Visualization of Smoothed Particle Hydrodynamics

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Abstract: Today smoothed particle hydrodynamics (SPH) play an important role in the computation of gas dynamical processes appearing in astrophysics and other fields. Nevertheless the visualization techniques for this particle based model are still rudimentary. In order to visualize the density and pressure appearing in the simulation correctly we investigated the simulation in more detail. By this investigation we show that a splatting based visualization technique displays the volume density and pressure used in the simulation exactly. Furthermore we conclude that in our situation splatting is the only reasonable technique and is superior to other techniques such as raycasting. Finally, our visualization method enables the scientist to control the result of the simulation especially with few particles and gives high quality rendering at every state of the research process.

Introduction

The visualization of concentrations or density values is one of the main tasks of volume rendering systems. Various fields of natur sciences produce three dimensional data which need to be visualized. The data produced can have various structures. Besides Cartesian grids where all the cells are identical

*Supported by the Deutsche Forschungsgemeinschaft, SFB 382

axis-aligned cubes there exist other structered data on regular, rectilinear, structured and unstructured grids. Finally there exist data where the values are assigned to scattered points in the volume (cf. [Elv92]). Scattered data are produced from measurements in, e.g., geophysics and chemistry. Also SPH simulations produce data with no underlying grid since it is a particle based model.

SPH Simulation

Gas dynamical processes play an important role in the evolution of astrophysical systems. In order to verify model assumptions on an astrophysical phenomen the scientist are often using numerical calculations. SPH was used in the last years with great success in the field of astrophysics (cf., e.g., [GM77] and [FMH+94]).
One reason for that success was the fact that finite-difference code on a Cartesian grid was not able to handle the necessary resolution. This was mainly due to memory and cpu limitation. SPH offered a new method to overcome these problems and was introduced by Gingold-Monaghan [GM77]. In SPH the fluid is modeled as a collection of fluid elements and is represented as particles with interactions. The dynamical equations are obtained from the Lagrangian formulation of the hydrodynamic conservation laws (cf. [Mon82], [FMH+94] and [HK94]).
We briefly present the way how the discretization of the thermodynamical equations works. This step is called smoothing procedure.
Suppose

$$f : D \subset \mathbb{R}^3 \to \mathbb{R}^n$$

is given. We approximate f by the kernel estimate

$$< f(r) >= f_h(r) = \int_D W(r, r', h) f(r')\, dr', \tag{1}$$

where W is the so called *smoothing kernel* and h the *smoothing length.* The smoothing kernel has to be normalized, i.e. the volume integral of W over its support equals 1

$$\int W(r, r', h)\, dr' = 1.$$

Furthermore W has to be choosen in a proper way such that

$$f_h(r) \stackrel{h \to 0}{\longrightarrow} f(r).$$

All this can be performed by kernels which are of the form

$$W(r, r', h) = W(r - r', h)$$

and because this greatly simplifies the discussion most of the authors use this representation.

If we consider the values of f only at a finite number of points, distributed with the number density $n(r) = \sum_{i=1}^{N} \delta(r - r_i)$, we can multiply the integrant in Equation 1 by $n(r')/ < n(r') >$ and obtain the following discrete approximation

$$f(r) \approx < f(r) > \approx \sum_i \frac{f(r_i)}{< n(r_i) >} W(r - r_i, h) \quad (2)$$

where $< n(r) > = \sum_i W(r - r_i)$.
In the special case of mass density ρ necessary in gas dynamical simulations we have to replace the function f by ρ, $< n(r_i) >$ by $\rho(r_i)/m_i$ and obtain the following final approximation

$$\rho(r) = \sum_i m_i W(r - r_i, h)$$

where m_i denotes the mass of the pseudo-particle i.
We can interpret this final equation in such a way that the mass of the particles is distributed in space according to the values of the kernel.
All interesting thermodynamical quantities can be approximated by the same procedure with the same kernel W. Since in most applications the kernel is choosen with compact support the smoothing length also determines the interaction radius of the particles.
Indeed we have to deal with scattered data for our visualization but we have to keep in mind that the discrete data are coordinates in a vector space of scalar valued functions.

3. Volume Function and its Visualization

In the field of volume rendering most works implicitly suppose that the volume data is already given in a discrete form. Indeed numerical computations and measurements yield this kind of data. Nevertheless often we have the discrete datasets given either as sample points of continuous volume functions or as coordinates in finite dimensional function spaces and the linear combination of the basis functions forms the continuous function.
From this observation we can assume that the volume data always correspond to a volume function f given by

$$f : D \subset \mathbb{R}^3 \rightarrow \mathbb{R}^n$$

From this point of view we can now apply the same smoothing procedure to the volume function as done in the previous section for the density function in the SPH simulation. So we are able to approximate the volume function in the first step with smoothing kernel $W(r - r', h)$ as before and obtain

$$< f(r) > = f_h(r) = \int_D W(r - r', h) f(r')\, dr'.$$

We have

$$f_h(r) \stackrel{h \to 0}{\longrightarrow} f(r)$$

and now discretize f for given sample points r_i. We obtain the following approximation

$$< f(r) > \approx \sum_i \frac{f(r_i)}{< n(r_i) >} W(r - r_i, h)$$

with $n(r) = \sum_i \delta(r - r_i)$.
To model the process of rendering the volume function, we assume the following. Each infinitesimal volume element yields a contribution to a plane. Assuming that these contributions are additive and only along one coordinate in a suitable coordinate system we formulate the rendering step in form of an integration along the z-coordinate, i.e.,

$$\begin{aligned} f_{2D}(x, y) &= \int_{\mathbb{R}} \sum_i \frac{f(r_i)}{< n(r_i) >} W(r - r_i, h)\, dz \\ &= \sum_i \frac{f(r_i)}{< n(r_i) >} \int_{\mathbb{R}} W(r - r_i, h)\, dz \end{aligned}$$

with $r = (x, y, z)$. Note that a similiar approach was done by Westover in [Wes90] in his paper on splatting.
In the special situation of SPH we have to render the density function, resp. the pressure function. Applying our model to this function we have to evaluate the picture function

$$\rho_{2D}(x, y) = \sum_i m_i \int_{\mathbb{R}} W(r - r_i, h)\, dz$$

with kernel W used in the simulation.
In the situation of identical masses for each particle the picture function ρ_{2D} is of the simple form

$$\rho_{2D}(x, y) = \sum_i \int_{\mathbb{R}} W(r - r_i, h)\, dz. \tag{3}$$

The important fact is that our picture function uses the same smoothing kernel and the same smoothing length as used in the simulation. Thus we picture the density function used in the simulation exactly. Since our rendering model coincides with the simulation model the display of ρ_{2D} will give insight in the data produced by the SPH simulation without any artefacts.
Performing the integration and summation will now be the task of the rendering algorithm. The scalar values of ρ_{2D} will be coded by the intensity of a RGB color.

4. Volume Rendering

According to Westover [Wes90] volume rendering is the direct display of data sampled in three dimensions and there are two principle approaches to this: backward mapping algorithms (raycasting) that map the image plane onto the data by shooting rays from each pixel in the scene (cf. [Lev88], [Lev90], [UK88]), and forward mapping algorithms (splatting) that map the data onto the image plane (cf. [Wes90] and [LH91]).
With respect to our picture function ρ_{2D} this means that we have either to compute the value of $\rho_{2D}(x_j, y_j)$ for each pixel (x_j, y_j) or to compute first the contribution of each datapoint by evaluating the function

$$\rho_{2D,i}(x, y) = \int_{\mathbb{R}} W(r - r_i, h)\, dz.$$

This second method is the so-called splatting method introduced by Westover [Wes90].
Since $\rho_{2D,i}(x, y)$ does not depend on the on the z-coordinate of the sample i we only have to evaluate once the footprint function

$$foot(x, y) = \int_{\mathbb{R}} W((x, y, z), h)\, dz$$

and obtain

$$\rho_{2D,i}(x, y) = foot(x - x_i, y - y_i).$$

The advantage of Westover's method is that the integration has to be done only once (cf. [Wes90]). Another important advantage of splatting is that we do not need any kind of structured grid in our datasets in order to trace the rays in the three dimensional scene.
Since in our situation the coordinates of the data samples are scattered we have to compute first the values on a structered grid in order to apply ray-casting algorithms.
Comparing raycasting with splatting we can use the same arguments as in the comparison of grid-based simulations of hydrodynamics with the SPH simulation. Since finite-difference methods are not able to treat the necessary resolution, raycasting will not be able to display the data with sufficient resolution.
Therefore the simplest and only realizable method to display equation (3) is splatting.

5. Implementational Aspects

To allow perspective viewing of the volume data and to be able to render hybrid structures composed out of volume and surface data we used a polygonal based splatting technique.

This means that we performed the integration of the footprint function in Cartesian coordinates once and approximated this result by a planar mesh with different transparency values at the vertices.
To every coordinate value of the samples we then translated this planar mesh with normal orthogonal to the viewing plane. The summation was then performed using α-blending.
In the situation with additional surface data we used the z-buffer algorithm to eliminate hidden surfaces. In this case we had to sort the sample points with respect to the distance to the view plane in order to obtain correct blending results.

6. Simulation Data

The simulation data have been supplied by Peter Kroll, Theoretische Astrophysik Tübingen. The group of Hanns Ruder investigated the model assumption that Be star disks are formed by ejection of stellar matter from a point source at the equator rotating at critical velocity (cf. [KHRR94]).
For smoothing kernel they used a spline given by

$$W(q) = \begin{cases} \frac{8}{\pi h^3}(1 - 6q^2 + 6q^3) & : \quad 0 \leq q < \frac{1}{2} \\ \frac{16}{\pi h^3}(1 - q)^3 & : \quad \frac{1}{2} \leq q < 1 \\ 0 & : \quad 1 \leq q \end{cases}$$

where $q = |r|/h$. For the simulation they used a smoothing length h of $5 \cdot 10^{10}$ (see figures).

7. Conclusions

We investigated the SPH simulations and constructed a visualization model which coincides with the model used in SPH. We have been able to show that the same assumptions can be made for the simulation and the visualization. We concluded that in our situation splatting is the only reasonable technique and is superior to other techniques such as raycasting.
This was shown with the same arguments as used in the comparison of finite-difference methods with SPH simulations.
Since our visualization model coincides with the simulation assumptions we produce relyable images which give the scientist new, correct and deeper insight in the simulation data.

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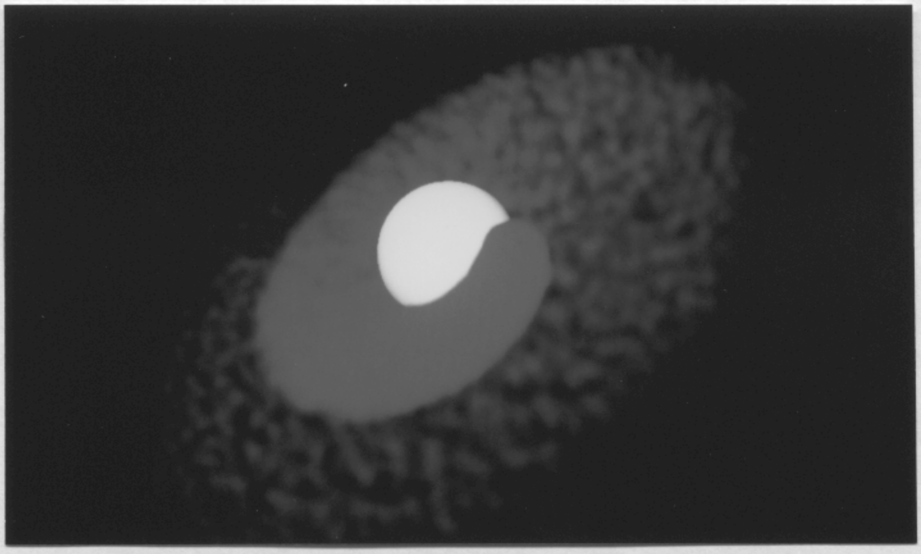
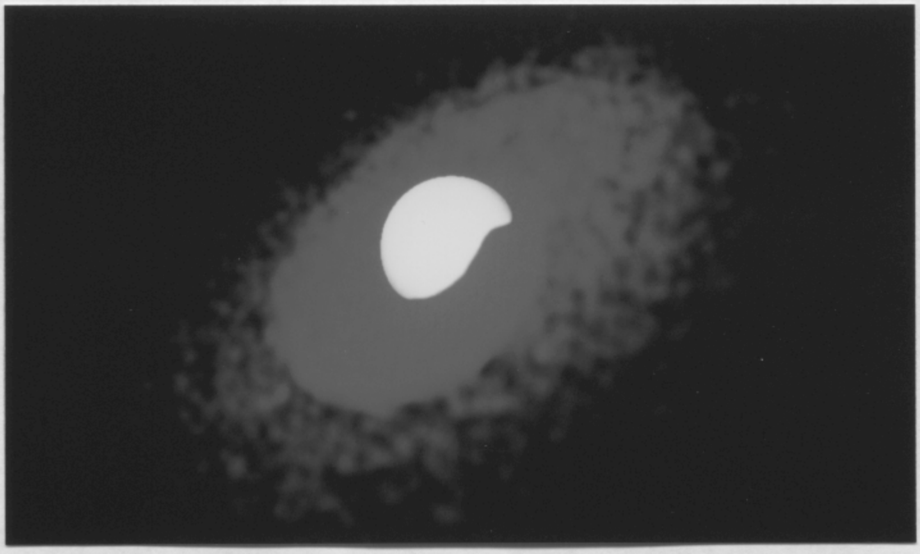
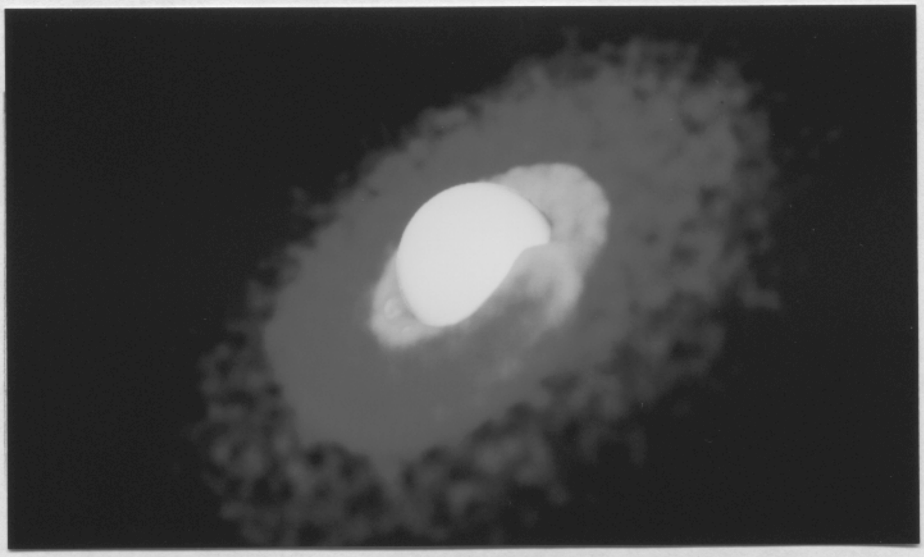


SPH simulation of the formation of a Be star disk. Smoothing length $h \approx 1/30R_{star}$. Time=200. Mass density visualized with smoothing kernel.



Time=500. Mass density visualized with smoothing kernel.



The same situation as above. Mass density visualized with smoothing kernel and color coding.