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Active Integral Vibration Control of Elastic Bodies

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Abstract

This paper deals with the design and analysis of active integral vibration control of elastic beam using piezoactuator. The concept of integral control means that multiple distributed piezoapatches are controlled using one integral feedback signal. This concept has been verified by simulations. However, it was detected that the dual concept of integral sensing, i.e. to integrate the signals from multiple piezopatches as sensors, fails in the case of position feedback due to the mutual interaction of piezopatches as sensors and actuators.

Keywords: piezoactuator, beam, vibration control, integral control, MIMO, SISO

1. Introduction

Piezopatches are attractive actuators for vibration and position control of elastic bodies. The problem is that their straightforward usage leads to the necessity of synthesis of Multiple Input Multiple Output (MIMO) control systems with tens, hundreds or thousands of inputs and outputs. Such synthesis is not an easy task. Therefore the results describe in [1, 2] are very attractive. Instead of large MIMO systems the set of multiple piezopatches can be controlled just as Single Input Single Output (SISO) system by a single control input. The single input and single output are achieved by integration of multiple signals, thus it is spoken about integral control. This paper describes the simulation study of application of integral control to the active vibration control of elastic beams using multiple piezopatches.

2. Integral control

The concept of integral control originates in the paper [1] that investigates the shape of electrodes of composite piezoelectric beams and their usage for vibration control. Assuming the validity of Euler-Bernoulli beam theory it is proven that using suitable shape of electrodes (Fig. 1 it is possible to achieve the same bending deformation as the loading by external force causes. It is possible to achieve the same beam deformation however with opposite sign as the deformation by external load. If a beam is loaded by an external force and simultaneously the inner deformation by piezoelectric phenomenon are equal but opposite, it is possible to eliminate the beam deflection. Further it is described that sensor with the same shape of electrode as the actuator is collocated with the actuator and thus the stability of the control is guaranteed. This can be used for perfect compensation of external load (Fig. 2) and vibration control. This concept has been further developed and applied in [2].

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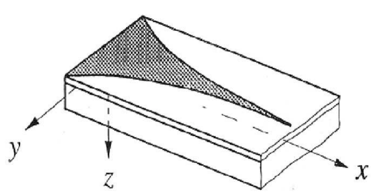

Fig. 1. Suitable shape of electrodes on piezoelectric layers

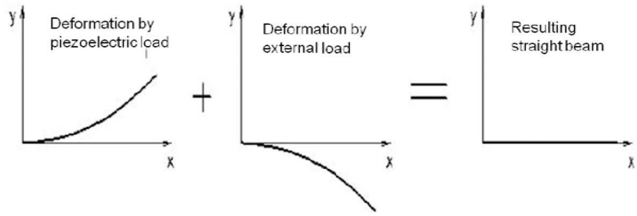

Fig. 2. Exact piezoalectric compensation of external load

In order to apply this approach in practice it is necessary to replace the continuously shaped electrodes of actuator and sensor by a sequence of piezoelectric patches. The shape of electrodes can be replaced by a shaped function represented by an input weighting function $[{}^aU_1, {}^aU_2, \ldots, {}^aU_n] = f(v)$. This function transforms one input into the system using the weights into multiple inputs to the particular actuators (Fig. 3). This function is a discretization of the shape of electrodes in [1]. The variable deformation effect along the beam length is achieved by variable voltage applied on the piezoactuators. Similarly the variable influence of beam deformation on the resulting deflection of the beam is represented by the output weighting function $y = g({}^sU_1, {}^sU_2, \ldots, {}^sU_n)$. It transforms the multiple outputs from particular sensors into one output. The complete concept of integral control of a beam with sensors and actuators loaded by external force F and simple control loop with weighting functions is in Fig. 3. The key choice of weighting functions is done according to [1, 2] by the static deformation of the beam loaded by the external force F.

According to [3] the force effect of piezoactuator on the beam can be using the validity of Euler-Bernoulli beam theory replaced by a couple of moments located on the ends of the

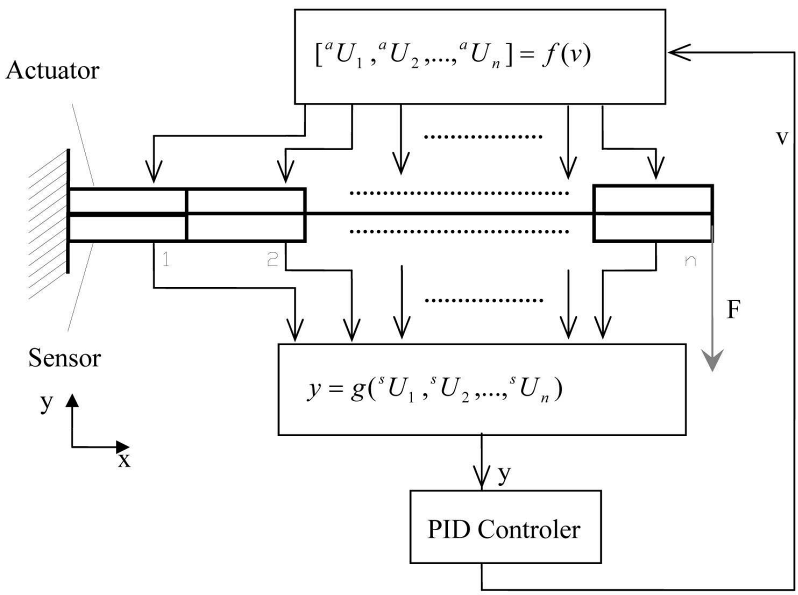

Fig. 3. Concept of integral control of a beam with piezoactuators and piezosensors

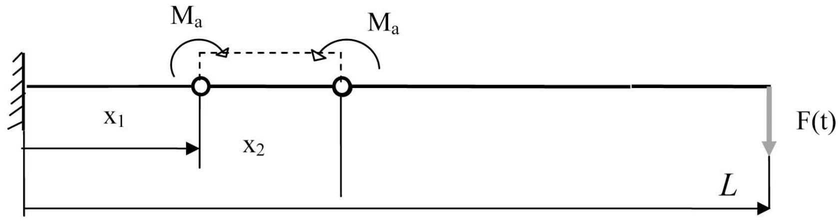

Fig. 4. Replacement of the force effect of piezoactuator on the beam by moments located on the ends of the actuator

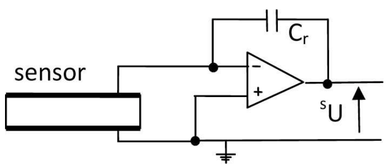

Fig. 5. Charge amplifier

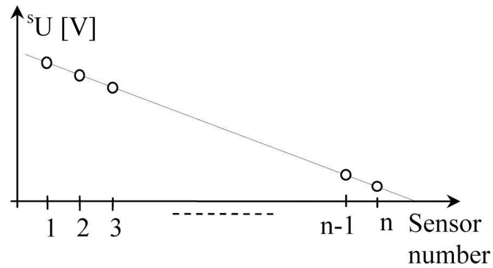

Fig. 6. Course of voltage on sensors

actuator (Fig. 4) with the size

$$M_a = e_{31} \cdot {}^{a}U \cdot b \cdot \frac{h}{2} \tag{1}$$

where e_{31} is the piezoelectric coefficient, b is the width and h the height of the beam, ${}^{a}U$ is the applied voltage on the actuator. The voltage generated by charge amplifier (Fig. 5) can be expressed according to [3] as

$${}^{s}U = \frac{h \cdot e_{31} \cdot b}{2 \cdot C_r}[w'(x_2) - w'(x_1)]. \tag{2}$$

Here $w'(x)$ is the course of rotation of deformed beam and x_1, x_2 are the coordinates of terminal points of the sensor. The consequence of the equation (2) is that the voltage on the sensor is a linear function of rotation difference and thus of beam curvature and hence

$$w'' = \frac{M_y}{I_y E} = -\frac{F \cdot (L - x)}{I_y E}, \tag{3}$$

where I_y is the quadratic moment of cross-section with respect to the y axis, E_n is the elasticity module and L the length of the beam. The equations (2) and (3) describes the linear course of voltages on sensors loaded by unite force on the beam end (Fig. 6).

If the bending deformation of the beam is to be minimum, then the actuators must create such bending moment along the beam that its sum with the moment course caused by the external force F is minimum (Fig. 6). According to the linear course of moment of the force F the necessary electrical voltage on actuators is also linear.

Based on the previous considerations the choice of weighting functions is

$$\{{}^{a}U\} = f(v) = [L - X_1, L - X_2, \ldots, L - X_{n-1}, L - X_n]^T \cdot v = \{V\} \cdot v, \tag{4}$$

$$y = g(\{{}^{s}U\}) = [L - X_1, L - X_2, \ldots, L - X_{n-1}, L - X_n] \cdot \{{}^{s}U\} = \sum_{i=1}^{n} V_i \cdot {}^{s}U_i. \tag{5}$$

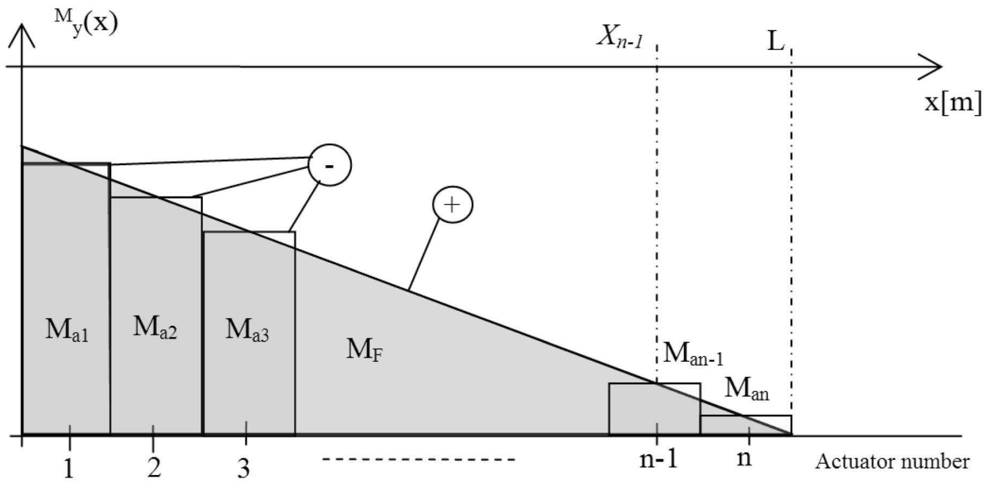

Fig. 7. Course of moment along the beam length caused by n piezoactuators for the best elimination of the moment caused by external force F

where L is the beam length and X_i is the position of the center of i-th actuator in the x direction. Using the weighting functions the MIMO system is reduced to SISO system that can be controlled by simple regulator. Another advantage is the possibility of commanding of all actuators by the usage of just single amplifier together with electrical circuit for voltage distribution on particular actuators.

3. Simulation model

In order to verify the concept of integral control of elastic bodies several simulation models based on FEM were created. The simulation models were derived based on the results of [4] where the way of modeling of structures with active piezoelectric elements is described. An example of application of this modeling approach is in [5] where the model of elastic membrane with piezoelectric actuators is investigated.

Two models were created. The first one is a simplified model developed under the assumption of validity of Euler-Bernoulli beam theory. The beam was modeled using FEM elements of the type BEAM in the ANSYS package. The mass and stiffness matrices were exported into MATLAB and transformed into state space description. The piezoelectric actuators and sensors were introduced into the model using the equations (2)–(3). The mass and stiffness of actuators were neglected. The simulation experiments using this model were successful and the influence of control on vibration damping was quite good. The resulting amplitude — frequency characteristics with the control on/off are in Fig. 8.

The second model was created using spatial FEM elements of the type SOLID45 for the beam and spatial FEM elements of the type SOLID5 for piezoelectric patches in the ANSYS package according to [5] in order to consider the mass and stiffness of piezoelectric patches. This model was again exported from ANSYS to MATLAB using special program Mor4ansys [6].

The linear dynamic FEM model for systems with piezoelectric elements used in ANSYS can be written as

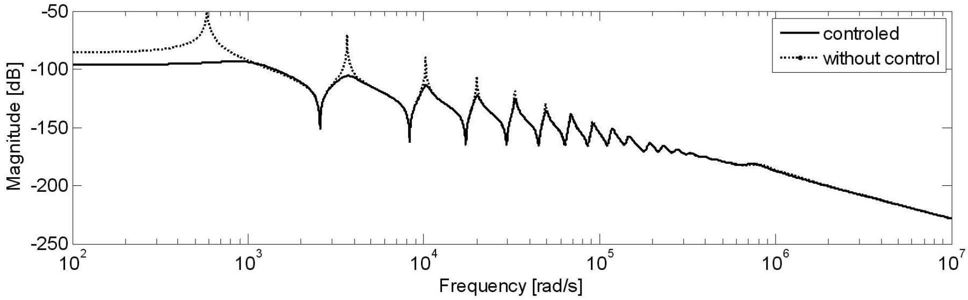

Fig. 8. Amplitude — frequency characteristics of simplified model with the control on/off

$$\begin{bmatrix} [M_{uu}] & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \{\ddot{\delta}\} \\ \{\ddot{\phi}\} \end{Bmatrix} + \begin{bmatrix} [B_{uu}] & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \{\dot{\delta}\} \\ \{\dot{\phi}\} \end{Bmatrix} + \begin{bmatrix} [K_{uu}] & [K_{u\phi}] \\ [K_{\phi u}] & [K_{\phi\phi}] \end{bmatrix} \begin{Bmatrix} \{\delta\} \\ \{\phi\} \end{Bmatrix} = \begin{Bmatrix} \{f\} \\ \{g\} \end{Bmatrix}. \tag{6}$$

Here $\{\delta\}$ is the vector of nodal displacements, $\{\phi\}$ is the vector of nodal potentials, $\{f\}$ is the vector of external nodal forces and $\{g\}$ is the vector of nodal charges imported or exported from the system. Further the matrices $[Muu]$, $[Buu]$, $[Kii]$ represent the submatrices of global mass, damping and stiffness matrices.

After splitting the electrical potentials into the free $\{{}^p\dot{\phi}\}$ and forced $\{{}^f\dot{\phi}\}$ ones the dynamic equations of the system can be written as

$$\begin{bmatrix} [M_{uu}] & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \{\ddot{\delta}\} \\ \{{}^p\ddot{\phi}\} \\ \{{}^f\ddot{\phi}\} \end{Bmatrix} + \begin{bmatrix} [B_{uu}] & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \{\dot{\delta}\} \\ \{{}^p\dot{\phi}\} \\ \{{}^f\dot{\phi}\} \end{Bmatrix} + \begin{bmatrix} [K_{uu}] & [{}^pK_{u\phi}] & [{}^fK_{u\phi}] \\ [{}^pK_{\phi u}] & [{}^{pp}K_{\phi\phi}] & [{}^{pf}K_{\phi\phi}] \\ [{}^fK_{\phi u}] & [{}^{fp}K_{\phi\phi}] & [{}^{ff}K_{\phi\phi}] \end{bmatrix} \begin{Bmatrix} \{\delta\} \\ \{{}^p\phi\} \\ \{{}^f\phi\} \end{Bmatrix} = \begin{Bmatrix} \{f\} \\ \{{}^pg\} \\ \{{}^fg\} \end{Bmatrix}. \tag{7}$$

The forced electrical potentials are those which values are given by boundary conditions or controls. The other potentials are considered as free. Because the total number of DOFs is large and the solution of resulting equations of motion is very demanding, the system was transformed into the state space description

$$\begin{matrix} \{\dot{z}\} = [A] \cdot \{z\} + [B] \cdot \{v\} \\ \{y\} = [C] \cdot \{x\} + [D] \cdot \{v\} \end{matrix}, \quad \{z\} = \begin{bmatrix} \{q\} \\ \{\dot{q}\} \end{bmatrix}, \quad \{v\} = \begin{bmatrix} \{f\} \\ \{{}^fg\} \\ \{{}^p\phi\} \end{bmatrix}, \quad \{\hat{y}\} = \begin{bmatrix} \{\delta\} \\ \{{}^f\phi\} \end{bmatrix}. \tag{8}$$

where the modal transformation was used. Only the first n eigen modes were considered. In order to create more accurate reduced order model the quasistatic correction for residual eigenmodes was applied [3]. By this way the system was reduced into $2n$ equations. The details of this derivation are described in [4, 5].

4. Interaction between sensors and actuators

Unfortunately the simulation experiments with the second model differ from the results with the first model. The reason was the interaction between the piezoelectric sensors and actuators.

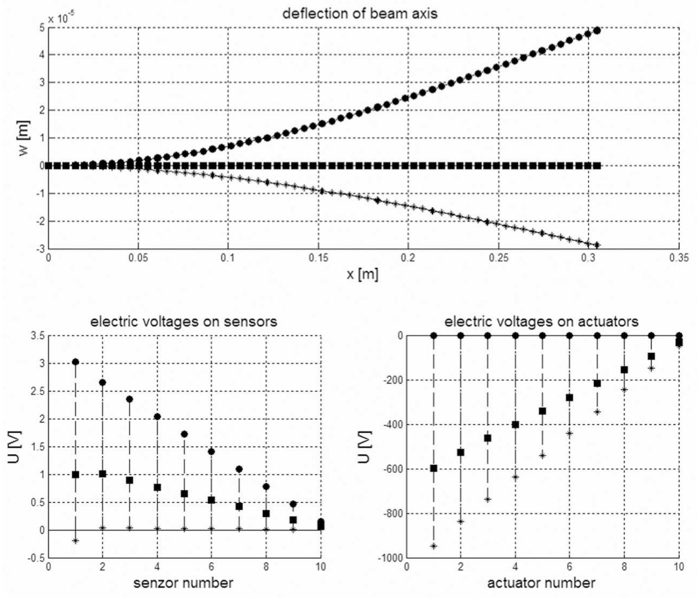

Fig. 9. Course of beam deflection and electrical voltages on sensors and actuators during the gradual increase of input voltage to piezoavtuators on the beam loaded by the force F

It was found out that the output from piezoelectric sensors is directly proportional to the beam deflection only in the case of turned off piezoelectric actuators. It was carried out the following simulation experiment. The beam was loaded by the force F and the input to the integral control of piezoelectric actuators was gradually increased, i.e. the gradually increased voltage was distributed on the particular actuators according to the weighting functions (4). For the certain input voltage on the actuators the beam was undeformed, i.e. the deflection caused by the force F was compensated by the action of piezoactuators, however then the output of piezosensors was nonzero. If the input voltage was further increased, then the output of piezosensors became indeed minimum — almost zero, but in this state the beam is significantly deflected to the opposite side (Fig. 9).

During the design of integral control it was assumed the negligibility of the influence of all mechanical stresses with the exception of stresses in the x-axis direction due to the bending moments both on the beam deflection and on the electrical voltages generated by the piezopatches used as sensor. Exactly this influence on the generated electrical voltages is not negligible and despite the action of piezoactuators removes the beam deflection the sensors were exposed to the mechanical stresses and correctly generated the electrical voltages. Thus the simplified assumption of validity of Euler-Bernoulli beam theory was wrong. This means that the simplified modeling according to Fig. 4 and Fig. 10a is not valid and the more complex modeling of interaction between the piezoelectric patch and the substrate material according to Fig. 10b must be applied.

5. Modified control concept

According to the simulation results in the previous section the control concept from the Fig. 3 has been changed to the Fig. 11. The piezoelectric sensors were replaced by the absolute sen-

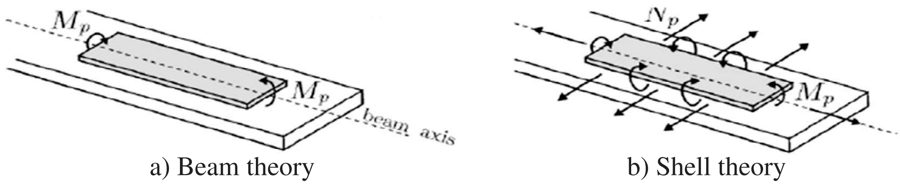

Fig. 10. Two ways of force interaction between piezoelectric patch and the substrate material (according to[3])

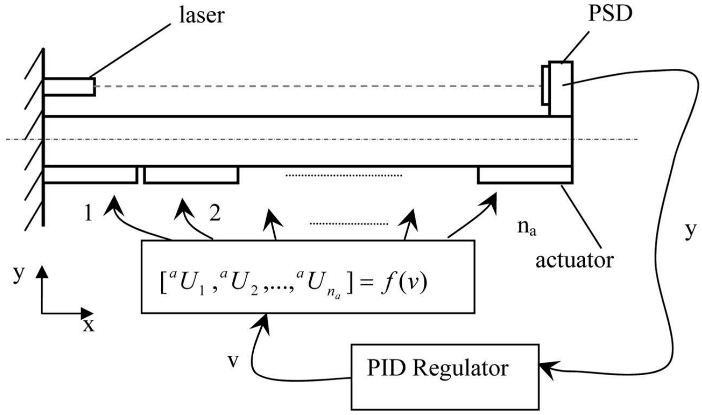

Fig. 11. New concept of integral control of a beam with piezoactuators and piezosensors replaced by PSD sensors

sor of position of beam end realized for example by laser beam and PSD (position sensitive detector) sensor that detects the position of center of incident laser beam (Fig. 11).

It is obvious that the output weighting is not used and the input of PID regulator is directly the output from PSD sensor.

6. Simulation experiments

It was assembled a simulation model of a beam with the length $L = 0.345$ [m], square cross-section $h = b = 0.01$ [m]. On the beam 10 piezoactuators of the length $La = 0.03$ [m] and the height $hp = 0.000\,4$ [m] are mounted. The actuators were modeled with the gaps $Lg = 0.003$ [m] (Fig. 12).

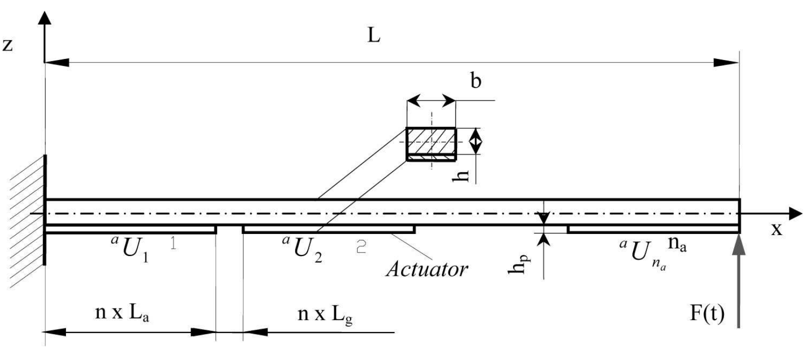

Fig. 12. Geometry of simulated beam with piezoactuators

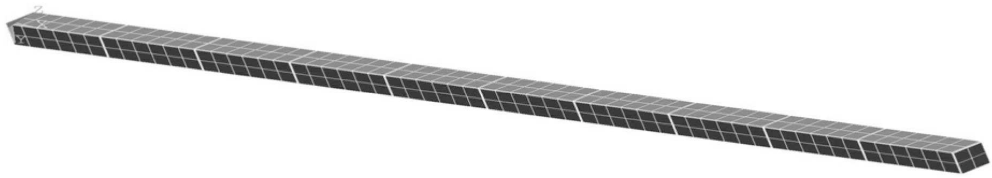

Fig. 13. FEM model of the beam in ANSYS

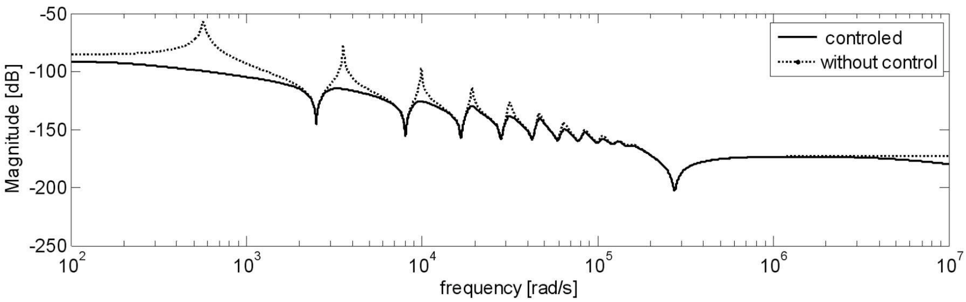

Fig. 14. Amplitude — frequency characteristics with the PID control on/off

The considered material of the beam was steel and of the piezoactuators the material PIC181 from the catalogue of Physik Instrumente. The density of FEM mesh could not be high because of transfer to MATLAB. The created FEM model (Fig. 13) had 3210 DOFs and the reduced order model included the first 30 eigenmodes.

The amplitude – frequency characteristics with the PID control on/off is in Fig. 14. The first three resonancies are almost completely suppressed and the positive influence on the next eigenfrequencies is visible.

The response of position of point at the beam end to the unit step of the force F is in the next Fig. 15. This demonstrates that this control not only suppressed the vibrations but also eliminates the deflection of the beam due to the loading by static force F. However it is necessary to state this control action required to apply to the actuators the peak voltage of about 600 [V] that would impose quite high requirements on the amplifier for real implementation. This limits the maximum applicable force that could be compensated by the described integral control.

Finally the response of position of the same point of the beam excited by harmonic force with the amplitude 0.1 [N] at the first eigenfrequency is in Fig. 16. The beam was at first excited with the controller off and after the transient response the control was switched on. It is visible that the active integral control reduced the resulting vibration significantly.

7. Conclusion

The proposed integral control has demonstrated excellent regulation properties and the broadband suppression of resonance peaks. Moreover this control enables to compensate the static deflection of the beam loaded by external force.

An experimental verification of this simulation study is being prepared. Especially critical issue is the development of suitable and powerful electrical circuit for voltage distribution from

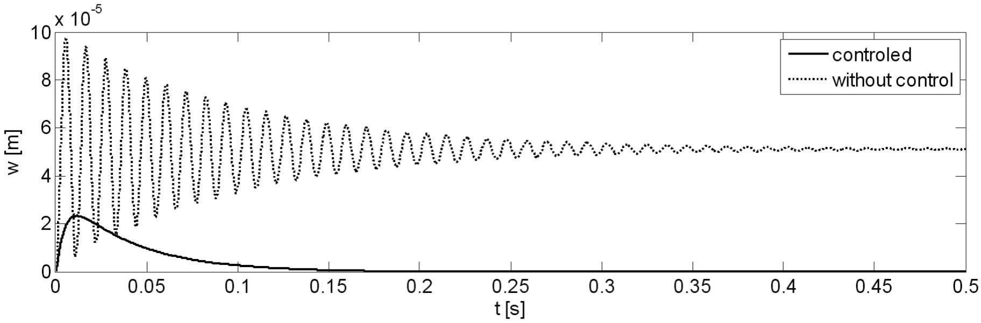

Fig. 15. Response to unit step with the PID control on/off

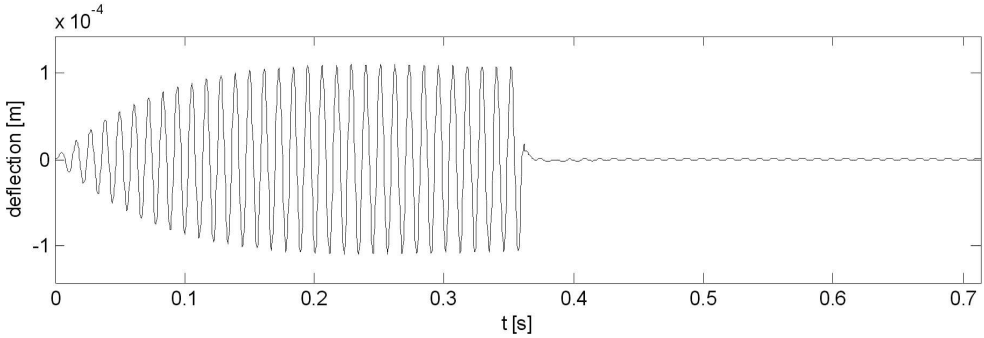

Fig. 16. Response of position of the end point of the beam excited by harmonic force before and after closing the control

one amplifier on particular actuators. In previous implementations [2] multiple amplifiers were used that is quite demanding.

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