EXTENDING THE ZONAL METHOD TO SPECULAR SURFACES

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ABSTRACT

This paper focuses on how to calculate, with radiosity techniques, images associated to different scenes which take into account both an isotropic diffuse participating media and specular surfaces. To do so, we propose a new expression of the form factors defined in the zonal method.
The first part of this article briefly quotes the expression of energetic exchanges by using operators and zonal method equations. The second part deals with the different changes brought to the zonal method in order to express it under the form of operators first and then redefine its form factors. Finally, we will treat of computational considerations and see how different images enable us to show the many possibilities of our algorithm.

Keywords: Radiosity, participating media, specular reflection, zonal method, extended form factors.

1. INTRODUCTION

The rendering algorithms by the so-called radiosity method have quickly changed to generate images more and more realistic. The initial techniques [Goral84] only made it possible exchanges of perfectly diffuse energy between the different surface elements of a scene.
The quality of the images was much improved in ten years time. The techniques usually called hierarchical techniques based on regular or irregular subdivisions of surfaces enable to optimise both the computation time and the quality of shadow limits (cf. [Hanra91] [Campb90] [Langu92]).
The definition of a bidirectional reflection function (BRDF) more and more precise is, at present, one of the main rendering algorithmic challenges (in terms of calculation time but mainly in terms of memory space). The two-pass algorithms [Silli89] or [Walla87] take into account the effects of specular reflections or translucent materials [Rushm90] in the radiosity algorithm without any excessive memory space needs contrary to the methods which sample the emission directions [Immel86].
Taking into account the participating media (air, smoke, fog, …) between surfaces certainly adds quality to generated images. In [Rushm87], H.E. Rushmeier and K.E. Torrance have been the first to suggest the use of the zonal method in the field of image synthesis [Siege92] to calculate isotropic energetic exchanges in participating media.
This paper improves this method to account for specular reflections on surfaces, initially not treated in the zonal method.
Consider Kajiya rendering equation [Kajiy86] expressed in terms of energetic radiance: (1)

$$L(x,\omega) = L_e(x,\omega) + \int_{\omega' \in \mathrm{hem.}} f_r(x,\omega' \to \omega) L(x',\omega') \cos\theta \, d\omega'$$

$L = L(x,\omega)$ is the radiance emitted from the point x of a surface in the direction ω, $L_e = L_e(x,\omega)$ is the self-emitted radiance in the point x and $f_r = f_r(x,\omega' \to \omega)$ the BRDF of the surface. See Fig. 1 for geometric representation of Eq. 1.

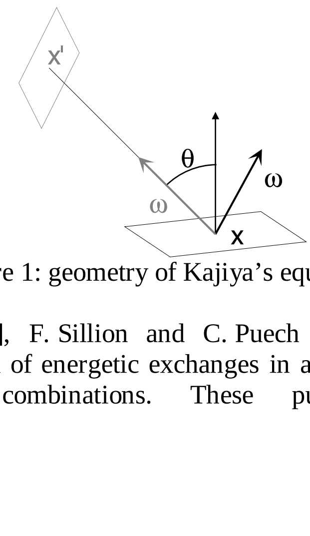


Figure 1: geometry of Kajiya's equation.

In [Silli89], F. Sillion and C. Puech introduce a formulation of energetic exchanges in a scene using operator combinations. These pure formal manipulations contribute to a better understanding of the different illumination models. They propose to separate f_r into the sum of a diffuse component $f_{rd} = f_{rd}(x)$ and a specular component $f_{rs} = f_{rs}(x,\omega'\to\omega)$: $f_r = f_{rd} + f_{rs}$. Then, they define two operators: D for the diffuse reflection and S for the specular reflection by:

$$DL = \int_{\omega'\in hem.} f_{rd}(x)L(x',\omega')\cos\theta\, d\omega'$$

$$SL = \int_{\omega'\in hem.} f_{rs}(x,\omega'\to\omega)L(x',\omega')\cos\theta\, d\omega'$$

The energetic radiance L become $L = L_d + L_s$ (2)

with $L_d = L_e + DL$ (3)

and $L_s = SL$ (4)

The suffixes d and s for L_d and L_s simplify the notations and obviously correspond to diffuse and specular terms.

- **Application to the classical radiosity algorithm:**

In classical radiosity, we only treat ideal diffuse surfaces ($L_s = 0$). Thus:

$L = L_d = L_e + DL_d$

We rewrite this expression as $(I - D)L_d = L_e$, then $L_d = (I - D)^{-1}L_e = D^*L_e$

Or $L_d = L_e + D(D^*L_e)$ (5)

We may explain this equation as follows: the diffuse energy (L_d) emitted from a point of a surface is calculated as the sum of its self-emitted energy (L_e) and the energy received from the points of the other surfaces after multiple diffuse reflections (D^*L_e). When the scene is subdivided in patches we calculate L_d for each patch, and the operator D is associated to the form factor calculating the energy exchanged between two patches.

Rq. We intentionally keep the expression Eq. 5 to compare it with future equations.

- **Application to the two-pass method and to the extended form factors:**

In the paper [Silli89], Sillion and Puech consider specular surfaces ($L = L_d + L_s$). Then they express L_d to associate only one value to each patch, Eq. 3 becomes:

$L_d = L_e + DL$ (with $L = L_d + SL$).

We may rewrite it:

$L_d = L_e + DS^*L_d$ (6)

Then, using the subdivision of the surfaces in patches, Sillion and Puech express the extended form factor to simulate the operator DS^*: an undefined number of specular reflections between two diffuse reflections.

The zonal method is useful to include a participating medium to the classical radiosity. Like this model, the surfaces are subdivided in patches whereas the surrounding medium is discretised into small volume elements. We consider that the medium is able to emit its own energy (by its heath in infrared frequencies, by combustion in visible frequencies…) and scatter some of the energy it gets. Thus the characteristics of the medium are similar to those of a diffuse surface. The extinction factor K_t that characterises the medium is equal to the sum of an absorption component K_a and a scattering component K_s.

The albedo $\Omega_k = K_s(V_k) / K_t(V_k)$ corresponds to the proportion of energy scattered by the volume element V_k and may be compared to the diffuse reflection coefficient ρ_i representing the proportion of energy diffused by the patch A_i. We solve the energetic equations system using each patch and each volume as it is done in classical radiosity. But here, the reflected energy by a surface or volume comes from the energy emitted by both the other surfaces and volumes. We get the two following equation systems:

For each patch A_i:

$$B_i = E_i + \rho_i\left(\sum_{\text{surface } A_j} B_j F_{A_iA_j} + \sum_{\text{volume } V_k} B_k F_{A_iV_k}\right)$$

For each volume V_k:

$$B_k = \frac{K_a}{4K_t}E_k + \frac{\Omega_k}{4}\left(\sum_{\text{surface } A_j} B_j F_{V_kA_j} + \sum_{\text{volume } V_m} B_k F_{V_kV_m}\right)$$

These expressions use four different types of form factors: surface to surface, surface to volume, volume to surface and volume to volume.

$$F_{A_iA_j} = \frac{1}{A_i}\int_{A_i}\int_{A_j}\tau(r)\frac{\cos\theta_i\cos\theta_j}{\pi r^2}dA_i\, dA_j$$

$$F_{A_iV_k} = \frac{1}{A_i}\int_{V_k}\int_{A_i}\tau(r)\frac{K_t(V_k)\cos\theta_i}{\pi r^2}dA_i\, dV_k$$

$$F_{V_kA_j} = \frac{1}{V_k}\int_{V_k}\int_{A_j}\tau(r)\frac{K_t(V_k)\cos\theta_j}{\pi r^2}dA_j\, dV_k$$

$$F_{V_kV_m} = \frac{1}{V_k}\int_{V_k}\int_{V_m}\tau(r)\frac{K_t(V_k)K_t(V_m)}{\pi r^2}dV_m\, dV_k$$

These form factors include a transmittance term $\tau(r) = \tau(AB)$ which takes into account both the visibility of point B from point A and energy absorption in the way AB:

$$\tau(AB) = \begin{cases} e^{-\int_{AB} K_t(x)\,dx} & \text{if A see B} \\ 0 & \text{else} \end{cases}$$

The zonal method treats only lambertian energetic diffusion on the surfaces and isotropic scattering into the volumes. In the next section, we extend it to include surfaces with specular behaviour.

2. ZONAL METHOD AND EXTENDED FORM FACTORS

We express first the zonal method equations under the form of operators, and then we complete this model in order to include specular reflection on surfaces. To do so, we modify the form factors of the zonal method.

2.1. Zonal method and operators

The zonal method treats two types of elements: surfaces and volumes. To simplify future notations, we use the suffix P for the radiance linked to patches and the suffix V for the radiance associated to volume elements. The radiances L_P and L_V are defined by:

$$L_{Pd}(x) = L_{Pe}(x) + \int_{\omega' \in hem.} f_{rd}(x) H(x) \cos\theta \, d\omega' \qquad (7)$$

$$L_{Vd}(x) = L_{Ve}(x) + \int_{\omega' \in sphere} \frac{\Omega_{rd}(x)}{4} H(x) \, d\omega' \qquad (8)$$

with $H(x) = L_{Pd}(x_0)\tau(x_0, x) + \int_{x_0}^{x} \tau(x', x) L_{Vd}(x') dx'$

Where x_0 is the abscissa of the first surface cut on the way defined by x and the incident direction ω' (cf. Fig. 2) and $L_{Ve}(x) = (1-\Omega_{rd})\, E(x)$ (E(x) represents the emitted radiance of the volume).

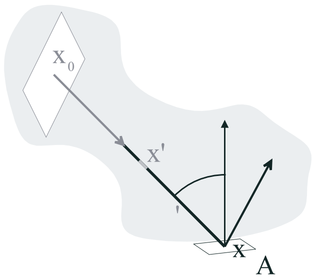


Figure 2: energy received by the patch A from a direction ω'.

We express in Eq. 7 that the radiance emitted from a point x of a patch A, which is constant for each diffusion direction (under our assumption of lambertian diffusion), is the sum of three terms:

1. A self-emitted radiance $L_{Pe}(x)$.
2. The re-emission, after reflection on the patch A, of all the radiance received from all the visible patches placed in the half-space over A (taking into account the absorption $\tau(x_0,x)$) expressed by:

$$D_a L_{Pd} = \int_{\omega' \in hem.} f_{rd}(x)\, L_{Pd}(x_0)\tau(x_0, x) \cos\theta \, d\omega'$$

Rq. This expression obviously generalises the classical radiosity equation (Eq. 1).

3. The re-emission, after reflection on the patch A, of all the energies received from all the visible volumes placed in the half-space over A expressed by:

$$D_a L_{Vd} = \int_{\omega' \in hem.} f_{rd}(x) \left(\int_{x_0}^{x} L_{Vd}(x')\tau(x', x) dx' \right) \cos\theta \, d\omega'$$

The equation Eq. 8 may be interpreted similarly, and induces the operator D'_a applied to L_{Pd} or L_{Vd}. Then we can write the previous equations under the form of operators:

$L_{Pd} = L_{Pe} + D_a(L_{Pd} + L_{Vd})$
$L_{Vd} = L_{Ve} + D'_a(L_{Pd} + L_{Vd})$

Let us express now $L_{Pd} + L_{Vd}$:

$L_{Pd} + L_{Vd} = L_{Pe} + L_{Ve} + (D_a + D'_a)(L_{Pd} + L_{Vd})$
$= (D_a + D'_a)^*(L_{Pe} + L_{Ve})$

This equation express the energy $(L_{Pd} + L_{Vd})$ received on a point x of a patch and coming from a point x_0 of another patch P along the path $\omega' = (x_0,x)$ (cf Fig. 2). This energy is the sum of the self-emitted radiance of the patch P (L_{pe}), the self-emitted radiance of the voxels (L_{Ve}) along ω' and the energy $(L_{Pd} + L_{Vd})$ reaching from all directions, P and each voxel along ω' and re-emitted (op. D_a+D'_a) in the direction ω'.

We use the last equation to finally express L_{Pd} et L_{Vd}:

$L_{Pd} = L_{Pe} + D_a(D_a + D'_a)^*(L_{Pe} + L_{Ve})$
$L_{Vd} = L_{Ve} + D'_a (D_a + D'_a)^*(L_{Pe} + L_{Ve})$

These equations may be compared to the equation Eq. 5 of the classical radiosity $L_d = L_e + D(D^*L_e)$. We only express here the fact that the radiance of a surface (respectively a volume) is equal to the sum of three components: its own radiance, some energy coming from diffuse reflections on the surfaces and some energy from the isotopic scattering on the volumes.

2.2. Specular reflection process in the zonal method

We now improve the zonal method by treating the specular reflections on surfaces (cf. Fig. 3). We suppose that only surfaces have a specular behaviour (added to the diffuse one). Like in the previous section, we introduce the operator S_a representing a specular reflection associated to a specular coefficient ρ_s.

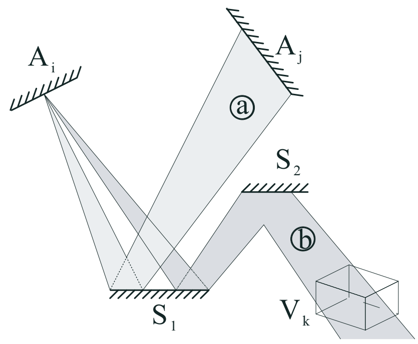


Figure 3: energetic exchanges using specular surfaces: a) surface/surface, b) surface/volume.

Thus we write:

$L_P = L_{Pd} + L_{Ps}$
with $L_{Pd} = L_{Pe} + D_a(L_P + L_V)$ and $L_{Ps} = S_a(L_P + L_V)$
Then
$L_P = L_{Pd} + S_a(L_P + L_V)$
Using $L_V = L_{Vd} = L_{Ve} + D'_a(L_P + L_V)$ (volumes are perfectly diffuse), we can deduce:
$L_P + L_V = L_{Pd} + L_{Vd} + S_a(L_P + L_V)$ and
$L_P + L_V = S_a^*(L_{Pd} + L_{Vd})$
Thus
$L_{Pd} = L_{Pe} + D_a\, S_a^*(L_{Pd} + L_{Vd})$

$L_{Vd} = L_{Ve} + D'_a S_a^*(L_{Pd} + L_{Vd})$

We may compare these two equations with those of Sillion and Puech (Eq. 6) which define extended form factors. Then, we rewrite the zonal method form factors by calculating the energy exchanged by two elements X and Y (patch or volume element) thanks to any number of specular reflections on the surfaces $S_1 \dots S_n$. As a consequence, it is necessary to complete the surface/surface, surface/volume and volume/volume form factors of the zonal method to integrate this new origin of energy. We define the intermediary form factor $F_{XS_1 \dots S_n Y}$ which represents the proportion of energy exchanged between two elements (patches or volume elements X and Y) by means of one or several specular surfaces ($S_1 \dots S_n$):

$$\begin{aligned}\overline{F}_{XY} &= F_{XY} + \sum_{S_1} F_{XS_1Y} + \sum_{S_1,S_2} F_{XS_1S_2Y} + \dots + \sum_{S_1 \dots S_n} F_{XS_1 \dots S_n Y} \\ &= F_{XY} + \sum_{n \geq 1} \sum_{S_1 \dots S_n} F_{XS_1 \dots S_n Y}\end{aligned}$$

This extension to the algorithm of the zonal method is useful to simulate every energetic exchange in a scene which contains absorbing/scattering media and surfaces admitting diffuse and specular behaviour.

3. IMPLEMENTATION OF THE EXTENDED FORM FACTORS

3.1. A particular case of one level specular reflection

An easy method which makes it possible to simulate specular reflections on flat surfaces consists in replacing the specular surface by a visualisation window. This classical method [Rushm90] [Silli94] is a good alternative method to classical methods based on the Monte Carlo algorithm [Malle88].

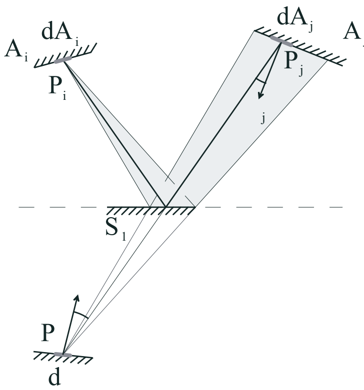


Figure 4: Calculation of the extended form factor

We shall now consider how we calculate the intermediary form factor $F_{A_iS_1A_j}$ which uses the specular property of the surface S_1 (Fig. 4): we call it the intermediary form factor of level 1. The other form factors (surface/surface and surface/volume) are given by similar expression. We shall write $\sigma = \sigma_{S_1}(A_i)$ the symmetric of A_i in relation to S_1. The amount of energy exchanged between A_i and A_j using the specular surface S_1 is equal to the energy exchanged between $\sigma_{S_1}(A_i)$ and A_j through the window S_1. The intermediary form factor $F_{A_iS_1A_j}$ is expressed by:

$$F_{A_iS_1A_j} = \frac{\rho_s(S_1)}{A_i} \int_{\sigma} \int_{A_j} \tau(r') \overline{V}(r, S_1) \frac{\cos\theta_\sigma \cos\theta_j}{\pi r^2} dA_j \, d\sigma$$

with r representing the distance between P_σ and P_j (respectively center of $d\sigma$ and dA_j), r' represents the real way between P_i (center of dA_i) and P_j via S_1 where the energy is absorbed and

$$\overline{V}(r, S_1) = \begin{cases} 1 & \text{if the segment } [P_\sigma, P_j] \text{ cuts } S_1 \\ 0 & \text{else} \end{cases}$$

3.2. General expression

We shall now consider the calculation of the intermediary form factor of level 2: $F_{A_iS_1S_2A_j}$ (Fig. 5). The problem is how to consider the symmetric of the previous situation in relation to S_2. Consequently we calculate at the same time the symmetric of $\sigma_{S_1}(A_i)$ in relation to S_2 (we call it $\sigma_{S_1S_2}(A_i)$) and the symmetric of the window S_1 in order to keep the notion of successive windows. We obtain:

$$F_{A_iS_1S_2A_j} = \frac{\rho_s(S_1)\rho_s(S_2)}{A_i} \int_{\sigma_{S_1S_2}(A_i)} \int_{A_j} \tau(r') \, \overline{\upsilon}(r) \frac{\cos\theta_\sigma \cos\theta_j}{\pi r^2} dA_j \, d\sigma$$

with $\overline{\upsilon} = \overline{V}\left(r, \sigma_{S_2}(S_1)\right) \overline{V}(r, S_2)$

The real way r' is calculated using the points of intersection P_1 and P_2 between $[P_\sigma, P_j]$ and the surfaces $\sigma_{S_2}(S_1)$ and S_2. r' pass by P_i, Q_1 (symmetric of P_1 in relation to S_2 and S_1) P_2 and P_j.

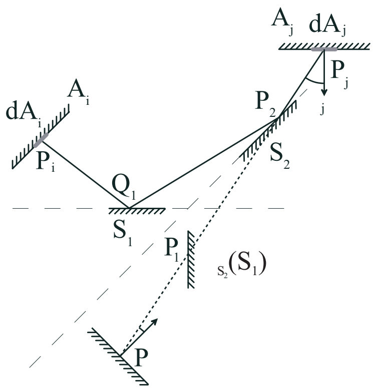


Figure 5: Calculation of a form factor of level 2.

Then, it is easy to generalise the previous expression (§ 3.1) to n specular reflections. Defining $\sigma_{S_1 \dots S_n}(A_i)$ as the symmetric of A_i in relation to $S_1, S_2 \dots S_n$, we obtain a general expression of the intermediary form factor for n specular reflections:

$$F_{A_i S_1 \ldots S_n A_j} =$$

$$\frac{\prod_{k=1}^{n} \rho_s(S_k)}{A_i} \int_{\sigma_{S1 \ldots S_n}(A_i)} \int_{A_j} \tau(r') \, \overline{\upsilon}(r) \frac{\cos\theta_\sigma \cos\theta_j}{\pi r^2} dA_j \, d\sigma$$

with $\overline{\upsilon}(r) = \overline{V}(r, S_n) \prod_{k=1}^{n-1} \overline{V}\left(r, {}_{S_{k+1} \ldots S_n}(S_k)\right)$

We generalise the expression of the way r' which goes by the points $P_i \; Q_1 \ldots Q_{n-1} \; P_n \; P_j$, with Q_k defined by: $Q_k = \sigma_{S_n \ldots S_{k-1}}(P_k)$

4. ALGORITHM AND COMPLEXITY

Scenes are discretised in patches and voxels (volume elements). The software developed uses a two-pass algorithm. The first pass solves the radiosity equation system with a progressive refinement algorithm. It uses classical form factors and extended form factors of level 1 and 2 (treating textured patches). The second pass is a ray tracing which permits to render the radiance (of the patches and the volume elements) calculated by the first pass. The specular reflections and the textured patches are visualised too by this ray-tracer. This software is coded in C++ on Unix workstation (Pentium Pro under Linux).

4.1. Algorithm of calculation of the intermediary form factor of level 1

The calculation of the form factors implies to evaluate a double integral over the surfaces of the patches (respectively over voxels volumes). These integrals are evaluated by a classical numeric method using a regular discretisation of the patches and the voxels. The following algorithm presents the method which calculates the intermediary form factors (IFF) of level 1 between two patches A_i and A_j. It uses the function WellOriented(A,S) which returns True if the surface S is oriented towards A.

```
Function IFF(Ai,Aj)
 IFF = 0
 For each specular surface S1
  If WellOriented(Ai,S1) and WellOriented(Aj,S1) then
   For each surface element dAi of Ai
    pAi = center of dAi
    For each surface element dAj of Aj
     pAj = center of dAj
     pAi’ = symmetric of pAi in relation to S1
     P =intersection point between [pAi’pAj] and S1
     If P exists then
      tau=transmittance(pAi,P)*transmittance(P,pAj)
      r = norm(pAi’pAj)
      cosThetai = ([pAiP] * [normal(Ai)])/r
      cosThetaj = ([pAjP] * [normal(Aj)])/r
      IFF = IFF+rho(S1)*tau*surface(pAj)*cosThetai*
            cosThetaj/(PI*r*r)
     End if
    End for
   End for
  End if
 End for
End Function
```

4.2. Complexity of the radiosity pass.

Time complexity is mainly due to the calculation of the extended form factors between all the elements of the scene. Let N be the total number of elements (patches or voxels), k the number of specular patches ($k \le N$) and r the level of successive specular reflections treated in the extended form factors.

For a classical form factor, time complexity is O(N) because of the evaluation of the visibility term. So for a step of the progressive refinement algorithm, we can deduce that the complexity is:

$O(N^2)$ for the calculus of the N zonal method form factors.

$O(N^2k)$ for the intermediary form factors of level 1 (for each element, we test k specular patches).

$O(N^2k^2)$ for the intermediary form factors of level 2.

⋮ ⋮ ⋮ ⋮ ⋮

$O(N^2k^r)$ for the intermediary form factors of level r.

Summing these r partial complexity gives:

$$N^2\left(1 + k + k^2 + \ldots + k^r\right) = N^2 \frac{k^{r+1} - 1}{k - 1}$$

As a consequence, the global complexity, for a step of the progressive refinement algorithm, is $O(N^2k^r)$.

For example, the scene of the laser (proposed on Fig. 7) admits N=46.000, k=2 and r=2.

The complexity of the second pass is the classical complexity of the ray-tracing algorithm.

5. RESULTS AND COMMENTS

The patchwork of images on Fig. 6 shows the same scene rendered by four different radiosity algorithms. To generate these images, we use the same ray-tracing algorithm for the second pass, but we use several algorithms to achieve the first pass (radiosity). The scene shows a light source (the laser), two mirrors put on 45° in the light beam and a screen on the left. These objects, put on a table, are lit by a second light source situated above (justifying the shadows of the mirrors and the screen), and are plunged into a participating media.

On Fig. 6a, the pass of radiosity is the same as the classical zonal method [Rushm87]. In this method, no specular surfaces are taking into account, it just treats the diffuse property of the mirrors. We can see just one light beam materialised by the voxels directly lighted by the source. The others are much less luminous because they just receive diffused energy.

On Fig. 6b, the radiosity algorithm uses classical extended form factor [Silli89] of level 1 and 2. This algorithm does not take into account the participating media but the observations of the bright disc on the left screen and the reflections on the table testify the treatment of the specular reflections during the radiosity pass.

The images Fig. 6c and Fig. 6d are generated by the method presented in this paper which combines both the zonal method and the extended form factors. We notice the effects of the intermediary form factors of level 1 (Fig. 6c) and of level 1 and 2 (Fig. 6d) which visualise the light beam in the participating media and the reflections of the second light source on the table.

The images presented on Fig. 7 show the previous scene (Fig. 6d) on a bigger size.

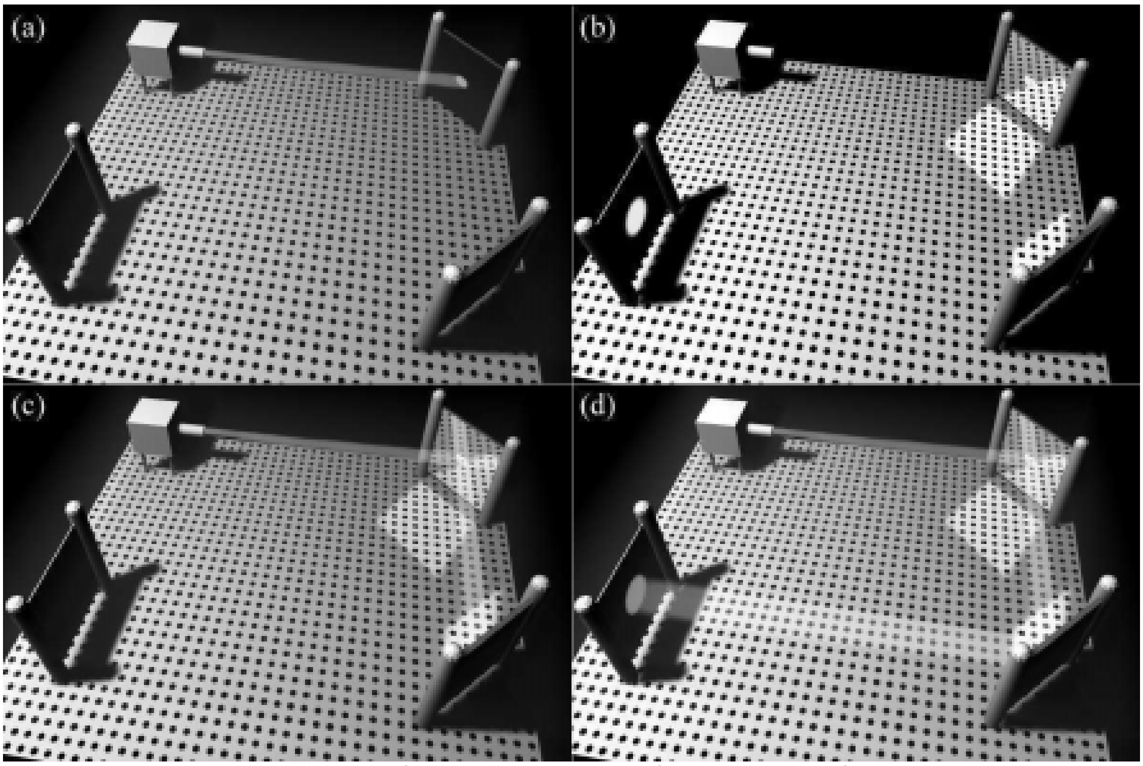


Figure 6: a scene of a laser represented by four algorithms of radiosity.

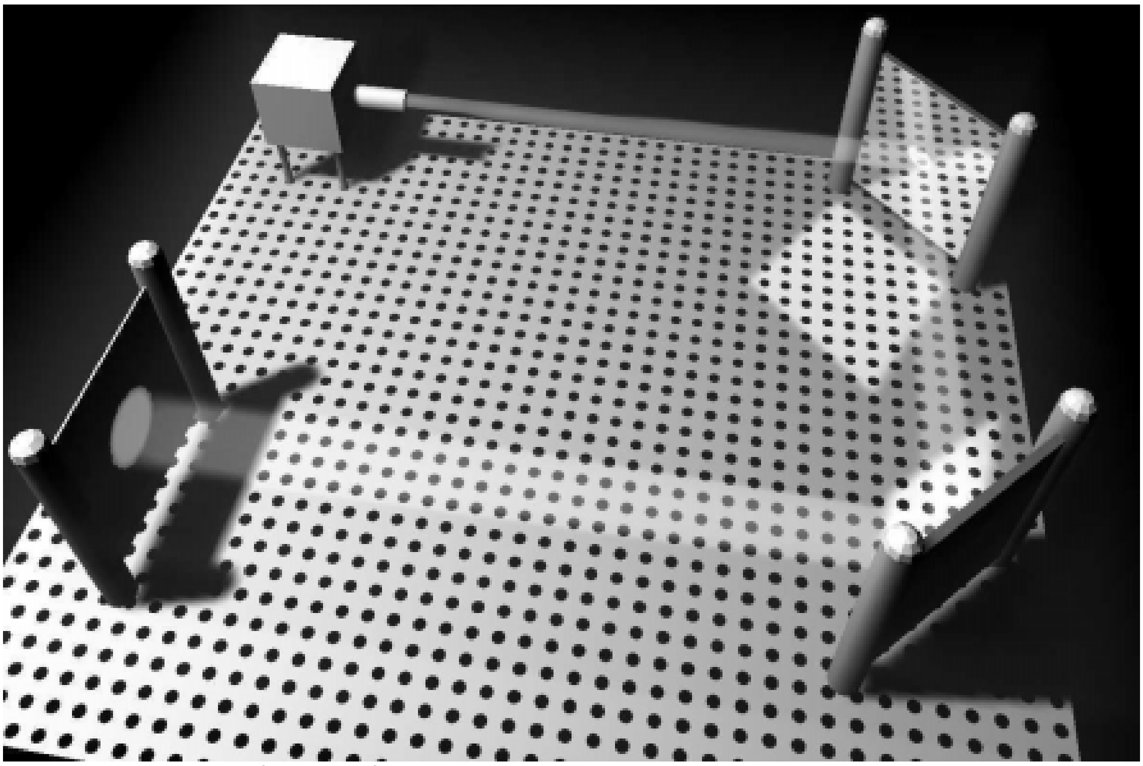


Figure 7: multiple reflections of a laser beam on mirrors, 46 000 patches (2 mirrors) and 140 000 voxels. Calculation time for 50 steps of progressive refinement: about 43 hours.

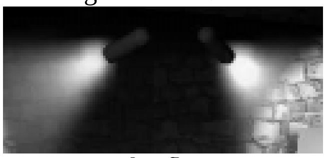
The scene Fig. 8 contains a sphere built with small mirrors put together and lit by three directional lamps. The spotlights illuminate the participating media directly and indirectly by specular reflection on the sphere, it explains the round form of the light beams. The bright zones on the walls and on the ceiling are the reflections of the spots on the mirror. A simple animation using this scene has been realised consisting in the rotation of the sphere around the vertical axis.

Figure 8: the discotheque, 18 000 patches (1 152 mirrors) and 216 000 voxels.
Calculation time for 3 steps of progressive refinement: about 3 hours.

Figure 9: reflective painting lighted by spots, 21 000 patches (2 mirrors) and 46 000 volume elements (voxels).
Calculation time for 50 steps of progressive refinement: about 18 hours.

The image proposed on Fig. 9 contains at the same time specular reflections (the glasses on the pictures) and a participating media. The noticeable effects are mainly:

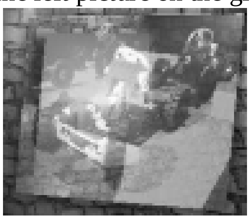
- The materialisation of the light beam through the participating media, it is a consequence of exchanges between the sources (patches) and the voxels constituting the media.



- The bright zones on the floor are generated by the specular reflections of the spots on the mirrors. We notice the shadows produced by these reflected lights under the bench. These energies are linked to the intermediary form factors of level 1 between patches.

  With the classical radiosity algorithm, the lambertian reflection on the wall would have drawn a bright zone on the floor much larger and circular.

- We notice the specular reflections on the pictures generated by the ray-tracing, for example the blue zone in the right picture (the car) is a specular reflection of the left picture on the glass.



6. CONCLUSION

We have introduced an extension of the zonal method which enables to treat the scenes containing specular surfaces. This method is a mean to add a specular component to the bi-directional reflection definition function, and thus shows phenomena, which can not be obtained neither by the classical zonal method nor by the ray-tracing algorithms. To do so, we define new form factors to treat specular energetic exchanges between surfaces and volumes in a scene. The presentation of a few images with simple or multiple reflections through participating media have enabled us to underline these energetic exchanges. This paper belongs to the successive extensions brought to the image synthesis algorithms whose aim is to sharpen the realism of the generated images taking into account more and more complex light/material interactions. Other improvements will be found in refining the BRDF or implementing new optical phenomena (diffraction…).

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