# Semiregular Grids in 2D and 3D

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**Abstract**

Advantages of grids used in finite element analysis (FEM), which are similar to regular, are described in [5]. We show possibilities of generation and transformation of such grids and we make comparison of them in 2D. We show difficulties arising in 3D.

## 1 Introduction

When the practical problems are solved using partial differential equations, geometry of the problem must be input. FEM divides the area to *elements*. We suppose the simplexes, i.e. triangles in 2D and tetrahedrons in 3D. There are many algorithms which make triangulation of area. These triangulations are, however, general. It means that nodes of such grids have different coincidency. It is unfavourable for computation and pre- and post-processing too. That is the reason to use regular grids or at least the grids, which don't differ very much from the regular ones.

## 2 Regularly structured grids

In the first step we shall analyse 2D problem.
*Regular grid* is the grid, which we obtain from rectangle by regular dividing:

$$x(i_x, i_y) = x_{min} + (x_{max} - x_{min})\frac{i_x}{n_x}, \quad y(i_x, i_y) = y_{min} + (y_{max} - y_{min})\frac{i_y}{n_y}$$

*Regularly structured grid* is the grid obtained from the regular one moving the nodes, while their coincidency isn't concerned.

Set of nodes with one constant index is named *index line* - each node corespondes with two index lines.

Regularly structured grids are generated by described steps:

1. input of *essential nodes* which determine geometry of the problem
2. computing intersections of index lines of essential nodes
3. computing the coordinates of the other nodes in the same index line of the grid
4. completing the rest of nodes of the grid

Detailled description of deformation of the grid is in [1], [3].

This scheme enables a lot of variants, dependent on the choice of the steps 2.–4.
In the step 3. and 4. we use regular interpolation, in the second step we can choose one of the next variants:



**linear:** index lines of essential nodes are polylines, which break in the essential nodes. Therefore intersections of index lines of essential nodes are intersections of the relevant segments.

**index:** $y$-coordinate of nodes on the index line with constant $x$-index are regularly interpolated between the pair of essential nodes or between an essential node and a boundary node. Similarly, $x$-coordinate of nodes on the index line with constant $y$-index are regularly interpolated between pair of essential nodes or between an essential node and a boundary node — this way the intersection of index lines of essential nodes is determined unambiguously.

**local:** modification of coordinates of essential nodes doesn't touch the other nodes.

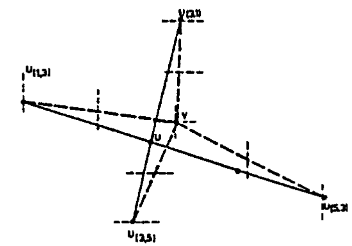

Fig. 1. Difference between linear and index variant of computing the intersection of index lines of essential nodes. $u(1,3)$, $u(5,3)$, $u(3,1)$, $u(3,5)$. U – linear variant, V– index variant.

## 3 Triangulations

The triangulation we obtain from the above described grid dividing tetragons by diagonals. Choice of diagonal in some cells is given by the shape of the solving problem. There, where the diagonal isn't determined, choice of diagonal is independent on the diagonals in the neighbour cells.

Number of nodes doesn't change, number of elements doubles, what enables to describe geometry in more detailled way. On the other hand, it is a new source of "unregularity" of the grid (nodes have various degree of coincidency).
Described variants of grid deformation were tested in two examples: generation of circle – Fig. 2 and generation of two perpendicular lines, with the inclination of $22,5°$ to the coordinate system – Fig.3. The shorter diagonal is chosen.

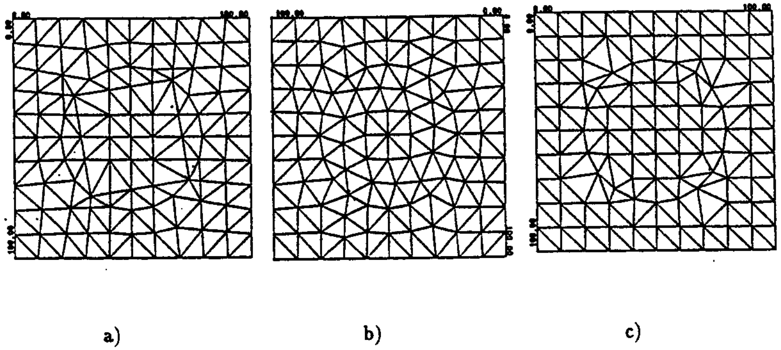


Fig. 2. Test example I. Generation of circle using regularly structured grids. a) linear variant b) index variant c) local variant.

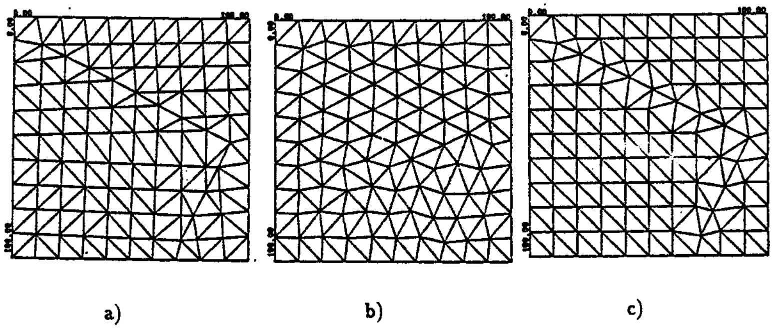


Fig. 3. Test example II. Generation of "turned" perpendicular segments using regularly structured grids. a) linear variant b) index variant c) local variant.



## 4 Characteristics of grids

For the comparison of the grids we use strategy [6] (page 68)

$$\{\min_{j=1,2,3} \sin\theta_{j\Delta} \longrightarrow \max\}_{\Delta\in S}$$

where $\theta_{j\Delta}$ $j = 1,2,3$ are angles of triangle $\Delta$ of grid $S$. Maximum is received in the equilateral triangle so we choose next characteristics

$$h(\Delta) = \sum_{j=1}^{3} |\theta_{j\Delta} - 60|.$$

Fig. 4 resp. 5 show distribution of value $h(\Delta)$ on the whole grid for testing examples I., II. from Fig. 2 resp. Fig. 3. We can see that the best results of the strategy

$$\{h(\Delta) \longrightarrow 0\}_{\Delta\in S}$$

are obtained in both testing examples using the index variant for evaluating the intersections of index lines of essential points.

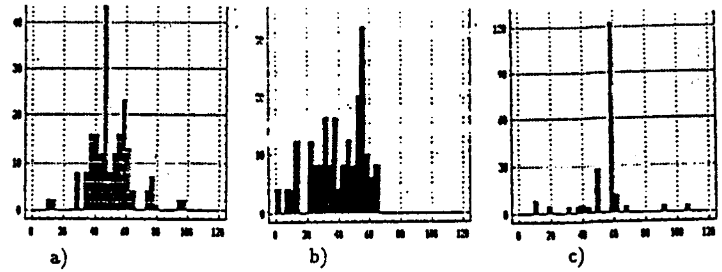


Fig. 4. Histogram of characteristic $h(\Delta)$ for test example I. a) linear b) index c) local variant.

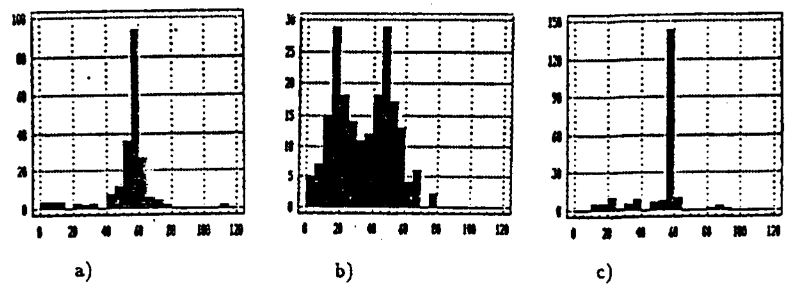


Fig. 5. Histogram of characteristic $h(\Delta)$ for test example II. a) linear b) index c) local variant.

## 5 Situation in 3D

All mentioned above concerning the structured grids is used in 3D too. So we obtain a grid which consists of octaeder elements. Problem arises when we divide octaeders to tetrahedrons. We need decomposition to be *conform*, [4] i.e. the intersection of neighbour tetrahedrons is node or whole edge or the whole face. In 2D all decompositions are automatically conform, in 3D it is not so, see Fig. 6. Moreover, there exist combinations of diagonals on the faces, for which the decomposition to the tetrahedrons does not exist, see Fig.7.

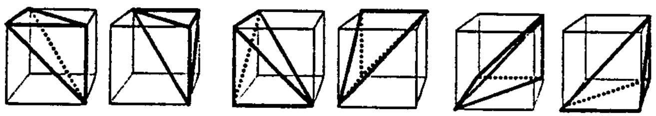

Fig. 6. Example of nonconform decomposition of octaeder to tetrahedrons.

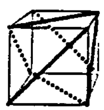

Fig. 7. Example of combination of face diagonals for which decomposition to tetrahedrons does not exists.

All conform decompositions can be classified using the opposite pairs of face diagonals, [2]:

1. all three pairs of diagonals are mutually parallel
2. exactly two pairs of diagonals are mutually parallel
3. exactly one pair of diagonals is parallel
4. no pair of diagonals is parallel

In the first case we obtain two nonisomorphic decompositions (isomorphism we suppose rotation) see Fig. 8a), 8b). Analogically in the fourth case – Fig. 8e), 8f). In the other cases we obtain one nonisomorphic decomposition in each case – Fig. 8c), 8d). All decompositions can be obtained from these six ones using rotation. There are 74 possible decompositions altogether.



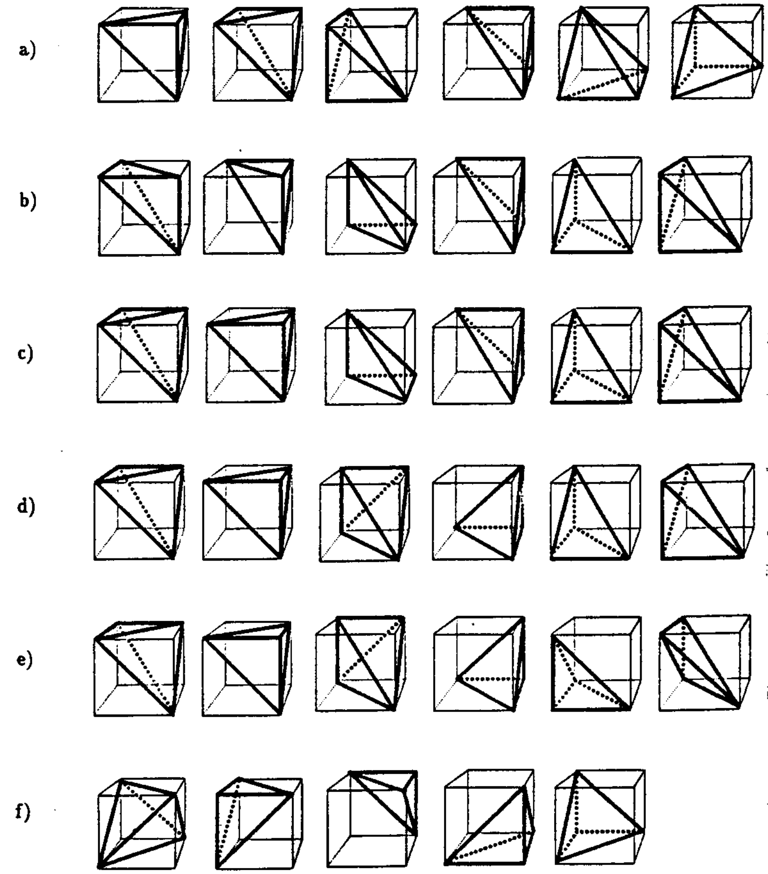


Fig. 8. List of all non-isomorphic conform decompositions of octaeder to tetrahedrons.

Decomposition 8b) consists of six tetrahedrons, three of them are identical, the other are plane symetrical. All decompositions except 8f) consist of six tetrahedrons – decomposition 8f) consists of five tetrahedrons.

## 6 Local refinement

The weakest part of above described analysis is, that regularly structured grids cannot express the situation well, when the part of an explored object is much more complicated then the rest of it. One of the possibilities how to make the grid more dense in given part of area is to input the essential nodes in adequate way, see Fig.9.

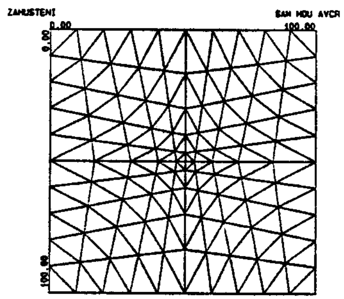

Fig. 9. Increasing of density in one node in structured grid.

Because of regular grid structure this way of grid deformation cannot satisfy us in general. However, we can supplement the primary grid by a set of sub-grids, each of them regularly structured too, see Fig. 10. Resulting grid is nonregular, but we have regularity of grid at any point.

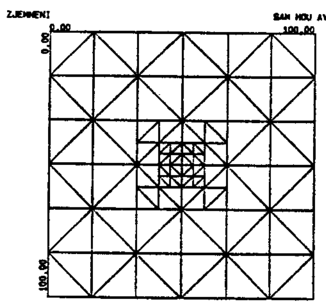

Fig. 10. 2-level local refinement of structured grid.



## 7 Conclusions

We described grids, which preserve usefull properties of regular grids, on the other hand they enable to concern much more difficult situations. We can construct such grids is in three steps:

1. regularly structured grids – geometry is modified but coincidency of grid nodes is conserved
2. triangulation – coincidence of grid nodes is increasing, but reciprocal position of nodes doesn't change
3. local refinement – primary grid is supplemented by set of grids, each of them is regularly structured too.

It was shown that the first step is practically the same in the case 2D and 3D; in the second step in 3D arise the new phenomenon – conformity of decomposition; the third step is common for 2D and 3D and it allows more detailled analysis in any part of explored grid.

## References


[1] Kolcun, A.: *Properties of grid generator for FEM system GEM22*, in Modern mathematical methods in technology, VŠB, Ostrava 1993. (in Slovak)

[2] Kolcun, A.: *Octaeder decompositions to tetrahedrons*, ÚGN AVČR, Ostrava 1993. (in Slovak).

[3] Slíva, J., Kolcun, A., Malík, J.: *User's guide of system GEMIN*, ÚGN AVČR, Ostrava 1993. (in Czech)

[4] Blaheta, R., Dostál, Z.: *Discretization of 3D stress problem — theory and algorithms*, in Acta of Mining Institute Cz. Ac. Sci. HOÚ ČSAV, Ostrava 1987.

[5] Blaheta, R.: *Iterative techniques for local grid refinement in FEM*, in Modern mathematical methods in technology, VŠB, Ostrava 1993. (in Czech)

[6] Shajdurov, V.,V.: *Polygrid methods of finite elements*, Nauka, Moscow 1989. (in Russian)