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Modal parameters of a rotating multiple-disk-shaft system from simulated frequency response data

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Abstract

Modal parameters of a rotating multiple disk-shaft system are estimated in Multiple Input/Multiple Output (MIMO) scheme. The response at multiple output degrees of freedom (dofs) and excitations at multiple input (reference) dofs are related through the Frequency Response Function (FRF) matrix. The corresponding Impulse Response Function (IRF) matrix is obtained by Inverse Fast Fourier Transform (IFFT) of the FRF matrix. The resulting FRF matrix is not symmetric due to the gyroscopic effects introduced by rotation. The Eigensystem Realization Algorithm (ERA) and its equivalent low order time domain algorithm, based on the Unified Matrix Polynomial Approach (UMPA) are employed to estimate the desired modal parameters, i.e., system eigenvalues and the associated right hand and left hand eigenvectors. The right hand vectors are estimated from multiple columns of the FRF matrix with the structure rotating in one direction, and the left hand vectors are estimated from the multiple rows of the FRF matrix, which are calculated as the transpose of the same multiple columns of the FRF matrix, estimated with rotation in the opposite direction. The obtained results are found to be in excellent agreement with results obtained from Theoretical Modal Analysis (TMA).

Keywords: modal parameters, mechanical vibration, structural dynamics, rotor dynamics

1. Introduction

The multiple disk-shaft system is found in numerous mechanical and aerospace applications, such as compressors, turbines, and hard disk drives. Stringent requirements on such systems which operate at high rotational speeds, resulted in strong coupling between modes of constituent components, i.e., between modes of the shaft and individual disks. It is therefore of great importance to accurately predict their modal parameters to come up with a reliable design, free from resonance vibration during operation. This subject was examined by several researchers, who employed different theoretical, numerical and experimental approaches to address the problem [6–8, 14, 15, 20, 21, 23].

Modal parameters of multiple disk shaft system were theoretically and experimentally estimated [11], where peaks in the FRF were used to identify the desired damped natural frequencies. However, it is known that the considered system has closely coupled or repeated frequencies, and it is essential to employ a MIMO estimation scheme to predict the modal parameters of such systems. This estimation technique was employed to estimate the modal parameters of the coupled vibration of a stationary flexible disk-flexible shaft system from theoretically generated FRF matrix [12, 13]. To account for rotation, the present work simulates multiple reference testing of a flexible shaft carrying more than one flexible disk and rotating at a constant angular speed. Impulse forces are assumed to excite the system at a number of excitation points N_i, and

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the resulting response at a number of output points N_o is calculated. The assumed excitations and corresponding responses are used to estimate $N_o \times N_i$ FRF matrix, which relates multiple inputs and multiple outputs in the frequency domain.

The considered structure has isotropic rotating components and rotates at constant rotational speed. This results in Linear Time Invariant system, which depends only on the rotational speed [4] through the gyroscopic effects. The corresponding eigenvalue problem is non self-adjoint and the corresponding FRF matrix is non-symmetric. Along with system eigenvalues, both right hand and left hand eigenvectors are required to completely define the modal model of the system. According to Nordmann [16], the right hand and left hand eigenvectors can be estimated from a column and row of the FRF matrix, respectively. Since measuring a row of the FRF matrix is not practical, Gutiérrez and Ewins [5] suggested an alternative way to estimate the left hand eigenvector from a row of the FRF matrix. It was suggested to use a column of the FRF matrix to estimate right hand eigenvectors with rotation in one direction and to use the transpose of the same column of the FRF matrix, but with rotation in the opposite direction to estimate the left hand eigenvectors. This is true because all system matrices of the considered structure are symmetric, except for the gyroscopic matrix, which is skew-symmetric. This means that the FRF matrix of a structure spinning in one direction is the transpose of the FRF matrix, obtained with the structure spinning in the opposite direction.

The considered structure is known to have closely coupled and/or repeated modes due to its isotropy and circular symmetry, it is therefore essential to employ MIMO estimation algorithms, so as not to miss these modes. Therefore, multiple columns of the FRF matrix with rotation in one direction are used to estimate the right hand eigenvectors, and the transpose of the same multiple columns of the FRF matrix with rotation in opposite direction are used to estimate the left hand eigenvectors.

The IRF matrix is obtained by IFFT of an FRF matrix. ERA [9, 10] and its equivalent first order UMPA time domain algorithm [2, 3, 19] are employed to estimate the desired modal parameters of the considered system.

The considered simulation is based on the theoretical model described in [11], where Lagrange's equation was combined with the assumed modes method to derive equations of motion. The obtained results are found to be in excellent agreement with results from TMA.

2. Theoretical Analysis

2.1. FRF Matrix of the multiple disk shaft system

The examined rotor consists of two flexible disks, attached to a fixed-free flexible shaft. The shaft is modelled by a slender beam with circular cross section and uniformly distributed mass and stiffness. The flexible disk is modelled by an annular thin plate, with uniformly distributed mass and bending rigidity, and clamped at its inner radius to the outer radius of the shaft, as shown in Fig. 1.

Vibratory motion of the considered rotating multiple disk-shaft system is discretized by the application of the assumed modes method, where the flexible deformation of the disk or shaft is represented by summation of a number of time-dependent generalized coordinates, multiplied by assumed functions, as given bellow:

$$U_s(z,t) = \sum_m U_m(z) a_m(t), \tag{1}$$

$$V_s(z,t) = \sum_m V_m(z) b_m(t), \tag{2}$$

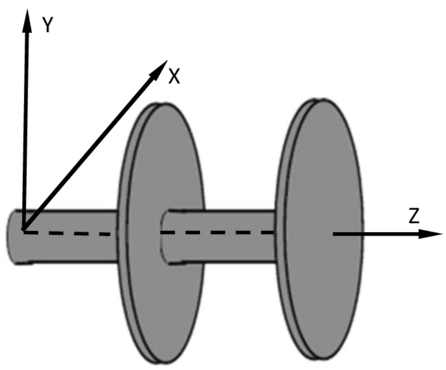

Fig. 1. Two flexible disk flexible shaft system

$$w(r,\theta,t) = \sum_m [w_m(r)\sin\theta q_{ms}(t) + w_m(r)\cos\theta q_{mc}(t)], \tag{3}$$

where U_s and V_s are flexible shaft deformations along X and Y directions in a fixed coordinate system, and w is the out-of-plane disk deformation in an attached to the disk coordinate system. Mode shapes of individual flexible disk or flexible shaft are taken as the assumed functions. In this study, where the coupled disk-shaft modes are examined, only one nodal diametral disk modes are accounted for, because they are the only flexible disk modes that couple with flexible shaft deformations. It is known that transverse flexible disk vibration takes place around the disk according to $\sin(n\theta)$ and $\cos(n\theta)$, where n denotes the number of nodal lines in a given mode. Modes shapes with $n > 1$ are known as reactionless modes because they don't result in net force or moment. This is not the case in modes with $n = 1$, where resulting forces and moments produce coupling between these flexible disk modes and its pitching and translation on the supporting flexible shaft.

The assumed modes method is combined with Lagrange's equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_j}\right) + \left(\frac{\partial D}{\partial \dot{q}_j}\right) - \left(\frac{\partial L}{\partial q_j}\right) = Q_j \tag{4}$$

to derive the governing equations of motion for the complete system, where $L = T - U$, D is the dissipation function, and q_j, Q_j, T and U are the j-th generalized coordinate, the j-th generalized force, total kinetic energy, and total strain energy, respectively. With assumed proportional damping, equations of motion of the considered system are expressed in the following matrix form:

$$[M]\{\ddot{q}\} + ([C] + [G])\{\dot{q}\} + [K]\{q\} = \{Q\}, \tag{5}$$

where, $[M]$, $[C]$, $[G]$, and $[K]$ are the mass, damping, gyroscopic, and stiffness matrices, and $\{q\}$ and $\{Q\}$ are vectors of the generalized coordinates and generalized forces, respectively.

When proportional damping is considered, the damping matrix $[C]$ is assumed to be a linear combination of the mass and stiffness matrices, i.e. $[C] = \alpha[M] + \beta[K]$, where α and β are constants. The FRF matrix, which relates the generalized coordinates to the generalized forces, can be expressed by:

$$[H]_{qq} = \left[[K] + j\omega\left([C] + [G]\right) - \omega^2[M]\right]^{-1}. \tag{6}$$

Knowing that the generalized forces are related to the physical forces by the transformation $\{Q\} = [\Psi_{\text{in}}]\{f\}$, where $[\Psi_{\text{in}}]$ is the transformation matrix constructed from the assumed functions of system components and the position of excitation points (references). Similarly, the displacements at response dofs can be expressed in terms of the generalized coordinates as: $\{x\} = [\Psi_{\text{out}}]\{q\}$, where $[\Psi_{\text{out}}]$ is the transformation matrix constructed from the assumed functions of system components and the position of response points. Details about these matrix transformations can be found in [13].

Using the given above relationships between the generalized coordinates and generalized forces on one side, and the physical displacements and physical excitation forces, on the other, one can write the FRF matrix that relates the response $\{X\}_{N_o\times 1}$ at N_o output dofs with the excitations $\{F\}_{N_i\times 1}$ at N_i input dofs in the form:

$$\{X(\omega)\}_{N_o\times 1} = [H(\omega)]_{N_o\times N_i}\,\{F(\omega)\}_{N_i\times 1}\,. \tag{7}$$

This FRF matrix (or the corresponding IRF matrix) can be subsequently employed in a MIMO estimation algorithm to extract the desired modal parameters of the considered system, as discussed in the following section.

2.2. *Modal parameters of the multiple disk shaft system*

In the formulated FRF matrix, the output dofs consist of shaft points with two orthogonal deformations for each point, and out-of-plane responses points for each disk. Similarly, the input dofs consist of transfer excitations at a number of shaft points and out-of-plane excitations at a number of points for each disk. Both input and output dofs will be discussed in details in the results section.

The Complex Mode Indicator Function (CMIF) of the estimated FRF matrix is examined first, which is a plot of the singular values obtained from Singular Value Decomposition (SVD) of the estimated FRF matrix for each spectral line [1, 22] according to:

$$[H(\omega_k)]_{(N_o\times N_i)} = [U_k]_{(N_o\times N_i)}[\Sigma_k]_{(N_i\times N_i)}[V_k]^H_{(N_i\times N_i)}, \tag{8}$$

where $[U_k]_{(N_o\times N_i)}$ is the matrix of left singular vectors, $[\Sigma_k]_{(N_i\times N_i)}$ is a diagonal matrix of the singular values, and $[V_k]_{(N_o\times N_i)}$ is the matrix of right singular vectors at the k-th spectral line. Peaks in the CMIF curves occur at the damped natural frequencies of the considered structure, and the left and right singular vectors associated with these peaks give approximation to the corresponding mode shapes and modal participation factors, respectively.

ERA and its equivalent low order time-domain UMPA algorithm are used to extract the desired modal parameters from the estimated IRF matrix. The ERA algorithm was originally developed by NASA to construct a state space model from MIMO test data, and UMPA was developed as a general estimation approach, with the different estimation algorithms being special cases of this general approach in both time and frequency domains.

Formulation of the UMPA time domain is based on the following general equation:

$$\sum_{k=0}^{n}[\tilde{\alpha}_k]_{N_o\times N_o}[h_{t+k}]_{N_o\times N_i} = [0] \quad \text{for} \quad t = 0,\ldots,N, \tag{9}$$

where $[\tilde{\alpha}_k]$ is the k-th polynomial coefficient matrix and $[h_{t+k}]$ is the IRF at the k-th time shift. In the empolyed first order UMPA model, only $[\tilde{\alpha}_0]$ and $[\tilde{\alpha}_1]$ are used in the expansion. Additional time shifts are added to this basic form to make sure the size of the coefficient matrix is enough to determine the desired number of modes, which results in following expanded form of the equation:

$$[\alpha_0]\begin{bmatrix} h_0 & h_1 & h_2 & \cdots & \cdots & \cdots & h_N \\ h_1 & h_2 & \iddots & \cdots & \cdots & h_N & h_{N+1} \\ \vdots & \iddots & & & & & \vdots \\ h_{n-1} & & & & & & h_{N+n-1} \end{bmatrix} = -[\alpha_1]\begin{bmatrix} h_1 & h_2 & h_3 & \cdots & \cdots & \cdots & h_{N+1} \\ h_2 & h_3 & \iddots & \cdots & \cdots & h_{N+1} & h_{N+2} \\ \vdots & \iddots & & & & & \vdots \\ h_n & & & & & & h_{N+n} \end{bmatrix} \qquad (10)$$

or in compact form:

$$[\alpha_0][H_0] = -[\alpha_1][H_1], \qquad (11)$$

$[\alpha_0]$ and $[\alpha_1]$ are rectangular matrices of order $n \times N_o$, and $[H_0]$ and $[H_1]$ are the Hankel matrices, constructed from the IRF matrix $[h(t)]_{N_o \times N_i}$. The corresponding matrix coeffiecient characteristic polynomial is given by:

$$[\alpha_1]z^{(1)} + [\alpha_0]z^{(0)} = 0, \qquad (12)$$

where $z = \mathrm{e}^{\lambda \Delta t}$ and Δt is the time between consecutive samples.

For high order normalization, i.e., $[\alpha_1] = [I]$, the companion matrix is simply $[\alpha_0]$ and the resulting eigenvalue problem is:

$$[\alpha_0]\{\phi\} = \lambda_z\{\phi\}, \qquad (13)$$

where $\{\phi\} = \begin{Bmatrix} \lambda_z^{n-1}\psi \\ \lambda_z^{n-2}\psi \\ \vdots \\ \lambda_z\psi \\ \psi \end{Bmatrix}$.

On the other hand, when low order normalization is used, the resulting eigenvalue problem is:

$$[I]\{\phi\} = \lambda_z[\alpha_1]\{\phi\}. \qquad (14)$$

The desired eigenvalues $\lambda_r = \sigma_r + j\omega_r$ of considered sytem are obtained from eigenvalues $(\lambda_z)_r$ of the formulated eigevalue problems above, with $\sigma_r = \mathrm{Re}\left(\frac{\ln(\lambda_z)_r}{\Delta t}\right)$, and $\omega_r = \mathrm{Im}\left(\frac{\ln(\lambda_z)_r}{\Delta t}\right)$.

3. Results and Discussion

The geometric and material properties of the shaft-disk system are:
$E_{\mathrm{disk}} = E_{\mathrm{shaft}} = 200$ GPa, $\nu_{\mathrm{disk}} = \nu_{\mathrm{shaft}} = 0.3$, $\rho_{\mathrm{disk}} = \rho_{\mathrm{shaft}} = 7\,800$ kg/m^3,
$\left(\frac{R_{\mathrm{in}}}{R_{\mathrm{out}}}\right)_{\mathrm{disk}} = 0.2$, $(R_{\mathrm{out}})_{\mathrm{disk}} = 0.25$ m, $h_{d_1} = 0.002$ m, $h_{d_2} = 0.002\,5$ m,
$Z_{d_1} = 0.5$ m, $Z_{d_2} = 0.75$ m, $L_{\mathrm{shaft}} = 0.75$ m, $(R_{\mathrm{out}})_{\mathrm{shaft}} = 0.05$ m, $(R_{\mathrm{in}})_{\mathrm{shaft}} = 0.048$ m,
where h_{d_k} and $Z_{d_k}$are the thickness and spanwise position of the k-th disk. An FRF matrix, which relates 86 response and 7 input dofs is estimated. The response dofs consist of two transverse orthogonal deformations at 7 shaft points, and 36 out-of-plane response dofs for each disk, located at the intersections of 6 radial and 6 circular lines, uniformly distributed over the disk. The excitation is assumed to take place at input (reference) points, which consist of a shaft excitation along the transverse X-direction (dof # 7), a shaft excitation along the transverse

Y-direction (dof # 11), shown in Fig. 2. The first disk is excited by two out of plane excitations at dofs # 17 and # 34, and the second disk is excited by three out-of-plane excitations at dofs # 51, # 68 and # 85, shown in Fig. 3. The driving point FRF, associated with dof # 34, located at the first disk, is presented in Fig. 4 for angular speed $\Omega = 300$ Hz. The cross point FRFs $H(34, 85)$ and $H(85, 34)$, which relate dof # 34 (located on the first disk) and dof # 85 (located on the second disk) are shown in Fig. 5. These FRFs are estimated at angular speed $\Omega = 300$ Hz with the same sense of rotation. The non-symmetry property of the FRF matrix is noticed from the shown phase plot. As discussed earlier, the skew symmetric nature of the gyroscopic matrix results in an FRF matrix for a rotating structure in one direction to be the transpose of the FRF matrix for the same structure when it is rotating in the opposite direction. This is demonstrated in Fig. 6, which shows the cross point FRF $H(34, 85)$ at $\Omega = 300$ Hz in one direction, and the cross point FRF $H(85, 34)$ at $\Omega = 300$ Hz in the opposite direction. The agreement between the presented cross point FRFs demonstrates the fact stated earlier, i. e., the FRF matrix with rotation in one direction is equivalent to the transpose of the FRF matrix with rotation in the opposite direction.

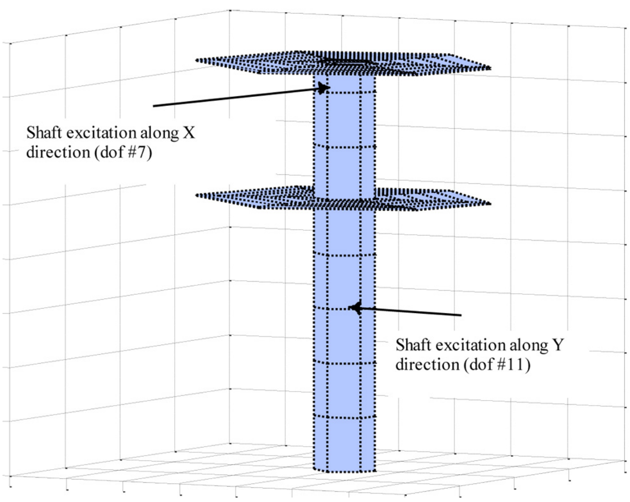

Fig. 2. Transverse shaft excitation dofs

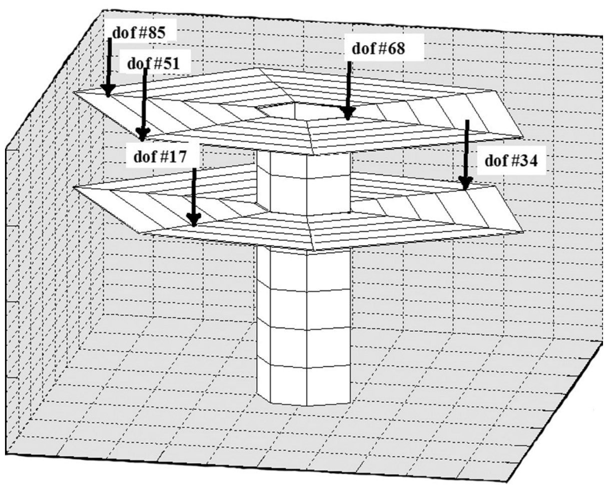

Fig. 3. Out of plane disk excitation dofs

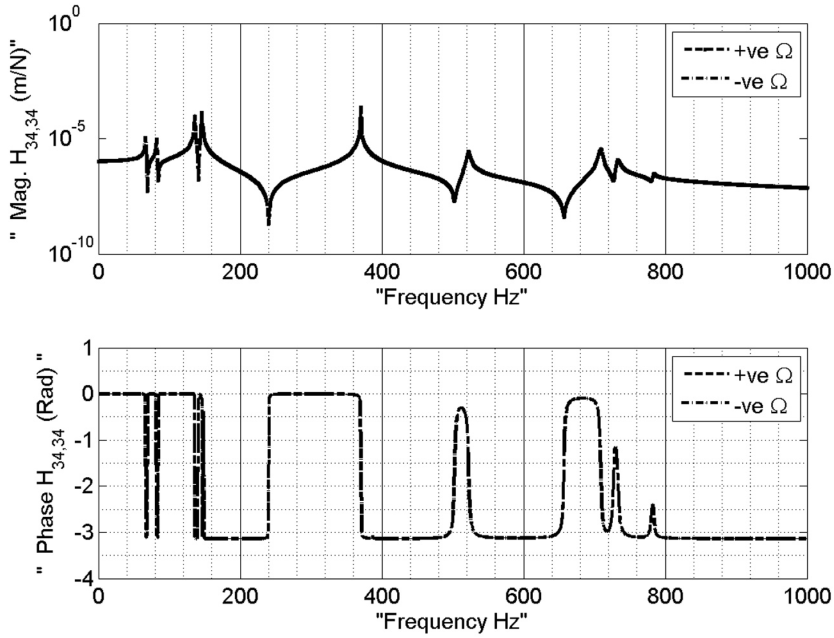

Fig. 4. Driving point FR $H(34, 34)$ with positive and negative Ω

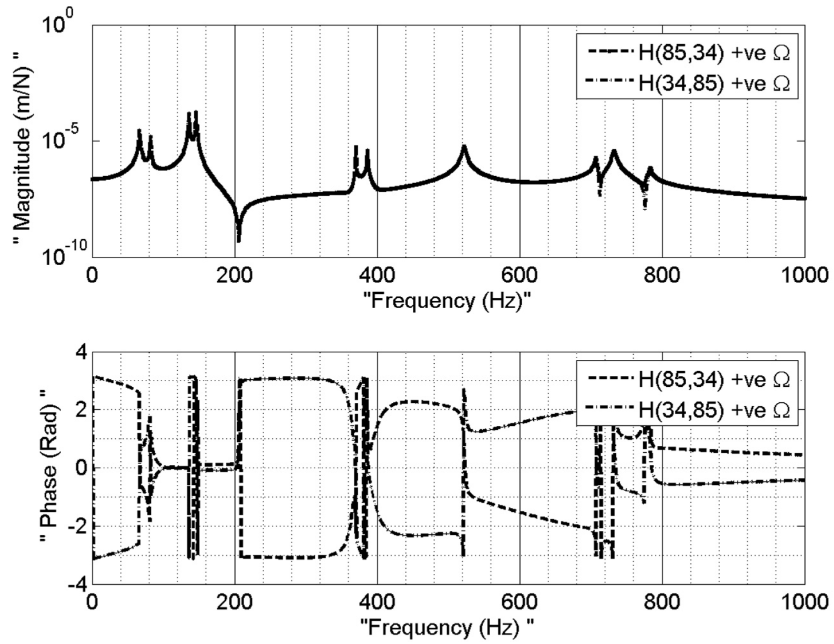

Fig. 5. Cross point FRFs $H(85, 34)$ and $H(34, 85)$ with positive Ω

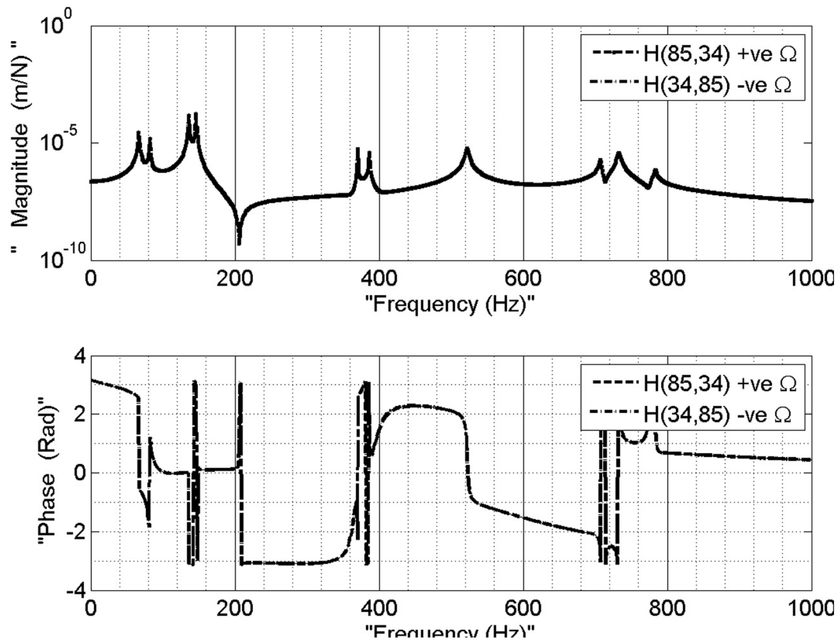

Fig. 6. Cross point FRFs $H(85, 34)$ with positive Ω and $H(34, 85)$ with negative Ω

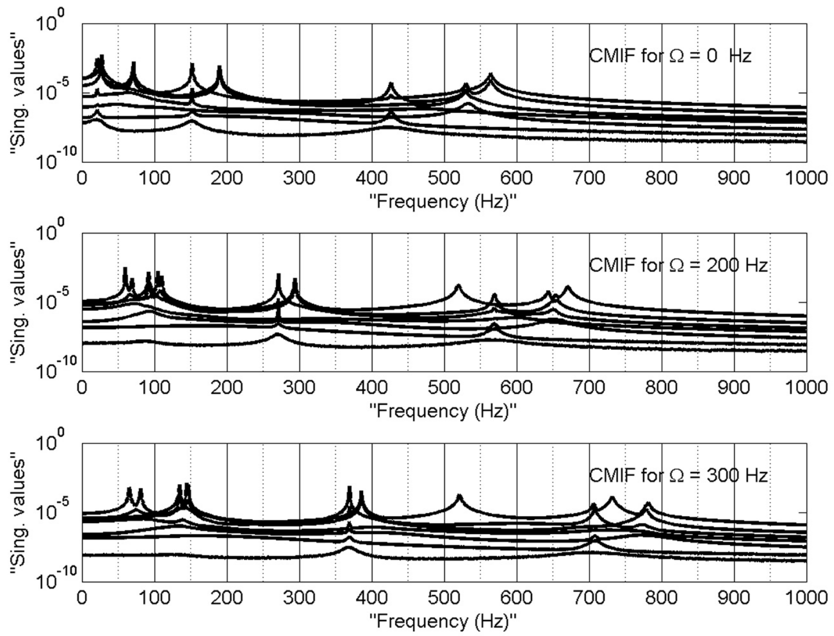

Fig. 7. CMIF of the FRF matrix at $\Omega = 0$, 200 and 300 Hz

Fig. 7 presents the CMIF of the estimated FRF matrix at rotor speeds $\Omega = 0$, 200 and 300 Hz, where split in the resulting natural frequencies due to the gyroscopic effect is demonstrated.

Since the eigenvalue problem associated with rotating structure is non self-adjoint and its FRF matrix is not symmetric, it is necessary to estimate both right hand and left hand eigenvectors, associated with the same set of eigenvalues. Multiple columns of the FRF matrix with rotation in a given direction are used to estimate eigenvalues and the corresponding right hand eigenvectors. Based on the discussion above, the multiple rows, required to estimate left hand eigenvectors, are obtained as the transpose of the same multiple columns of the FRF matrix, estimated with rotation in the opposite direction.

Thus, the same reference and same excitation points are used to obtain the required FRF matrix data, necessary to estimate the system eigenvalues and both right hand and left hand eigenvectors. The natural frequencies and damping ratios are estimated using both ERA and the first order UMPA time domain algorithms, and the results are shown in Table 1 along with the corresponding results from TMA, where an excellent agreement between the results is noticed. The available FRF data along with the estimated eigenvalues are used to estimate residues, which can be used to obtain normalized sets of the right hand and left hand eigenvectors.

Table 1. Estimated natural frequencies (Hz) by different estimation methods

Mode #	ERA	UMPA	TMA
1	65.391	65.391	65.391
2	80.726	80.728	80.726
3	134.755	134.756	134.755
4	134.775	134.775	134.775
5	145.140	145.140	145.140
6	147.113	147.113	147.113
7	369.011	369.010	369.011
8	369.057	369.058	369.058
9	385.694	385.696	385.694
10	386.039	386.039	386.039
11	521.193	521.192	521.193
12	706.340	706.336	706.341
13	708.175	708.171	708.175
14	731.825	731.825	731.825
15	777.624	777.624	777.624
16	781.672	781.670	781.672

In modal analysis, the FRF matrix at a given frequency ω_k is expressed as a superposition of the contribution of individual modes as:

$$[H(\omega_k)] = \sum_{r=1}^{N} \left(\frac{[A_r]}{j\omega_k - \lambda_r} + \frac{[A_r^*]}{j\omega_k - \lambda_r^*} \right), \tag{15}$$

where $[A_r]$ is the residue matrix and λ_r is the r-th eigenvalue associated with the r-th mode, and $(\ )^*$ denotes complex conjugate. The k-th vector of the residue matrix, associated with the r-th eigenvalue, can be expressed in terms of the normalized r-th right hand eigenvector and the k-th element of the normalized r-th left hand eigenvector by:

$$\{A_r\}_k = \{\phi_R\}_r (\phi_L)_{kr}, \tag{16}$$

where $\{\phi_R\}_r$ and $\{\phi_L\}_r$ are r-th right hand and left hand eigenvectors. If a unit value is assigned to the k-th element of the r-th left hand eigenvector, i.e., $(\phi_L)_{kr} = 1$, then the r-th right hand eigenvector $\{\phi_R\}_r$ is determined from the k-th vector of the residue matrix i.e., $\{\phi_R\}_r = \{A_r\}_k$ where the residue matrix $[A_r]$ can be estimated from the available FRF matrix data. Other elements of the left hand eigenvector can be estimated from the corresponding rows of the residue matrix.

The estimated modal parameters are used to synthesize the FRF matrix, which is then compared to the original FRF matrix as a way to check the accuracy of the estimated results. One driving point and two cross point original and synthesized FRFs are shown in Figs. 8–10. The presented data clearly demonstrate the agreement between these FRFs, which confirms the validity of the estimated eigenvalues and corresponding right hand and left hand eigenvectors.

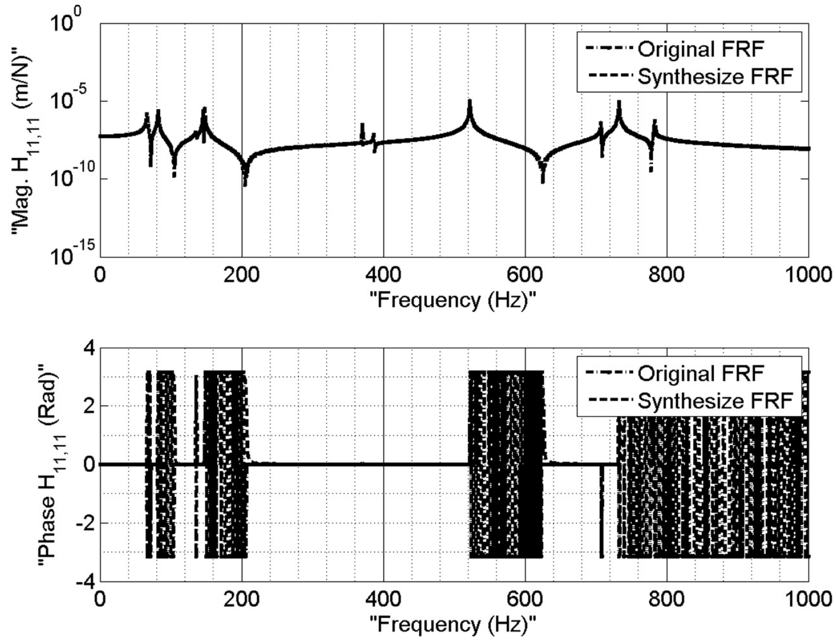

Fig. 8. Original and synthesized driving point FRF $H(11, 11)$

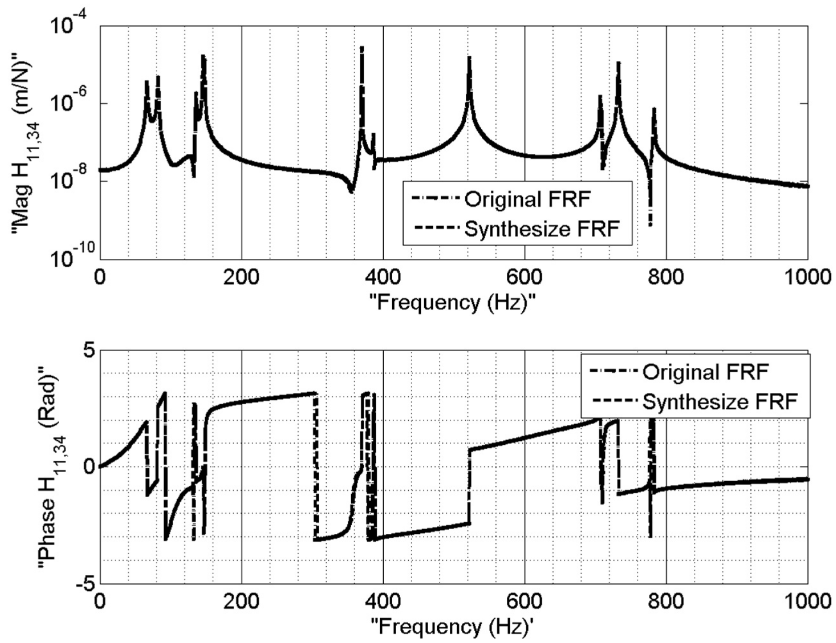

Fig. 9. Original and synthesized cross point FRF $H(11, 34)$

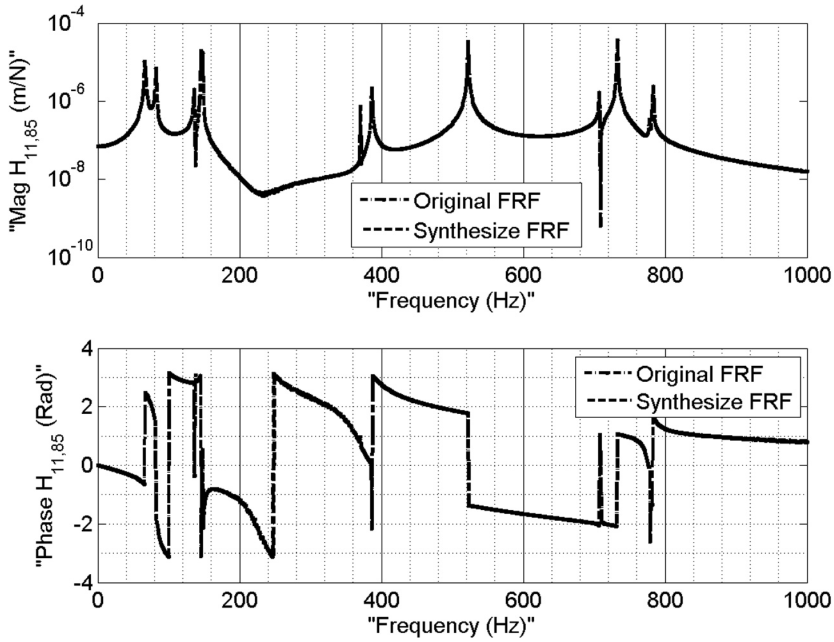

Fig. 10. Original and synthesized cross point FRF $H(11, 85)$

4. Conclusions

MIMO estimation algorithm is employed to determine modal parameters of a rotating multiple-disk-shaft system from theoretically generated FRF and IRF matrices. A first order time domain estimation algorithm is used to estimate modal frequencies. Residue matrix, obtained from an FRF matrix with rotation in one direction is used to estimate right hand eigenvectors, and residue matrix, obtained from an FRF matrix with rotation in the opposite direction is used to estimate left hand eigenvectors. With the adopted approach, the same excitation and response dofs can be used to estimate both right hand and left hand eigenvectors. The obtained eigenvalues and eigenvectors are in excellent agreement with the results from TMA, which confirms the ability of the presented method to handle experimental data.

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