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A computational method for determination of a frequency response characteristic of flexibly supported rigid rotors attenuated by short magnetorheological squeeze film dampers

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Abstract

Lateral vibration of rotors can be significantly reduced by inserting the damping elements between the shaft and the casing. The theoretical analysis, confirmed by computational simulations, shows that to achieve the optimum compromise between attenuation of the oscillation amplitude and magnitude of the forces transmitted through the coupling elements between the rotor and the stationary part, the damping effect must be controllable. For this purpose, the squeeze film dampers lubricated by magnetorheological fluid can be applied. The damping effect is controlled by the change of intensity of the magnetic field in the lubricating film. This article presents a procedure developed for investigation of the steady state response of rigid rotors coupled with the casing by flexible elements and short magnetorheological dampers. Their lateral vibration is governed by nonlinear (due to the damping forces) equations of motion. The steady state solution is obtained by application of a collocation method, which arrives at solving a set of nonlinear algebraic equations. The pressure distribution in the oil film is described by a Reynolds equation modified for the case of short dampers and Bingham fluid. Components of the damping force are calculated by integration of the pressure distribution around the circumference and along the length of the damper. The developed procedure makes possible to determine the steady state response of rotors excited by their unbalance, to determine magnitude of the forces transmitted through the coupling elements in the supports into the stationary part and is intended for proposing the control of the damping effect to achieve optimum performance of the dampers.

Keywords: rotors, magnetorheological dampers, steady state response, collocation method

1. Introduction

The unbalance forces and moments of rotating parts are one of the main sources of lateral vibration of rotors working in industrial devices or means of transport. Their excessive vibration reduces the service life of all components of rotating machines, increases their noise and the forces transmitted through the coupling elements between the rotor and the stationary part. Excessive oscillations produce large deflection of the shaft, which may lead to exceeding the limit state of deformation and to occurrence of impacts between the discs and the rotor casing.

The damping devices inserted between the rotor and the stationary part can considerably reduce the vibration amplitude and magnitude of the transmitted forces. To achieve their efficient work, the damping effect must be controllable to be possible to adapt performance of the dampers to the current operating conditions.

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2. The investigated rotor system

The investigated rotor (Fig. 1) consists of a shaft and of one disc. The rotor is mounted with rolling element bearings whose outer races are coupled with the casing by flexible elements. The system is symmetric relative to the middle plane of the disc. The rotor turns at constant angular speed and is loaded by its weight. In addition, it is excited by the centrifugal force produced by the disc unbalance.

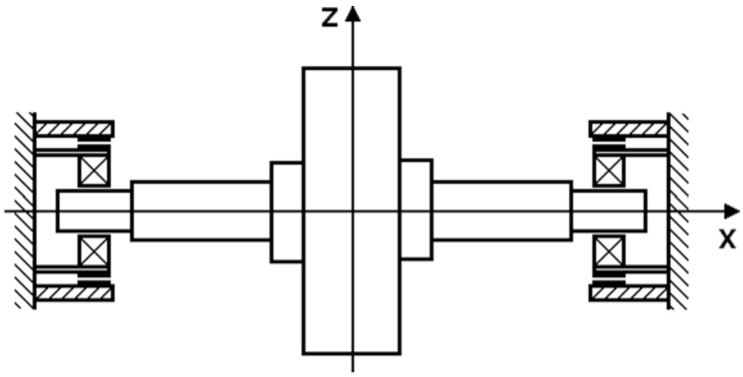

Fig. 1. Investigated rotor

To attenuate the rotor vibration, the damping devices should be inserted between the shaft journals and the casing. The task is to analyze their influence on the rotor steady state response. The attention should be focused on the dependence of amplitude of the vibration and magnitudes of the time varying forces transmitted through the coupling elements into the stationary part on the speed of the rotor rotation.

In the computational model, the rotor and the stationary part are considered as absolutely rigid and the spring elements supporting the rotor and the dampers as linear. Taking into account the system symmetry, lateral vibration of the rotor is described by two equations of motion

$$0.5 m_R \ddot{y} + (b_D + 0.5 b_P)\dot{y} + k_D y = 0.5 m_R e_T \omega^2 \cos(\omega t + \psi_o), \quad (1)$$

$$0.5 m_R \ddot{z} + (b_D + 0.5 b_P)\dot{z} + k_D z = 0.5 m_R e_T \omega^2 \sin(\omega t + \psi_o) - 0.5 m_R g. \quad (2)$$

m_R is the mass of the rotor, b_D is the coefficient of linear damping of the damper, b_P is the damping coefficient of external damping (damping caused by the environment), k_D is stiffness of the supporting spring, e_T is eccentricity of the rotor centre of gravity, ψ_o denotes the phase lag of the unbalance force, y, z are the horizontal and vertical displacements of the rotor centre, ω is the angular speed of the rotor rotation, t is the time, g is the gravity acceleration and $(\dot{\ })$ and $(\ddot{\ })$ denote the first and second derivatives with respect to time.

On these conditions, the steady state trajectory of the rotor centre is a circle whose centre is slightly shifted in the vertical direction. Radius of the orbit and amplitude of the force transmitted via the spring and the damping elements in the rotor support depend on amount of the damping and on angular velocity of the rotor rotation

$$r = e_T \frac{\eta^2}{\sqrt{(1-\eta^2)^2 + 4\xi^2\eta^2}}, \quad (3)$$

$$F_A = 0.5 m_R e_T \omega^2 \sqrt{\frac{1 + 4\xi^2\eta^2}{(1-\eta^2)^2 + 4\xi^2\eta^2}}, \quad (4)$$

where

$$\eta = \frac{\omega}{\Omega}, \qquad \Omega = \sqrt{\frac{2k_D}{m_R}}, \qquad \xi = \frac{b_D + 0.5b_P}{\sqrt{2k_D m_R}}. \tag{5}$$

r is the radius of the rotor centre trajectory, F_A is amplitude of the force transmitted through the coupling elements in each rotor support, Ω is the natural frequency of the rotor system, η is the frequency ratio and ξ is the damping ratio.

Analysis of relations (3) and (4) makes possible to draw several conclusions limiting application of passive and semiactive linear damping devices:

- rising damping always decreases amplitude of the rotor steady state vibration but for high revolutions the amplitude always approaches to eccentricity of the rotor unbalance and cannot be further reduced by the dampers,
- amplitude of the force transmitted via the coupling elements from the rotor into the stationary part with rising damping goes down if the frequency ratio η is lower than $\sqrt{2}$ and increases if η is greater than $\sqrt{2}$, but in this case its value is always less or equal to the centrifugal force caused by the rotor unbalance,
- for higher speeds of the rotor revolutions, amplitude of the rotor vibration is reduced only negligibly (is approximately equal to eccentricity of the rotor unbalance) but the forces transmitted through the coupling elements significantly rise.

It is evident that to achieve a compromise between attenuation of the amplitude of the rotor oscillation and magnitude of the force transmitted through the coupling elements the performance of the damper must be adaptable to the current operating conditions by means of the change of amount of damping in the supports.

3. Controllable magnetorheological squeeze film dampers

The control of the damping effect can be achieved by application of magnetorheological dampers. These damping devices are lubricated by magnetorheological liquids, which consist of the oil and of tiny ferromagnetic particles dispersed in it. If the magnetorheological liquid is not affected by magnetic field, it behaves as normal newtonian one. But if the magnetic field is applied, the flow begins only if the shear stress between two neighbouring layers exceeds the limit value (yield shear stress). In the areas where the limit value is not reached, the magnetorheological material forms a core in which the fluid behaves as a solid body.

In the mathematical models, the magnetorheological fluids are usually represented by Bingham or Bulkley-Herschel materials. Their properties, especially the relation between intensity of the magnetic field and the yielding shear stress, were studied e.g. by Kordonsky [5], Shulman et al. [6] and Si et al. [7].

The magnetorheological dampers consist of two rings, between which there is a thin film of magnetorheological liquid (Fig. 2). The rings are coupled with the casing of the rotating machine, the outer one directly, the inner ring by a squirrel spring. The shaft is supported by a rolling element bearing whose outer race is coupled with the inner ring of the damper. Vibration of the inner ring relative to the outer one squeezes the liquid in the lubricating layer, which produces the damping effect. In the stationary part of the damper, there are the coils, which are the source of magnetic field. Its intensity influences the resistance of the magnetorheological liquid against its flow and therefore the change of magnitude of the applied electric current can be used to control the damping effect.

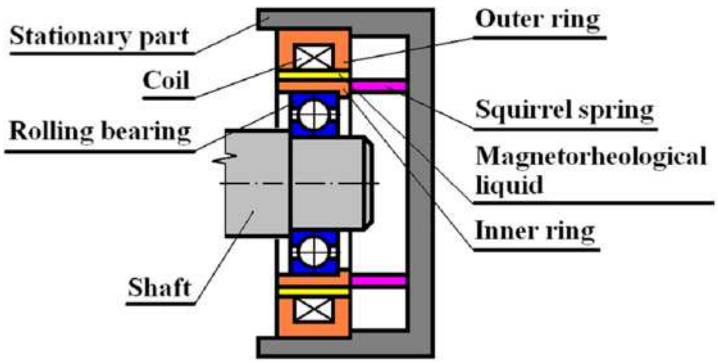

Fig. 2. Scheme of the squeeze film magnetorheological damper

The magnetorheological dampers have been a subject of intensive experimental and theoretical research since about the nineties of the 20th century. In [9], Wang et al. studied by means of experiments the vibration properties and the control method of a flexible rotor supported by a magnetorheological squeeze film damper. In [3, 4], Forte et al. presented results of the theoretical and experimental investigation of a long magnetorheological damper. In [8], Wang et al. developed a mathematical model of a long squeeze film magnetorheological damper based on modification of the Reynolds equation. The results of experiments carried out by Carmignani et al. with a squeeze film magnetorheological damper on a small test rotor rig were reported in [1, 2]. In [10] and [11], Zapomel and Ferfecki introduced the mathematical models of short and long squeeze film magnetorheological dampers. The developed model of a short damper was used for computational simulations of the transient response of a rigid rotor passing the critical speeds [12].

4. Mathematical modelling of a short magnetorheological squeeze film damper

In the developed mathematical model of a magnetorheological damper, it is assumed that (i) the inner and outer rings of the damper are absolutely rigid and smooth, (ii) the width of the damper gap is very small relative to the radii of both rings, (iii) ratio of the length of the damper to the diametre of its rings is small and the faces of the damper are not sealed (assumptions for a short damper), (iv) the lubricant behaves as Bingham liquid, (v) the yield shear stress depends on magnitude of the magnetic induction, (vi) the flow in the oil film (if occurs) is laminar and isothermal, (vii) the pressure of the lubricant in the radial direction is constant, (viii) the lubricant is considered to be massless, and (ix) the influence of the curvature of the oil film is negligible.

The thickness of the lubricating film depends on the position of the inner damper ring relative to the outer one

$$h = c - e_H \cos(\varphi - \gamma), \tag{6}$$

h is the thickness of the oil film, c is the width of the gap between the inner and outer rings of the damper, e_H is the journal eccentricity, φ is the circumferential coordinate, and γ is the position angle of the line of centres (Fig. 3).

The derivation, described in details in [11], arrives at relations for the pressure distribution in the lubricating layer

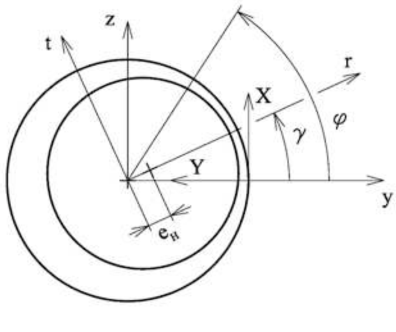

Fig. 3. The magnetorheological damper coordinate system

$$h^3 p'^3 + 3(h^2 \tau_y - 4\eta_B \dot{h} Z) p'^2 - 4\tau_y^3 = 0 \qquad \text{for} \quad p' < 0, \tag{7}$$

$$h^3 p'^3 - 3(h^2 \tau_y + 4\eta_B \dot{h} Z) p'^2 + 4\tau_y^3 = 0 \qquad \text{for} \quad p' > 0 \tag{8}$$

and for Y (radial) coordinate of the core boundary

$$h_1 = \frac{h}{2} + \frac{\tau_y}{p'} \qquad \text{for} \quad p' < 0, \tag{9}$$

$$h_1 = \frac{h}{2} - \frac{\tau_y}{p'} \qquad \text{for} \quad p' > 0. \tag{10}$$

p' denotes the pressure gradient in the axial direction, τ_y, η_B are the yield shear stress and viscosity of the Bingham liquid, h_1 is the radial coordinate of the core boundary on the side of the outer damper ring and Z is the axial coordinate.

The yielding shear stress τ_y depends on material properties and concentration of the ferromagnetic particles dispersed in the magnetorheological fluid, on intensity of the magnetic field in the damper gap and on several further parameters. Usually it is accepted

$$\tau_y = k_B H^{n_B}. \tag{11}$$

H denotes intensity of the magnetic field and k_B and n_B are the liquid material constants.

In the case of the simplest design of the damper, its inner and outer rings can be considered as a core of an electromagnet divided by two gaps and then the relation between the yield shear stress and the applied current in the coil can be expressed

$$\tau_y = k_d \left(\frac{I}{h} \right)^{n_B}, \tag{12}$$

where

$$k_d = k_B \left(\frac{N}{2} \right)^{n_B}. \tag{13}$$

N is the number of the coil turns, I is the current and k_d is a design parameter of the damper.

As evident from (12), the yield shear stress depends on the width of the damper gap and therefore, it changes around the circumference of the damper.

Determination of the pressure gradient for each value of the circumferential and axial coordinates requires solving cubic algebraic equations (7) and (8). Solution of each of them gives three roots. The one that has the physical meaning must satisfy the following three conditions

- it must be real (not complex),
- the conditions of validity of equations (7) and (8) must be satisfied, this means that the real roots obtained from (7) must be negative and the real ones obtained from (8) must be positive,
- $0 < h_1(p') < \frac{h}{2}$.

The pressure profile is calculated by integration of the pressure gradient

$$p = \int p' \, \mathrm{d}Z \tag{14}$$

with the boundary condition expressing that the pressure at the edge of the damper is equal to the atmospheric one

$$p = p_A \quad \text{for} \quad Z = \pm\frac{L}{2}. \tag{15}$$

p_A is the pressure in the surrounding space (atmospheric pressure) and L is the length of the damper.

If pressure at some location in the oil film drops to the critical level, a cavitation takes place. Further it is assumed that the cavitation occurs only in the area where the width of the damper gap increases with time and that pressure of the medium in cavitated areas is equal to the pressure in the ambient space. Then it holds with enough accuracy

$$p_d = p \qquad \text{for} \quad p \geq p_{CAV}, \tag{16}$$

$$p_d = p_{CAV} \qquad \text{for} \quad p < p_{CAV}. \tag{17}$$

p_d is the pressure distribution in the layer of lubricant and p_{CAV} is the pressure in the cavitated area. Differentiation of (6) with respect to time gives the equation for calculation of the circumferential coordinates of the borders of the cavitated area

$$\dot{e}_H \cos(\varphi_{CAV} - \gamma) + e_H \dot{\gamma} \sin(\varphi_{CAV} - \gamma) = 0. \tag{18}$$

Its solution gives two roots that define the angular coordinates $(\phi_{CAV1}, \phi_{CAV2})$ of the beginning and end edges of the cavitated region.

Assuming that the damper is symmetric relative to its middle plane perpendicular to the shaft centre line, components of the damping force are obtained by integration of the pressure distribution around the circumference and along the length of the damper

$$F_{dy} = -2R \int_0^{2\pi} \int_0^{\frac{L}{2}} p_d \cos\varphi \, \mathrm{d}Z \, \mathrm{d}\varphi, \tag{19}$$

$$F_{dz} = -2R \int_0^{2\pi} \int_0^{\frac{L}{2}} p_d \sin\varphi \, \mathrm{d}Z \, \mathrm{d}\varphi. \tag{20}$$

F_{dy}, F_{dz} are the y and z components of the damping force respectively and R denotes the inner ring radius.

5. The equations of motion of the investigated rotor system

To control the damping effect, the magnetorheological dampers are inserted between the spring elements, which are mounted with the outer race of the rolling element bearings, and the casing. The springs are prestressed in the vertical direction to eliminate their deflection caused by the weight of the rotor.

Lateral vibration of the investigated rotor system is then described (taking into account the system symmetry) by two equations of motion

$$0.5m_R\ddot{y} + 0.5b_P\dot{y} + k_D y = F_{dy}(y, z, \dot{y}, \dot{z}) + 0.5m_R e_T \omega^2 \cos(\omega t + \psi_o), \tag{21}$$

$$0.5m_R\ddot{z} + 0.5b_P\dot{z} + k_D z = F_{dz}(y, z, \dot{y}, \dot{z}) + F_{PS} + 0.5m_R e_T \omega^2 \sin(\omega t + \psi_o) - 0.5m_R g \tag{22}$$

that are nonlinear and mutually coupled due to the hydraulic damping forces. F_{PS} denotes the prestress force

$$F_{PS} = 0.5m_R g. \tag{23}$$

Because of the prestress of the spring elements, the stiffness and damping properties in the supports are isotropic and the direction of the damping force in the damper depends only on the direction of excitation caused by the centrifugal force due to the disc unbalance and turns with the same angular speed as the rotor rotates. Therefore, trajectory of the rotor centre has a circular form. Nevertheless, its radius is not proportional to the loading magnitude because of nonlinear character of the damping force.

This enables to assume the steady state solution of the equations of motion (21) and (22) in the form

$$y = r\cos(\omega t + \psi_r), \tag{24}$$

$$z = r\sin(\omega t + \psi_r). \tag{25}$$

r is the radius of the rotor centre trajectory and ψ_r is the phase lag. Introducing the substitutions

$$r_C = r\cos\psi_r, \tag{26}$$

$$r_S = r\sin\psi_r, \tag{27}$$

the relationships (24) and (25) take the form

$$y = r_C\cos\omega t - r_S\sin\omega t, \tag{28}$$

$$z = r_C\sin\omega t + r_S\cos\omega t. \tag{29}$$

The unknown values of coefficients r_C and r_S can be calculated by application of a collocation method. This requires to substitute (28), (29) and their first and second derivatives with respect to time into (21) and (22) and to express the resulting equations at the collocation points of time.

As the number of unknown parameters and the number of equations is two, only one collocation point (collocation point of time) is needed. Carrying out the mentioned manipulations for the collocation time equal to 0 s arrives at a set of two nonlinear algebraic equations whose solution gives the values of the unknown parameters r_C and r_S

$$(k_D - 0.5m_R\omega^2)r_C - 0.5\omega b_P r_S - 0.5m_R e_T\omega^2 - F_{dy}(r_C, r_S) = 0, \tag{30}$$

$$0.5\omega b_P r_C + (k_D - 0.5m_R\omega^2)r_S - F_{dz}(r_C, r_S) = 0. \tag{31}$$

6. Analysis of the investigated rotor with controllable magnetorheological dampers

The first task is to study amplitude of the rotor vibration (radius of the rotor centre trajectory) and of the force transmitted through the dampers and the flexible support elements into the stationary part during the rotor steady state running. The second task is to propose the dependence of magnitude of the electric current supplied into the coils on the speed of the rotor rotation so that the rotor could rotate at constant speed in the whole range of its working revolutions (including the resonances) and the time varying component of the force transmitted into the casing and its vibration amplitude could remain lower than 500 N and 0.2 mm.

Results of the computer simulations are evident from the figures. In Fig. 4 and 5, there are drawn the dependences of amplitude of the rotor vibration and amplitude of the force transmitted into the stationary part on angular velocity of the rotor rotation for four magnitudes of the applied electric current.

The results show that for rising rotor revolutions the increasing current contributes to reduction of the vibration amplitude only negligibly but the magnitude of the force transmitted into the rotor casing increases significantly. It is also evident that the increase of the current has a significant influence on the damping effect. If the damping is too strong, the supports behave as very stiff, the rotor almost does not oscillate and the force transmitted through the coupling

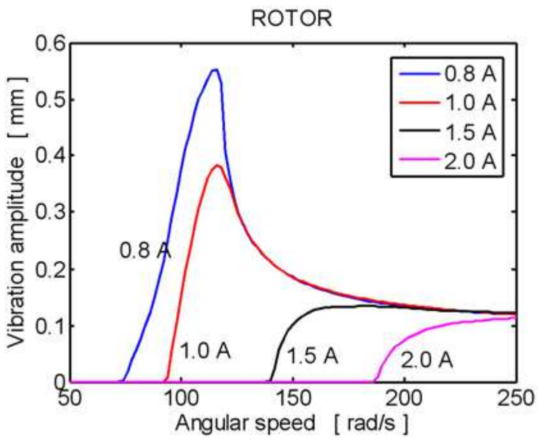

Fig. 4. Vibration amplitude — speed of rotation relation

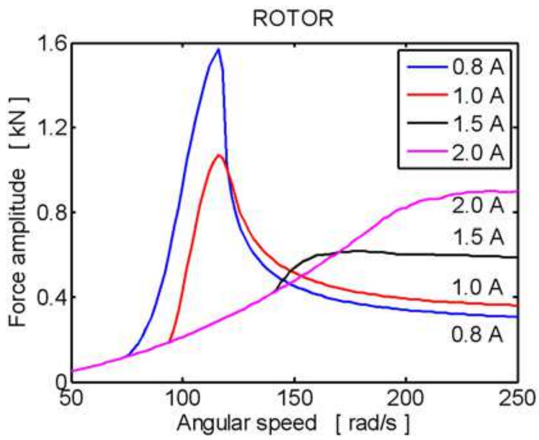

Fig. 5. Force amplitude — speed of rotation relation

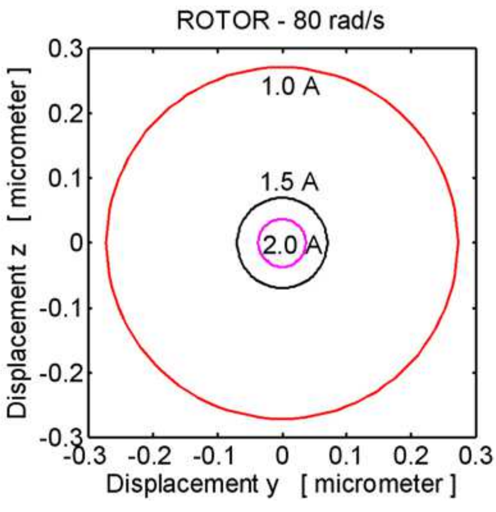

Fig. 6. Orbits of the rotor centre

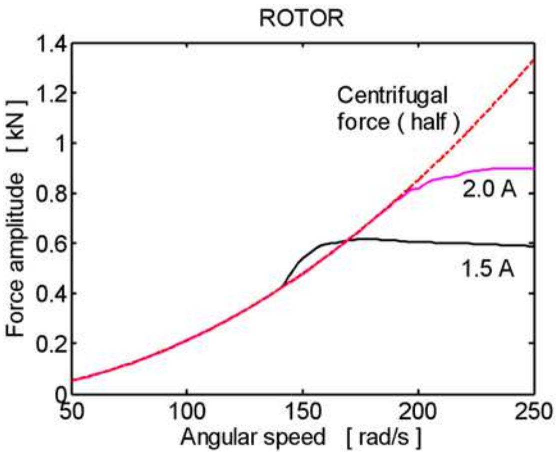

Fig. 7. Amplitude of the transmitted force

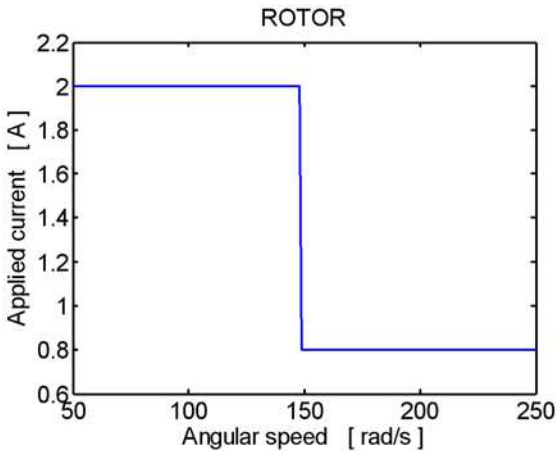

Fig. 8. Proposal of the current control

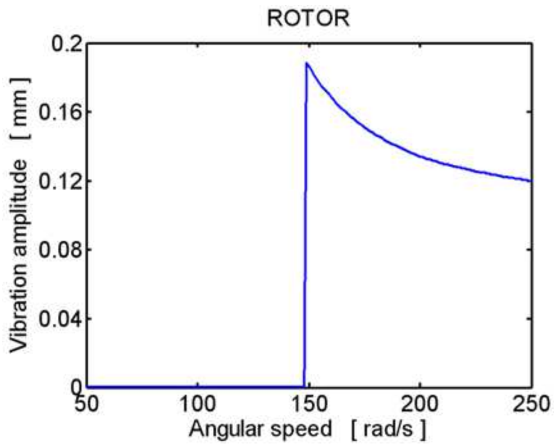

Fig. 9. Controlled vibration amplitude

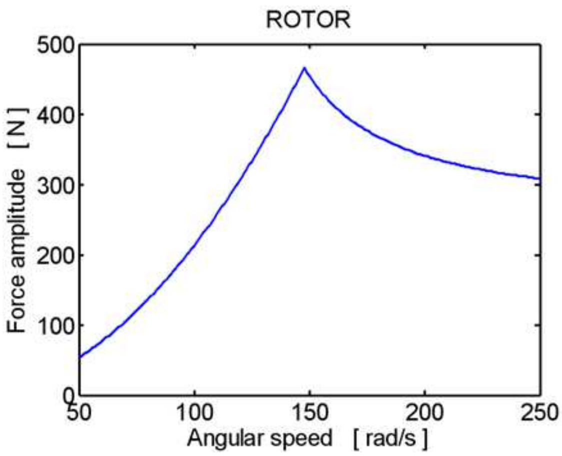

Fig. 10. Controlled force amplitude

elements is almost equal to the half of the whole centrifugal force produced by the rotor unbalance. This can be seen in Fig. 6 and 7. Fig. 8 shows the proposed dependence of the control current on the angular velocity of the rotor rotation to achieve the specified requirements. The corresponding amplitudes of the rotor vibration and of the transmitted forces are evident from Fig. 9 and 10.

7. Conclusions

The carried out analysis shows that to achieve efficient performance of the damping devices placed between the rotor and the stationary part and to reach the compromise between reduction of the rotor lateral vibration and magnitude of the forces transmitted via the rotor coupling elements, the damping effect must be controllable.

The approach described here represents a computational procedure for determination of the steady state response of a rigid symmetric rotor supported by flexible couplings combined with short magnetorheological squeeze film dampers. It is intended for determination of the steady state response of rotors excited by their unbalance and for determination of the magnitude of forces transmitted between the rotors and their casings through the coupling elements. The results can be used for judgement of the rotor limit state of deformation and for preparation of the input data for evaluation of the service life of the rotor components. From the mathematical point of view the presented procedure arrives at solving a set of nonlinear algebraic equations. For this purpose the Newton method was used. The pressure distribution in the lubricating film is described by a Reynolds equation modified for the case of short dampers and Bingham liquid. The damping forces are calculated by integration of the pressure distribution in the lubricating film utilizing the trapezoidal rule.

The carried out simulations show that there are some differences in behaviour of the rotors damped by classical linear and nonlinear magnetorheological squeeze film dampers and that the suitable change of the damping effect makes possible to satisfy the requirements put on maximum amplitude of the rotor vibration and of the forces transmitted to the stationary part.

Acknowledgements

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