simulation

The air surrounding an ice object is built from particles and like in the case of water, the SPH algorithm is used to simulate their motion. One should note, however, that we cannot use the basic form of SPH for this purpose. The main problem is that SPH has to handle particles representing physical objects with very different densities and viscosities[1]. In order to keep the simulation stable and produce realistic results, some changes must be made in the standard version of the algorithm.

First of all, in the spirit of [MSKG05], we need to average viscosity coefficients in the viscosity forces. This results in the new formulation of Eq. 4:

$$f_{vis} = \sum_{j \in N_i} \frac{\mu_i + \mu_j}{2} \frac{u_{ji}}{\rho_j} m_j \nabla^2 W(r_{ij}, h_F). \qquad (5)$$

This way the viscosity force will act differently in the interface area and, at the same time, remain unchanged for particles of the same sort.

Furthermore, as it was proved in [SP08], SPH technique does not operate well in the case when the density ratio of different fluids is larger than 10. Unfortunately, we cannot take advantage of the solution proposed in that paper, as we would like to take into account water droplets phenomenon. As a result, we just set the density of the air particles to 10 times smaller than the water density.

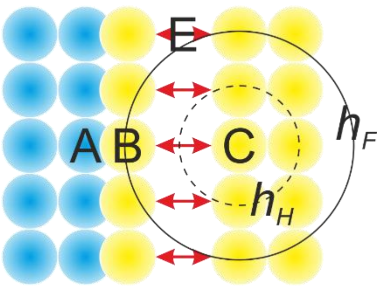

Figure 2. Thin layer of air particles is attracted to ice surface

[1] One should note that ice as a solid has no viscosity in a physical sense. Nevertheless, for the purpose of the SPH simulation of ice-water interactions, the ice particles are usually assigned the viscosity coefficient of water (see e.g. [TSK07]).

Additionally, we have to resolve the problem of spurious tension. As shown in Fig. 2, it is present in the interface between the fluids and its variation between ice and air, where a thin layer B of air particles is attracted to the ice surface A. The reason for this is the pressure force in the SPH algorithm. The air particles move from high to low dense areas on the basis of their current density value. When an air particle with a low rest density interacts with an ice particle whose mass is relatively very high, the air particle density exceeds its rest density very quickly. Nevertheless, in such a situation the ice particle and its relation between the current density and the rest density are involved in computation of the pressure force (Eq. 2). The ice particles are in a constant relation to the other particles that belong to the same ice object, and the particles on the boundary do not have enough amount of ice particle neighbors for their densities to reach the rest density of ice. As a result, some air particles are drawn into the ice surface to increase the density of ice particles. On the other hand, the remaining air particles cannot approach to the air particles on the ice surface due to their high calculated densities caused by the ice particles located nearby.

The mentioned situation is especially important, because such a free space E prevents the air particles B and C from exchanging their temperatures (Fig. 2), due to the shorter smoothing radius h_H of the kernel in Eq. 1 than the smoothing radius h_F in Eq. 2.

At first glance, a simple solution to this problem is to shrink the smoothing radius h_F (Fig. 2) of the pressure force equation (Eq. 2). However, according to [K06], such a short radius would not be usable with fluid simulation. Taking everything into consideration, we propose a different solution in which the pressures of particles located on the interface between different mediums are computed as:

$$p = \max(k(\rho - \rho_0), 0), \qquad (6)$$

rather than using the original equation (Eq. 3). Such a modification prevents the air particles from attracting to the ice surface by introducing a slight disturbance on the interface (Fig. 3). As a consequence, a relatively small time step is required to keep the simulation stable.

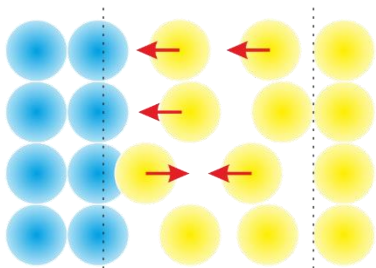

Figure 3. Disturbance on interface between air and other mediums

6.2. Heat transfer

Thanks to the particle structure of our air model, the air surrounding an ice object can be quite naturally incorporated into the heat transfer process described by the equation (1) (see Fig. 4). In addition, the model provides, without any additional cost, local changes in air temperature.

6.3. Natural air convection

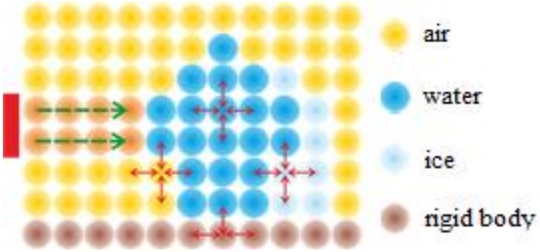

Figure 4. Heat transfer between particles of different mediums

As it was stated at the beginning, our goal is to achieve the simulation of ice as much realistic as it can be. According to laws of physics, volumes of fluid of different temperature move relative to each other due to the differences in their densities – the phenomenon called convection takes place. The proposed particle-based air model is well suited to enrich the simulation of ice melting with the influence of air convection, however, we must slightly alter the SPH algorithm.

First of all, we need to influence on rest densities of air depending on particles temperatures (similarly to [MSKG05]). We want the hot parts of air to transfer to the top areas and cold downwards. To achieve this, we propose the following equation:

$$\rho_0(T,\theta) = \frac{\rho_0}{1.0 + T * \theta} \tag{7}$$

where $\rho_0(T,\theta)$ the current rest density of an individual air particle, ρ_0 is the rest density of air particle in 0 degrees Celsius, θ is a parameter to steer the rate of change in the air rest density (experimentally set to 0.005).

Unfortunately, due to the relatively low gas constant of air together with its increased viscosity we chose in Sec. 6.1, the result of the application of the equation (7) is not as satisfactory as it was expected to be. Therefore to enhance the simulation of the phenomenon in question, we propose an additional, artificial force, which we include to the set of external forces of SPH technique. The force is calculated with the use of the equation:

$$F_b = \frac{\sigma}{T_{max} - T_i} v_{up} \tag{8}$$

where σ is an experimentally defined constant to steer the effect of the force, T_{max} is the maximum positive temperature allowed in simulation, T_i is the current temperature of particle i, and v_{up} is the unit upward vector.

7. EXTERNAL HEAT SOURCES

Yet another benefit of the particle-based model of air is that it allows us quite easily to affect ice melting with external heat sources.

In our simulation, an external heat source is modeled to act like hair dryer. It is built with a directional emitter, which effects on air particles in a specified area, so that both the particles' temperatures and their velocities are increased (Fig. 5). A stream of warm air is created, and a portion of its thermal energy is passed to the ice surface, locally altering its temperature.

We noticed that the shape of the air stream partially depends on the viscosity coefficient of air particles. The more "viscous" air is, the more the stream is concentrated, thereby resulting in lesser energy losses.

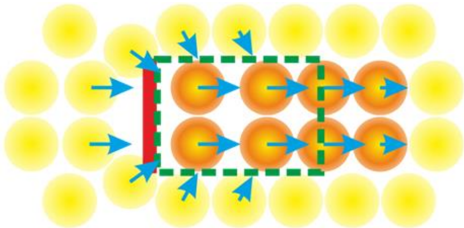

Figure 5. Motion of air particles due to external heat source

During experiments with our external heat sources, we successfully managed to influence on the ice surface in a number of ways: from a precise, shallow melting of a specified symbol, to dividing the ice object into two pieces.

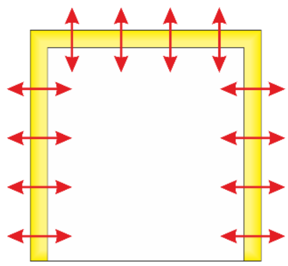

Figure 6. Dissipation of energy on the borders of simulation area

However, due to the finite area of simulation, the continuous application of an external heat source may easily alter the temperature of the entire volume of air. To prevent this we dissipate the excess energy through walls of the "simulation tank": For every particle located near the simulation boarders, we exchange the temperature of the particle with some ambient temperature. This way, we can provide an impression of unlimited area of air (Fig. 6).

8. RESULTS

The proposed method of ice melting simulation requires a significant number of particles for describing all mediums. Hence, the relevant computations could take large amount of time if executed sequentially on CPU. However, the majority of the required operations are performed on each particle using the same set of instructions. Moreover, once input data is delivered to simulator, there is no need to supply any more information. Therefore, in our implementation we can benefit from computational power of modern Graphics Processing Units (GPUs). For this goal, we use the OpenCL framework, which is still relatively new and under constant improvement.

All simulations discussed in this section where performed using a medium class mobile personal computer equipped with Radeon 6770M graphics card. Nevertheless, it allowed us to run the simulation at average speed of 10 FPS with 260k particles (this includes all the types of particles) on each scene.

In order to test the behavior of our air model, the simulations with the Utah teapot and the more complex model of the Stanford Asian Dragon's head were performed. In our opinion the results are more than satisfying. As it can be observed in Fig. 7 and Fig. 8, the ice sculptures are partially melted, mostly in areas with the highest exposure to the surrounding hot air, such as edges and thin elements like teapot ear, dragon horns and teeth.

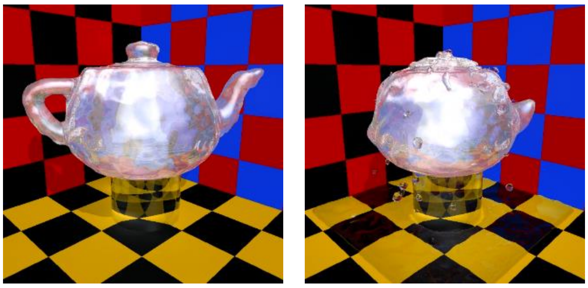

Figure 7. Melting ice teapot due to hot surrounding air

Figure 8. Melting Stanford Asian Dragon's head due to hot surrounding air

Furthermore, it should be noticed, that the amount of energy transferred between air particles themselves as well as air particles and ice particles, firmly depends on distances between particles. This is a result of the presence of the smoothing kernel in the heat transfer equation (1). Thus, by modifying the air viscosity coefficient one can influence the speed of the melting process.

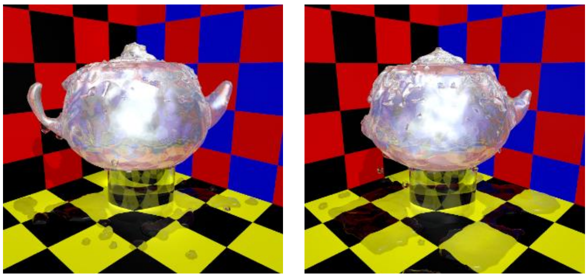

Figure 9. Different viscosity coefficients of air particles: on the left hand side viscosity is five times larger than on the right hand side

One can observe in Fig. 9 that for five times larger viscosity coefficient, the original ice teapot shape is better preserved after the same simulation time. It is a consequence of the lower movement speed of particles due to their higher viscosity, which prevents them from approaching each other and exchanging energy.

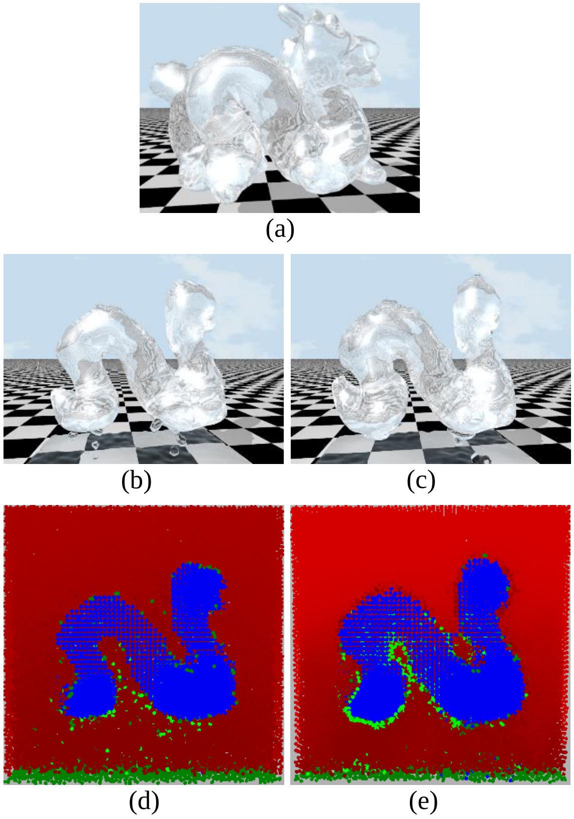

Figure 10. Melting of ice dragon due to hot air without (left column) and with (right column) the natural air convection

With regards to the natural air convection, the result of our efforts are presented in Fig. 10. The image (a) shows the original object before melting, and the left column and the right column show the results of simulation without and, respectively, with considering the natural air convection.

The difference between these two outcomes is subtle. Nevertheless, couple of differences should be pointed out. First, studying the particle views (Fig. 10 d, e), which informs of the particles' temperatures (with blue below 0 and red over 30 degrees), we can observe the behavior of particles with lower temperatures. In the simulation without convection (Fig. 10 d) those particles surround almost the entire ice sculpture. However, when the natural air convection is considered in simulation (Fig. 10 e) the "colder" particles move downwards and locate in the lower parts of the object. Secondly, there are differences in melting between the upper and lower parts of the object. It can be noticed that in the first case (Fig. 10 b) ice melts rather uniformly. Comparing to the second situation (Fig. 10 c), the upper part (where air was hot in both examples) seems to be in the same stadium of melting. However, the lower part with the natural air convection keeps more of its volume intact.

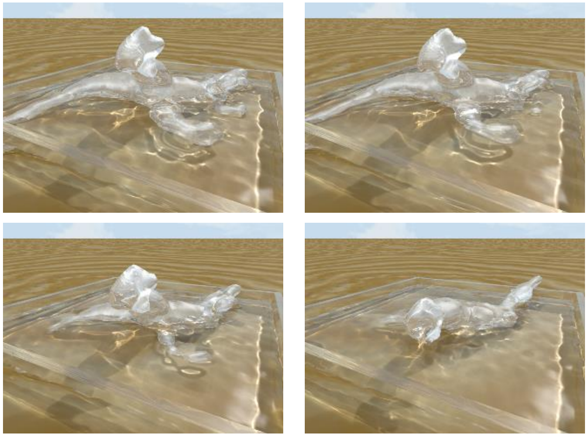

Figure 11. Melting of ice dinosaur partially submerged in water (water and air have the same temperature)

What is more, although the air particles have their density barely 10 times lower than the density of water and there is unnatural movement on the interface (due to the changes we made in the pressure equation), we observed that the heat transfer ratio between those two mediums and ice is very realistic. To confirm that, we conducted an experiment in which an ice dinosaur were partially submerged into a tank filled with water (Fig. 11). The temperature of both air and water was set to the same value. As a result, the submerged part of the object melted significantly faster than the other exposed to surrounding air. Such a phenomenon is consistent with the laws of physics.

As it was stated in Sec. 7, we also successfully managed to precisely alter the ice surface through melting process. The goal of the experiment was melting specified symbol in a block of ice with the use of a number of external heat sources – the results are depicted in Fig. 12.

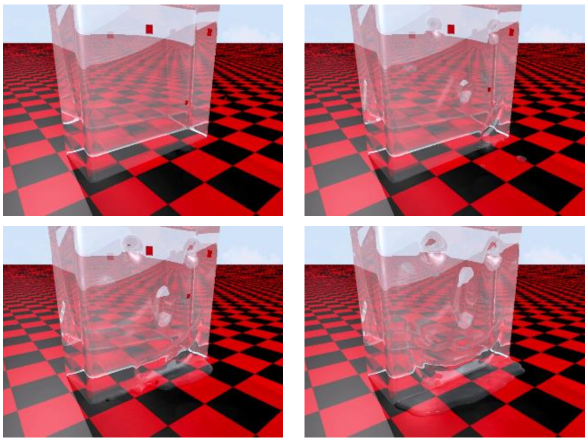

Figure 12. Melting symbol in block of ice with the use of external heat sources

Another test included the external heat source affecting on a number of ice objects. The purpose was to examine how the warm stream of air, created by the heat source, would behave after collision with a streamline object. Hence, we decided to model several different in shape icicles arranged in a line and direct the heat source to one of them.

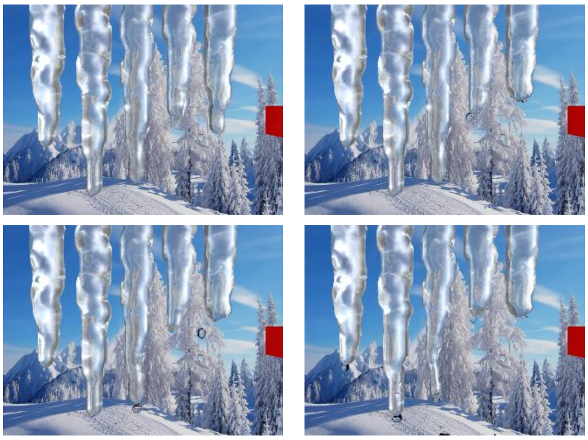

Figure 13. Melting icicles with an external heat source

One can observe in Fig. 13 that the heat source is affecting consecutive icicles respectively, melting one after another. The main volume of the warm stream of air, after collision with an icicle, is being spread sideways. It is also worth noticing that water droplets resulting from the melting process are blown away due to the motion of air.

9. CONCLUSIONS

In this paper we have proposed a simulation of ice melting based on particle representation and performed with the use of the SPH technique. The presented approach combines in a single method the capabilities of the previous approaches. Namely, it allows one to consider in ice melting simulation such phenomena like water, rigid body movement, heat transfer, and creation of water droplets. Moreover, we enhanced the simulation with a new particle-based air model which allows for considering local changes in air temperature, simulating the natural air convection, and controlled ice melting with the use of external heat sources. We have also shown on the example of the proposed air model how to handle multiphase environment. The effectiveness of our approach and the physical correctness of its outcomes was confirmed with a number of test examples. As a consequence, it seems that the presented method can be useful for both animation and physical simulation purposes. Furthermore, thanks to the GPU-based implementation the simulation can be run at interactive speed even on a medium class mobile personal computer.

10. REFERENCES

[CMIT02] Carlson M., Mucha P., III B. V. H., Turk G.: Melting and flowing. In Proc. ACM SIGGRAPH Symposium on Computer Animation (2002), pp. 167-174.

[FA01] Fujishiro I. and Aoki E.: Volume graphics modeling of ice thawing. In Volume Graphics 2001 (2001), Springer-Verlag, pp. 69-80.

[FM07] Fujisawa M. and Miura K. T.: Animation of ice melting phenomenon based on thermodynamics with termal radiation. In Proc. GRAPHITE (2007), pp. 249-256.

[G10] Green S.: Particle Simulation using CUDA [online]. Nvidia Corporation (2010).

[IUDN10] Iwasaki K., Uchida H., Dobashi Y., Nishita T.: Fast Particle-based Visual Simulation of Ice Melting. In Pacific Graphics 2010 (2010).

[K06] Kelager M.: Lagrangian Fluid Dynamics Using Smoothed Particle Hydrodynamics, MS Thesis, Univ. Copenhagen (2006).

[LC87] Lorensen W., Cline H.: Marching Cubes: A high resolution 3D surface construction algorithm. In Proceedings of the 14th annual conference on Computer graphics and interactive techniques (1987), vol. 21, pp. 163-169.

[MCG03] Müller M., Charypar D., Gross M.: Particle-based fluid simulation for interactive applications. In Proc ACM SIGGRAPH/Eurographics Symposium on Computer Animation (2003), pp.154-159.

[MSKG05] Müller M., Solenthaler B., Keiser R., Gross M.: Particle-based fluid-fluid interaction. In SCA '05: Proceedings of the 2005 ACM SIGGRAPH/Eurographics symposium on Computer animation (New York, USA, 2005), ACM, pp. 237-244.

[MT05] Matsumura M., Tsuruno R.: Visual Simulation of melting ice considering the natural convection. In SIGGRAPH 05: ACM SIGGRAPH 2005 Sketches (2005), Article No. 61.

[OCD13] Onderik J., Chládek M. and Ďurikovič R.: Animating multiple interacting miscible and immiscible fluids based on particle simulation, Journal of the Applied Mathematics, Statistics and Informatics, University of Saint Cyril and Metod Press, Trnava, Slovakia, De Gruyter, Landsberg, vol. 9, No.2 (2013), pp. 73-86.

[PPLT06] Paiva A., Petronetto F., Lewiner T. and Tavares G.: Particle-based non-Newtonian fluid animation for melting objects. In Sibgrapi 2006 (2006), pp. 78-85.

[SP08] Solenthaler B., Pajarola R.: Density Contrast SPH Interfaces. In SCA '08: Proceedings of the 2008 ACM SIGGRAPH/Eurographics Symposium on Computer Animation (2008), pp. 211-218.

[TSK07] Tanaka M., Sakaki M., Koshizuka S.: Particle-based rigid body simulation and coupling with fluid simulation. Transactions of Japan Society for Computational Engineering and Science, Paper No. 2007007 (2007).