Differential Geometric Concepts and Methods in CAGD

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Abstract

We report on new geometric results serving as a base for new methods in geometric modeling with free form surfaces. Those methods have all been implemented in the geometric modeling system **Praxiteles** of the MIT Design Laboratory.

The results concern two areas:

- Blendsurfaces,
- Distance Computations, i. e. computing nearest points.

First we report on practical criteria for second (and higher) order surface contact of blend surfaces. Those criteria use a minimal number of one - dimensional contact and curvature conditions (normal curvatures) to determine at contacts points all curvature values ore more generally higher order surface contact. Those criteria apply to the case of contact of two surfaces along a contact curve and to the case of contact of two surfaces in a single point.The single point contact result is applied to compute curvatures of certain surface patches with a degenerate parametrization hence solving an old problem. After that we explain how methods of local and global differential geometry can be combined to develop efficient algorithms for distance computations e.g. to trace a surface curve containing points nearest to a point moving on a space curve.

Those methods use:

- Tensorial differential equations
- Approximation of the inverse of the surface normal map
- Elimination of search areas
- Topological vector field index methods
- Global differential geometric methods based on the cut locus concept

Finally we sketch applications of those results in the area of distance computations. Those applications treat shape quality control (i.e. computing the shape deviation of a manufactured material piece from the prescribed design surface). The applications discuss also surface intersections and methods to compute equidistantial curves and the medial axis, the latter being used for shape classification and mesh generation.