A NEW RENDERING TECHNIQUE FOR WATER DROPLET USING METABALL IN THE GRAVITATION FORCE

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ABSTRACT

Till now there are many rendering models for water and other fluids and their dynamics. Especially in order to generate the curved surface of flexible objects such as water, jelly, and snow, the implicit metaball formulation is widely used in favor of its simplicity and flexibility. This paper proposes one novel method for generating water droplets, which would be deformed in a gravitation field. In previous works, a water droplet was simply represented by approximated curved surfaces of a symmetric and quite a simple metaball. So the finally rendered water droplet was far from a realistic droplet, because they did not consider the gravitation force in droplets attached on a surface. We give a new generalized metaball model for rendering water droplets placed on an arbitrary surface considering the gravitation and friction between droplets and the plate. Our new metaball model uses a new vector field isosurface function controlling the basic scalar metaball, with respect to the norm of gravitation force. In several experiments, we could render a photo-realistic water droplets with natural-looking shadows by applying ray-tracing techniques.

Keywords: mataball, droplet model, curved surface, rendering

1 INTRODUCTION

Since the 1980's, many methods have been developed for representing water droplets and other fluids[Kaned93a]. In the beginning of these works, researchers have mainly concentrated on representing a large mass of the fluids like a wave. Those methods did not consider the boundary between fluid and solid, but only study the structure of fluid dynamics in a free space. Some methods have been developed to represent the boundary(e.g., seashore) between fluid and solid. However, those previous methods had some technical problems to make a realistic model for small mass of the fluids like a water droplet[Nishi83a].

In order to represent a "soft object" like liquid, the metaball has widely been used since it was defined by a few simple implicit formula and it allows simple free-form deformation operations[Wyvil86a, Wyvil90a]. Especially most of human bodies(e.g. hands, arms, foot and muscular form) are highly adequate to be represented by a set of metaballs. In the metaball technique, a free-form surface is defined as an isosurface that has the same field value in space. The field value of each point is determined by distance from specified center point of metaball. The task of the user is to specify the center position of each metaball, radius and field function.

This metaball technique was first developed by Blinn, and he called it blobs. And this was improved by Nishimura[Nishi83a], Murakami[Murak87a] and Wyvill[Wyvil86a] et al.. They called it metaball and *soft objects*. The main differences in these previous works are in the shape of field functions and the methods solving for ray/isosurface intersections. For n different metaballs, the shape of the curved surface is defined by the points satisfying the following equation.

$$f(x, y, z) = \sum_{i=0}^{n} q_i \cdot f_i - C_0 = 0$$

where C_0 is a threshold constant, q_i the density values.

Previously Kaneda et al. have proposed the flow dynamics of water droplet on curved surface[Kaned93a, Kaned96b]. See Fig.1. The curved surface is divided into small meshes, and the flow of water droplets was calculated, based on probability. In the beginning, each new water droplet is put at each mesh point. And movement of each water droplets is destined by current state of each water droplets. Direction of movement is settled by using a random roulette to achieve a natural stream. Each water droplets are moved to next mesh points until movement is impossible because of the result of wet phenomenon.

Their work proposes a method for generating realistic animation of water droplets and their stream on curved surfaces, taking into account the dynamics that act on water droplets.

The finally rendered water droplet on a curved surface looks not realistic, since their water droplet model did not consider the gravitation force which makes the spherical droplet biased to the down direction. As was shown in Fig.1, the droplets scattered on the teapot seems a sticky liquid such as a honey or oil with high viscosity or merged droplets placed on a flat floor. In fact, neighboring droplets on a vertical surface should be merged into one bigger droplet and it finally runs down to the bottom of a teapot. We can easily check that Fig.1 shows one unnatural snapshot of water droplets on a teapot.

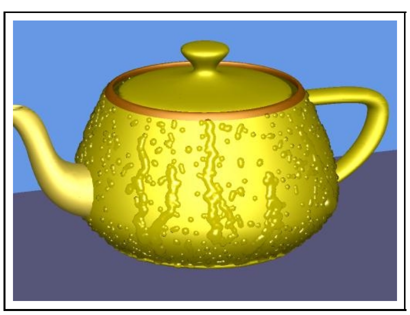

Figure 1: Droplets on a Teapot by [Kaned96b]

In this paper, we propose a new droplet model deformed in the gravitation field using metaball concept. Previous works on metaball have only concentrated on how to detect the isosurface fast and rendering speed of metaball, or how to do ray-tracing easily in metaball.

The shape of the metaball is symmetric, owing to the characteristics of scalar field of metaball. In a real world, water droplet, however, is deformed by the gravity, the frictional force between droplet and the surface, the viscosity of the droplet and the orientation of the surface(normal vector). Thus if we do not consider these factors, realistic droplet can not be obtained. In this paper we only consider the gravity and friction force because the viscosity for a droplet requires more information on a droplet physics.[Genne85a]

The main idea of our gravitation metaball is that we use a vector field isosurface rather than the previous scalar field isosurface, where each point of isosurface is settled by only the distance from the center point of a metaball. In our method, each point of isosurface is divided into three vector components ($\vec{x}$, $\vec{y}$ and $\vec{z}$). Each vector will be deformed in order to represent the gravity, friction and mass, respectively. And new field value of metaball is determined by the vector norm of the summation of three component vectors. On the plane plate, the height of water droplet is decreased and the width of water droplet is increased due to the gravitation force. On an inclined plate, since the mass of water droplet is moved to the inclined direction, the shape of water droplet can not be symmetric with respect to the metaball center. The following sections will explain how to consider these effects to make a realistic droplet using metaball.

2 DROPLET MODEL ON PLANE METAPLATE

When a water droplet lies on the plain surface horizontally, interfacial tensions and the contact angle, θ, of water and the surface satisfy the following equation(See Fig. 2):

$$\gamma_{SL} - \gamma_S + \gamma_L \cos\Theta = 0$$

where γ_{SL} is the interfacial tension between the surface and the water, and γ_S and γ_L are their respective surface tensions[Kaned93a]. When we consider this phenomenon, we can make the water droplet more realistic. However, we do not take into account it because we assume that the center of water droplet is on the metaplate.

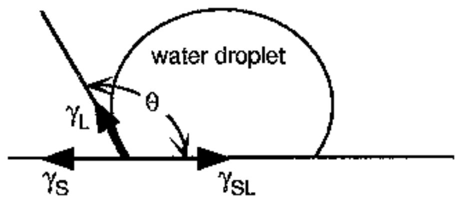

Figure 2: Tension between water droplet and surface

2.1 Vectorizing of Scalar Value

In this paper, we select Wyvill's the degree six polynomial for the field function among several field functions for the isosurface calculation, because higher degree field function makes isosurface of metaball more smooth when two metaballs are merged into each other.

We assume that the center of metaball is always on the metaplate. When a water droplet in a real world appears on the plate, the mass of metaball pushes the metaball itself to be in an equilibrium state against the interfacial tension of droplet. Thus the shape of metaball becomes hemispherical due to its mass gravitation to the bottom direction. Fig.3 shows the movement of the center of mass and deformation of metaball with the gravity. (a) is an original metaball without gravity and a movement of the center of mass. We assume that the radius of original metaball is unit distance 1. (b) shows the movement of the center of mass. In Fig.3 (b), the radius of deformed metaball becomes $2^{\frac{1}{3}}$ and the radius of isosurface decreases to $2^{\frac{1}{3}}/2$ by Wyvill's field function because we set the threshold as 0.5. (c) is a deformed metaball with a low gravity. (d) is a deformed metaball with a higher gravity than (c). Fig.4 shows a ray-traced image corresponding to droplets in Fig.3. Fig.5 explains the deforming process of metaball owing to the gravity force. In Fig.5, M_o is an original metaball and M_d is a deformed metaball. In our model, deformed metaball M_d is affected by three parameters in the following : $M_d = D_{M_o}(Gravity, Friction, Slope)$

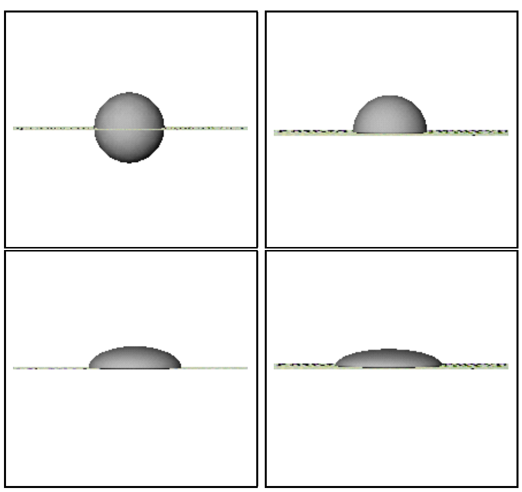

Figure 3: Gravity effect on plane metaplate: (a) a basic general metaball(left up). (b) a given metaball without gravity force(right up). (c) a metaball with low gravity(left down). (d) a metaball with high gravity(right down).

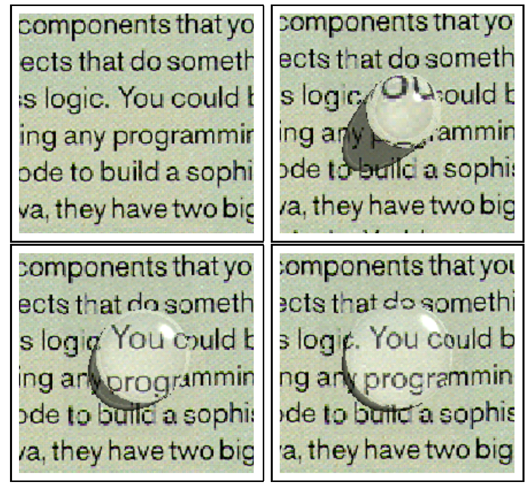

Figure 4: Ray-traced droplets of Fig.3.

Function D_{M_o} represents the deformation of metaball M_o controlled by the degree of gravity, friction and the surface slope. Field value of each point on M_o is settled by the distance from M_c. And it is a scalar value. Here we note that gravity G is a vector value with direction toward the ground. Then each point on metaball must be divided into vectors to deform the metaball. In Fig.5, vector force on a surface point o can be divided into $\vec{o_x}$ and $\vec{o_y}$. Vector $\vec{o_x}$ and $\vec{o_y}$ are divided into two sub-vectors $\vec{d_x}$ and $\vec{d_y}$ due to the gravity. Therefore, the point o will move to the direction of point d. Points on the metaball M_o also will move downwards of the metaball M_d through the same process. A deformed metaball with this gravitational model has a different field space. Fig.6 shows a field space of two metaballs. M_a and M_b share a center point with D_a and D_b, respectively. Metaball M_a and M_b have the isosurface M_{ia} and M_{ib}, respectively. M_a and M_b will be not merged, since the distance between two metaballs is greater than the threshold. Now we will make the deformed metaball D_a and D_b to be merged into one metaball.

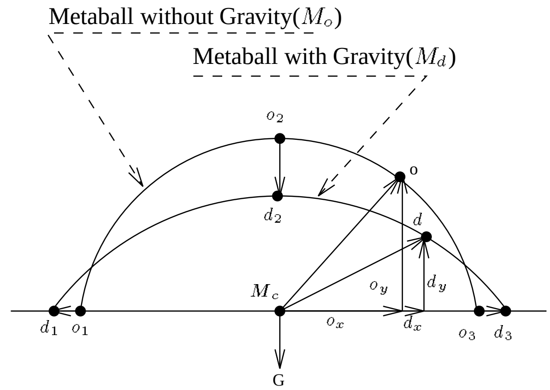

Figure 5: Deformation of metaball on plane metaplate with the vertical gravitation field.

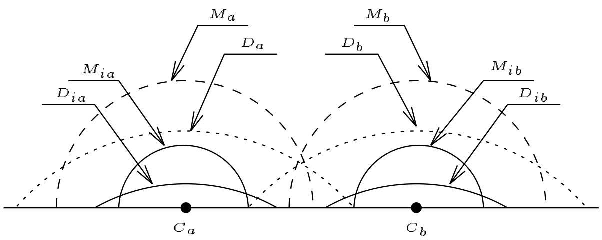

Figure 6: Merging step of two deformed gravitation metaballs, M_a and M_b.

Fig.7 and Fig.8 shows one merged droplet of two adjacent metaballs. Fig.7 shows one merged metaball from two neighboring metaballs separated in a large distance than Fig.8. Fig.9 shows ray-traced pictures of droplets in Fig.7 and Fig.8.

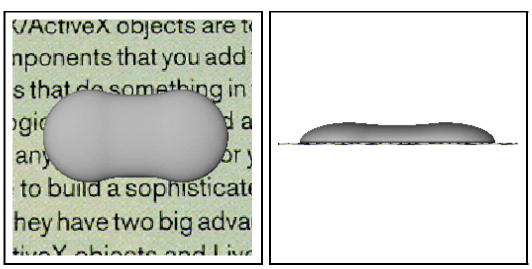

Figure 7: A merged metaball with two metaballs in a large distance. (a) see from up. (b) see from front.

2.2 Deforming the Shape of Water Droplet

Simply vectorizing scalar value makes a section of water droplet as a circle. However, in a real world, water droplets scattered on the surface would have various shapes of the section. We propose one method that deforms the shape of water droplet

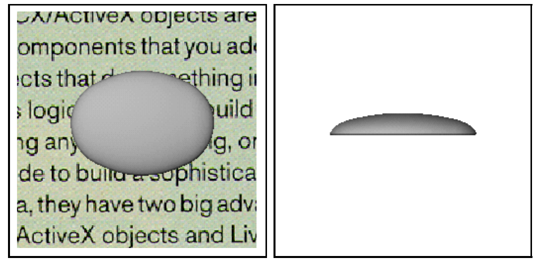

Figure 8: A merged metaball with two metaballs placed in a small distance. (a) see from up. (b) see from front.

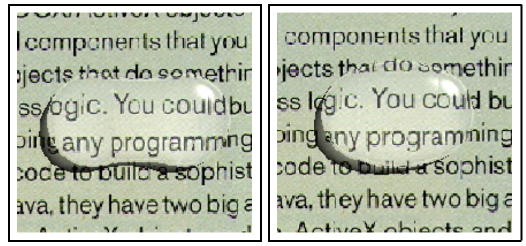

Figure 9: A merged metaball with a ray-tracing.

using control points. In this example, we use 12 control points. However, in order to describe the effect of control points, we use 4 control points in Fig.10. In order to distribute the control points uniformly, the angle between two control points locating nearby is $2\pi/n$, n is the number of control points. The length from the center point to a control point is determined interactively by user.

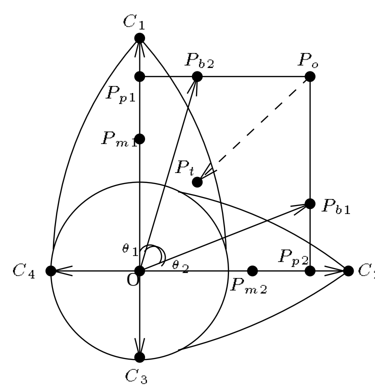

Figure 10: Deforming the shape of water droplet by using the control points.

In Fig.10, we calculate a new position of P_0 as follows:

1. Find points(P_{p1}, P_{p2}) that meet the line from P_0 to line(each control points, O) perpendicularly. Control points C_3, C_4 do not affect the movement of P_0 because C_3, C_4 lie on

the circle.

$$P_{p1} = \frac{\vec{C_1} \cdot \vec{P_o}}{||\vec{C_1}|| \cdot ||\vec{C_1}||}$$

$$P_{p2} = \frac{\vec{C_2} \cdot \vec{P_o}}{||\vec{C_2}|| \cdot ||\vec{C_2}||}$$

2. Calculate the ratio of control point to radius, and find the new locations of P_{p1}, P_{p2} affected by control points respectively.

$$P_{m1} = P_{p1}\frac{Radius}{||C_1||},\ P_{b1} = P_o - (P_{m1} - P_{p1})$$

$$P_{m2} = P_{p2}\frac{Radius}{||C_2||},\ P_{b2} = P_o - (P_{m2} - P_{p2})$$

3. Calculate the cosine value.

$$cos(\theta_1) = \frac{\vec{C_1} \cdot \vec{P_b 1}}{||\vec{C_1}|| \cdot ||\vec{P_b 1}||}$$

$$cos(\theta_2) = \frac{\vec{C_2} \cdot \vec{P_b 2}}{||\vec{C_2}|| \cdot ||\vec{P_b 2}||}$$

4. Sum the cosine value in each point, and find the new location of P_0 by using the ratio of the summation to each cosine value.

$$cos_{sum} = cos(\theta_1) + cos(\theta_2)$$

$$P_t = P_{b1}\frac{cos(\theta_1)}{cos_{sum}} + P_{b2}\frac{cos(\theta_2)}{cos_{sum}}$$

Fig.11 shows the shapes of water droplet deformed by control points. The lengths of control points are as follows:

- Left - 8, 0, 0, 6, 0, 0, 0, 0, 0, 0, 0, 0
- Right - 8, 0, 5, 6, 0, 5, 0, 6, 0, 0, 0, 0

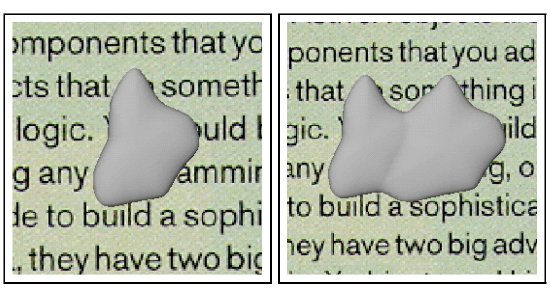

Figure 11: Water droplet deformed by 12 control points. (a) a water droplet (left), (b) two merged water droplets.

Fig.12 shows the ray-traced snapshots for a arbitrarily shaped droplet.

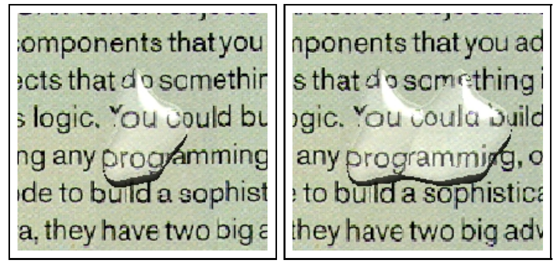

Figure 12: Ray-traced droplets deformed by control points.

3 DROPLET MODEL IN GRAVITATIONAL FIELD

Generally, when a water droplet lies on the inclined surface in a natural environment its shape is quite different from that of water droplet lying on the flat surface. The effecting elements that determine its final shape is the gravity, including the friction and the slope of inclined surface. When the gravity pushes the water droplet down, the upper part of water droplet leans downwards. In order to simulate this dynamics, we have to modify the norm of vectors: $\vec{u} = (u_x, u_y, u_z), \vec{v} = (v_x, v_y, v_z), \vec{w} = (w_x, w_y, w_z)$. Fig.13 shows that the shape of droplet is transforming on the inclined metaplate.

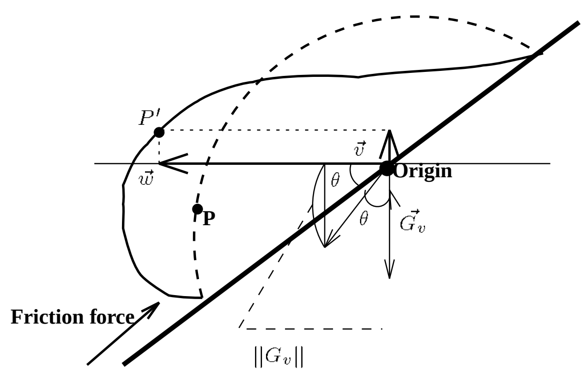

Figure 13: Droplet model on an inclined metaplate: dotted line is a droplet boundary without gravity.

The rendering procedure for the biased water droplets on an inclined metaplate is as follows:

1. Calculate the tangent of metaplate.
2. Calculate the friction force between the droplet and the surface.
3. Change the norm of vectors: $\vec{u}, \vec{v}, \vec{w}$ considering the degree of gravity pushing the plate perpendicularly.

4. Apply those norm vectors to the blending function.

The degree of skewness of metaball is dependent on the degree of the metaplate slope. In a real world, an object lying on a surface is affected by a frictional force. Therefore, we assume this friction as follows:

$$F_{friction} = \begin{cases} -(\|\vec{G_w}\| - \sin\theta \cdot \|\vec{v}\|)/R & \text{if } w_z < 0 \\ (\|\vec{G_w}\| - \sin\theta \cdot \|\vec{v}\|)/R & \text{,otherwise} \end{cases}$$

where θ is the slope of metaplate, R is a radius of metaball that controls the size of water droplet. The factor that determines the height in $P^{'}$ is the ratio of $\|\vec{w}\|$ to R. This ratio T is settled by the value of plate tangent, and calculated as follows:

$$T = \begin{cases} -\|\vec{w}\|/R & \text{if } w_z > 0 \\ \|\vec{w}\|/R & \text{,otherwise} \end{cases}$$

We newly define the field vector functions as follows:

$$h(T) = \frac{7}{8}T^3 + \frac{9}{4}T^2 + \frac{15}{8}T$$

$$\vec{u}' = \vec{u} \cdot (1 - \frac{\|\vec{G_v}\|}{3})$$

$$\vec{v}' = \vec{v} \cdot (1.3 + \|\vec{G_v}\| + h(T)\sin\theta)$$

$$\vec{w}' = \vec{w} \cdot (1 - \frac{\|\vec{G_v}\| + \|\vec{G_w}\| - F_{friction}}{3})$$

, where $h(T)$ is the amount of changes of length calculated by applying to the 3rd order function. The range of this value is $-\frac{1}{2} \leq h(T) \leq 5$ because T is $-1 \leq T \leq 1$.

Let $\|\vec{G_v}\|$ denote the norm of gravity. Since the amount of gravity is constant in every point in the real world, the shape of water droplet applied by the gravity is not destined by the amount of gravity, but destined by the direction of gravity and the slope of metaplate.

Therefore $\|\vec{G_v}\|$ has to preserve the different value according to the slope of metaplate in order to simulate it. The range of $\|\vec{G_v}\|$ is from the given amount of gravity (0.6 in experiment) to 0. And $h(T)\sin\theta$ denotes the movement of droplet's mass.

As the angle of metaplate's slope changes from $0°$ to $90°$, we can see that the shape of water droplet leans downwards fast in our experiments. Fig.14 shows the shape of water droplets with the slope's angle $30°, 45°$. Fig.15 shows the shape of water droplets with the slope angle $60°, 90°$. Fig.16 shows the water droplets with degree $45°, 90°$ after the ray-tracing step. Fig.17 shows the shapes of merged two droplets on an inclined metaplate where two water droplets are close to each other on a surface with the degree $45°, 90°$. The distance between two droplets in the left snapshot is larger than in the right one.

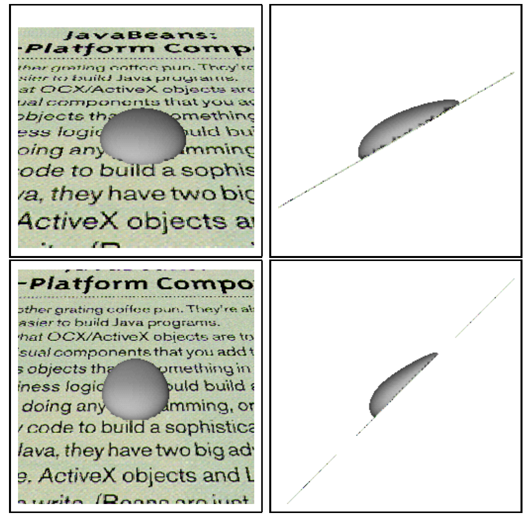

Figure 14: Shapes of water droplet on an inclined metaplate with degree $30°, 45°$.

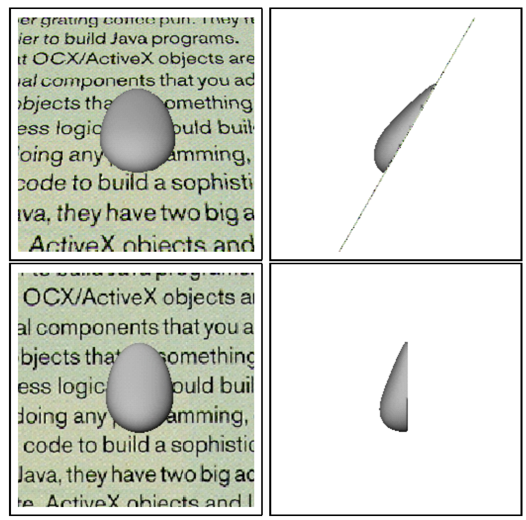

Figure 15: Shapes of water droplet on an inclined metaplate with degree $60°, 90°$.

In this paper, we only take into account of the situation where a water droplet sticks to the surface. In the future, we will consider the downward flow of water droplet when mass gravity is greater than the friction. Fig.18 shows the finally ray-traced images with the degree $45°, 90°$.

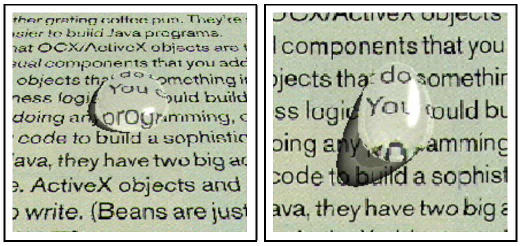

Figure 16: Ray tracing version of Fig.14,15.

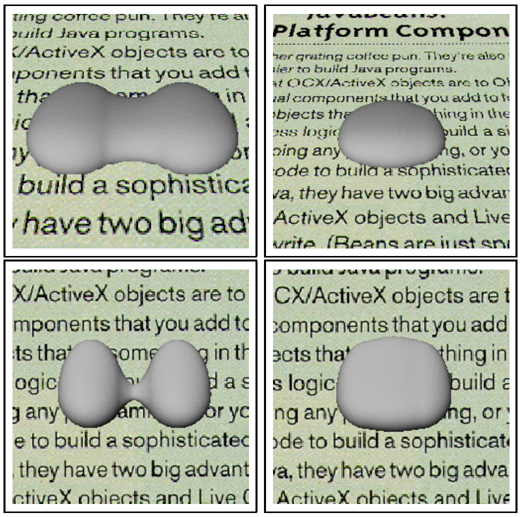

Figure 17: Shapes of merged two droplets on an inclined metaplate with degree $45°, 90°$.

4 EXPERIMENTS

We have conducted experiments for this gravitational water droplet model using *OpenInventor*, and rendered on a Silicon Graphics Indigo^2 IMPACT machine with 512×512 resolution. To improve reality, we adopt a ray-tracing and a shadow model. We produce a color of water by reflection-illumination model in Phong shading. The color contribution of water droplet surface are returned by the following ray tracing procedure:

$$I_T = I_{local} + k_{rg} I_{reflected} + k_{tg} I_{transmitted}$$

where I_T denotes the total color contribution of water droplet surface, I_{local} denotes the color of water droplet calculated by reflection-illumination, k_{rg} denotes the value to control the reflectivity of water droplet surface. And $I_{reflected}, k_{tg}, I_{transmitted}$ denotes the color of reflected ray, the value to control the scattering of light transmitted through the water droplet, the degree of refracted ray, respectively.

In a real world, light rays change its direction by refraction on entering or leaving the object. The light becomes dispersed, so that different regions

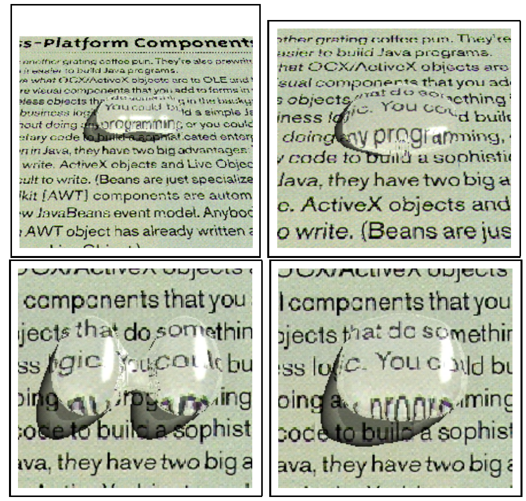

Figure 18: Ray tracing version of Fig.17.

of space contain greater concentrations of light than others. Any object placed in this region of 'bent' light reflects this variation of intensity producing so-called caustics on its surface. We adopt the backward method to make the shadow. For the every point on the metaplate, we find the intersection points between the points on the metaplate and a light source. And then there are two intersection points(P, P') and the distance between two points becomes less than the bound(0.7 in experiment), the color of point on the metaplate is decreased.

In addition, when light rays pass through the water droplet in the real world, the intersection of lower part between the water droplet and the metaplate is highlighted. We simulate this phenomenon by taking the large value of highlight in Phong shading. Fig.19 shows a rendered scene with 512×512 resolution. It consists of 15 water droplets with radius 1 in a vertical metaplate.

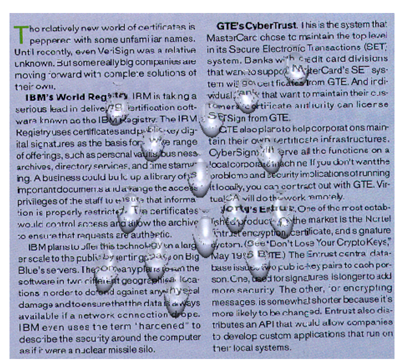

Figure 19: Droplets on the vertical metaplate. The point light source is located at the (0,20,20)

5 CONCLUSIONS

In this paper, we showed that this gravitational metaball is a very nice model to generate the biased water droplet reflecting the degree of the gravitation force and friction force. However, we have to solve another problems when we want an animation of water droplets. If two different droplets merged into one in an inclined plane, then that merged droplet would run into down direction leaving a random trace due to an interfacial tension. We do not know how the droplet runs down or sticks to a surface. Also we have to study how the interfacial tension deforms a little “wide” droplet. Let us summarize the contribution of our papers and future works.

- Our model can simulate a droplet that sticks to a surface and could be deformed considering the gravity and friction between droplet and surface.
- By what dynamics two droplets are merged into one merged droplet ? And when a merged droplet which sticks to a vertical surface
- After running a merged droplet down to the bottom, how the resulting water marks are left by the droplet generated ? For this, we have to study more about the interfacial tension dynamics of water on a specific surface.
- In our experiment, ray-tracing for our metaball needs lots of computation, since isosurface is defined implicitly. How can we detect fast the boundary of isosurface in our model ?

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