Vector-valued image restoration with applications to magnetic resonance velocity imaging

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ABSTRACT

The analysis of blood flow patterns and the interaction between salient topological flow features and cardiovascular structure plays an important role in the study of cardiovascular function. Flow velocity images acquired by Magnetic Resonance (MR) velocity imaging are generally subject to noise that are intrinsic to system hardware setup and those specific to patient movement in relation to imaging sequence designs. To improve the accuracy of the quantitative analysis of the evolution of topological flow features, it is essential to restore the original flow fields so that the associated critical points can be more reliably detected. In this study, we propose a total variation based variational method for the restoration of flow vector fields. The method is formulated as a constrained optimisation problem by minimizing the total variation energy of the normalized velocity field subject to a constraint that depends on the noise level. The effectiveness of this restoration method greatly depends on the choice of the regularization parameter in the formulation of the optimisation problem. A new computational algorithm based on the First Order Lagrangian method is proposed, which determines the optimal value of the regularization parameter while solving the minimisation problem. The proposed method has been validated with both simulated flow data and MR velocity maps acquired from patients with sequential MR examination following myocardial infarction.

Keywords

Image Restoration, Variational Method, Vector-Valued Image.

1. INTRODUCTION

The analysis of blood flow patterns and their interaction with cardiovascular structure plays an important role in the study of cardiovascular function [Yan98a]. In order to perform a systematic study and quantitative analysis of the flow patterns, several techniques for extracting important topological features depicted by MR velocity mapping techniques have been developed [Yan98b][Lod00]. The success of these techniques depends greatly on the noise level of the MR images. Flow velocity images acquired with MR velocity-mapping are subject to a certain amount of noise depending on the hardware of the MR system and the nature of the physiological movements of the patient during the imaging process. In practice, the signal-to-noise ratio (SNR) is frequently compromised in order to increase the speed of the image acquisition.

Previous research has shown that in order to achieve a comprehensive and integrated description of flow in health and disease, it is necessary to characterise and model both normal and abnormal flows and their effects [Yan98b]. This permits the establishment of links between blood flow patterns and the localized genesis and development of cardiovascular disease. To accommodate the diversity of flow patterns in relation to morphological and functional changes, the approach of detecting salient topological features prior to analytical analysis of dynamical indices of the fluid has been regarded an important way

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forward. To this end, critical points associated with salient flow features have to be extracted such that the remaining flow field and its geometry and topology can be compactly described. To improve the accuracy of the quantitative analysis of the evolution of topological flow features, it is essential to restore the original flow fields so that the associated critical points can be more reliably detected.

In recent years, restoration and denoising of vector-valued image has drawn a lot of interest due to the general applicability to a wide variety of applications. They include multi-valued image restoration (colour image in particular) [Blo98], regularization of optical flows and tensor fields [Per98][Cou01]. Many of the existing methods are based on functional minimisations via a variational approach [Cha00] [Tsc01a] [Tsc01b], and Total Variation (TV) based method is one of the most commonly used technique.

The selection of TV norm as the function to be minimised is due to the fact that it does not penalise discontinuities, thus edges and other topological features in the image can be preserved. TV-based restoration methods have been demonstrated to be effective and superior to linear filtering methods for scalar images, and several researchers have extended TV norm to vector-valued image [Blo98][Cha01]. In particular, Chan and Shen [Cha00] have used the definition of TV-norm for the restoration of non-flat image features that do not reside on Euclidean space. Examples of commonly encountered non-flat image features include vector distribution from flow images and chromaticity features from colour images. Coulon *et al.* [Cou01] has applied this method to restore the principal diffusion direction (PDD) of Diffusion Tensor Magnetic Resonance (DT-MR) images.

For restoration of the flow field reconstructed from MR flow velocity images, we attempt to restore the direction field (the phase of the velocity). We adopted the formulation proposed by Chan [Cha00] and formulated a new numerical scheme based on the First Order Lagrangian Method [Ber95] to restore the direction field with a much improved convergence behaviour.

2. TV-BASED VARIATIONAL METHOD

The restoration method in this study is formulated as a constrained optimisation problem that restores the original image by minimising the total variation energy of the normalized velocity field subject to a constraint depending on the noise level. It has been shown that the effectiveness of this restoration method greatly depends on the choice of a regularization parameter, which in practice is often conveniently fixed or determined empirically [Cha00]. This renders a major difficulty to the practical application of the technique as the restored image may converge to different results depending on the parameter settings. To avoid having to fix this parameter, we propose to use the First Order Lagrangian Method [Ber95] to derive the optimal value of this parameter while solving the minimisation problem.

Let $\mathbf{u_0}$ denotes the direction field of the noisy velocity images, $\mathbf{u}$ denotes the clean direction field we want to restore and that $\mathbf{u_0} = \mathbf{u} + \mathbf{n}$ where $\mathbf{n}$ denotes the additive noise. Assuming the variance of the noise $\mathbf{n}$ is σ^2 and that its value is known, the restoration of the direction field of a velocity field can be formulated as a constrained optimisation problem by minimizing the following term

$$E^{TV}(u) = \int_{\Omega} e(u;\alpha)$$

subject to the constraint h(u):

$$h(u) = \frac{1}{2}\left(\int_{\Omega}(u-u_0)^2 - |\Omega|\sigma^2\right) = 0$$

where E^{TV} denotes the total variation (TV) energy of the whole image, $e(u;\alpha)$ denotes the energy at pixel α, Ω denotes the image domain and $|\Omega|$ the size of the image domain.

This optimisation problem can be solved by solving the corresponding Tikhonov regularized [Tik97] unconstrained problem:

$$\min_{I} \int_{\Omega}(u-u_0)^2 + \gamma\int_{\Omega} e(u;\alpha)$$

or,

$$\min_{I} \lambda\int_{\Omega}(u-u_0)^2 + \int_{\Omega} e(u;\alpha)$$

where λ is a regularization parameter known as the Lagrange Multiplier of the constrained optimisation problem, γ is a positive regularization parameter and is inversely proportional to λ. This approach can be viewed as a penalty approach for the constrained optimisation problem. Several computational algorithms [Vog96][Li96] have been proposed to solve this unconstrained problem and in most cases, the value of the regularization parameter is conveniently fixed.

To solve the constrained optimisation problem, the First Order Lagrangian Method [Ber95] is used to solve the corresponding Lagrange system. This First Order Lagrangian method finds both the minimal solution and the associated Lagrange Multiplier for the constrained optimisation. The simplest of all Lagrange Methods is given by:

$$u^{n+1} = u^n - \delta_s \nabla_u L(u^n, \lambda^n)$$
$$\lambda^{n+1} = \lambda^n + \delta_s \nabla_\lambda L(u^n, \lambda^n)$$

where δ_s is a positive scalar step-size and L is the associated Lagrangian function

$$L(u;\lambda) = E^{TV}(u) + \lambda \cdot h(u) = \int_\Omega e(u;\alpha) + \frac{\lambda}{2}\int_\Omega \left[(u-u_0)^2 - |\Omega|\sigma^2\right]$$

and λ is the Lagrange Multiplier. Note that at the optimal solution, $L(u; \lambda)$ is same as E^{TV} because at the minimal point, $h(u)$ is equal to zero.

3. NUMERICAL SCHEME FOR DISCRETE IMAGE DOMAIN

The definition of the TV energy for direction (*i.e.* unit vector that lives on unit sphere $\mathbf{S}^2$) and the numerical scheme of our proposed method can be defined as follows. Let v denotes the original velocity vector and u denotes a normalized vector. We map the velocity vector v in R^3 to unit vector u in S^2 by setting $u = v/|v|$. Note that this mapping f: $\mathbf{R}^3 \to \mathbf{S}^2$ is valid for 2D vector as well as we can consider a 2D vector as a 3D vector with the z-component ignored. Therefore, the following formulation applies to both 2D and 3D vector fields.

The strength function $e(u; \alpha)$ at voxel α can be defined as:

$$e(u;\alpha) = \left[\sum_{\beta \in N\alpha} d_l^2(u_\beta, u_\alpha)\right]^{\frac{1}{2}}$$

where N_α denotes the neighbourhood of pixel α, d_l denotes the embedded Euclidean distance in $\mathbf{S}^2$ and is given by:

$$d_l(f,g) = \|f-g\|_{R^3}; \forall f, g \in S^2$$

The total variation (TV) energy of the direction field is then:

$$E^{TV} = \sum_\alpha \left[\sum_{\beta \in N\alpha} d_l^2(u_\beta, u_\alpha)\right]^{\frac{1}{2}}$$

Therefore, the constrained optimisation problem becomes:

$$\min E^{TV} = \sum_\alpha \left[\sum_{\beta \in N\alpha} d_l^2(u_\beta, u_\alpha)\right]^{\frac{1}{2}}$$

subject to the constraint of :

$$\frac{1}{2}\left[\sum_\alpha d_l^2(u_\alpha, u_\alpha^0) - |\Omega|\sigma^2\right] = 0$$

The corresponding Lagrange function, referred to as the constrained TV energy, is:

$$L(u;\lambda) = \sum_\alpha \left[\sum_{\beta \in N\alpha} d_l^2(u_\beta, u_\alpha)\right]^{\frac{1}{2}} + \frac{\lambda}{2}\left[\sum_\alpha d_l^2(u_\alpha, u_\alpha^0) - |\Omega|\sigma^2\right]$$

By computing the gradient of $L(u; \lambda)$ w.r.t. u, the following equation can be derived

$$\frac{\partial L}{\partial u_\alpha} = \sum_{\beta \in N\alpha}\left[\frac{\partial}{\partial u_\alpha} d_l^2(u_\beta, u_\alpha)\right]\left(\frac{1}{2}\left[\frac{1}{e(u;\alpha)} + \frac{1}{e(u;\beta)}\right]\right) + \frac{\lambda}{2}\left(\frac{\partial}{\partial u_\alpha} d_l^2(u_\alpha, u_\alpha^0)\right)$$

In order to compute the gradient of $G(u)$ on $\mathbf{S}^2$, the gradient of $G(u)$ on $\mathbf{R}^3$ is projected onto the plane that is orthogonal to u, *i.e.*,

$$\frac{\partial}{\partial u} G(u) = \Pi_u \, grad_{R^3} G(u)$$

Hence,

$$\frac{\partial}{\partial u_\alpha} d_l^2(u_\beta, u_\alpha) = \Pi_{u_\alpha} grad\left(\|u_\beta - u_\alpha\|^2\right) = -2\,\Pi_{u_\alpha}(u_\beta)$$

And the gradient of $L(u; \lambda)$ is:

$$\frac{\partial L}{\partial u_\alpha} = -\sum_{\beta \in N\alpha} \Pi_{u_\alpha}(u_\beta)\left(\frac{1}{e(u;\alpha)} + \frac{1}{e(u;\beta)}\right) - \lambda\,\Pi_{u_\alpha}(u_\alpha^0)$$

Accordingly, the gradient of $L(u; \lambda)$ w.r.t. λ becomes

$$\nabla_\lambda L(u;\lambda) = \frac{1}{2}\left[\sum_{\alpha \in \Omega} d_l^2(u_\alpha, u_\alpha^0) - |\Omega|\sigma^2\right]$$

Based on the above equations, the discrete form of the First Order Lagrangian Method can be written as follows:

$$u_\alpha^{n+1} = u_\alpha^n + \Delta t \cdot \Pi_{u_\alpha}\left[\sum_{\beta \in N\alpha} wt_\alpha^\beta u_\beta + \lambda^n u_\alpha^0\right]$$

$$\lambda^{n+1} = \lambda^n + \Delta t \cdot \frac{1}{2}\left[\sum_{\alpha \in \Omega} d_l^2(u_\alpha, u_\alpha^0) - |\Omega|\sigma^2\right]$$

where

$$wt_\alpha^\beta = \left(\frac{1}{e(u;\alpha)} + \frac{1}{e(u;\beta)}\right)$$

4. RESULTS AND DISCUSSIONS

In order to provide a detailed analysis of the performance of the proposed method, a synthetic data set simulating flow passing through a cylinder was used. Gaussian noise is then added to the velocity distribution for examining the restoration process.

To analyse the sensitivity of the regularization parameter, the existing variational method with various fixed lambda values was used to restore the noisy synthetic data. The results are shown in Figure 1. It is found that when the value of lambda is set to be the optimal value (which is 2.5 in this case), the restoration result is at its optimum and comparable to that of our newly proposed method. As expected, when the value of lambda is too small, the image is over-smoothed; whereas when the value of lambda is too big, the image is under-smoothed. The result clearly demonstrates the advantage of the proposed method in converging automatically to the optimal solution without explicitly presetting the regularization parameter.

To assess the convergence behaviour of the proposed technique, Figure 2 shows the results with different number of iterations. From these images, it is apparent that the algorithm converges at about 100 iterations. As a reference, the corresponding constrained and unconstrained energy is provided in Figure 3, which clearly indicates that both energy terms converge to the same value. The value of lambda is also plotted against the number of iterations, as shown in Figure 4.

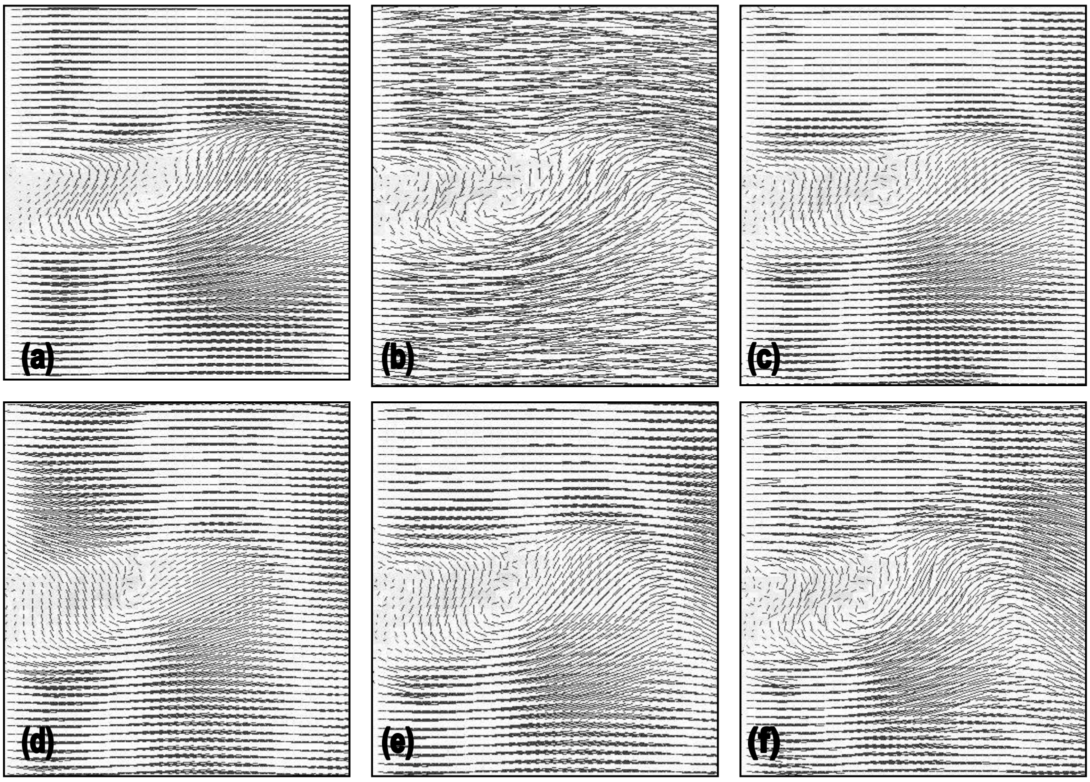

Figure 1 Comparison of TV-based variational method with different values of lambda using the synthetic dataset. (a) Original noise-free image; (b) with added noise; (c-f) images restored with TV-based variational method (c) with lambda determined by First Order Method, (d) with lambda = 0.1, (e) with lambda = 2.5 (optimal value) and (f) with lambda = 20.

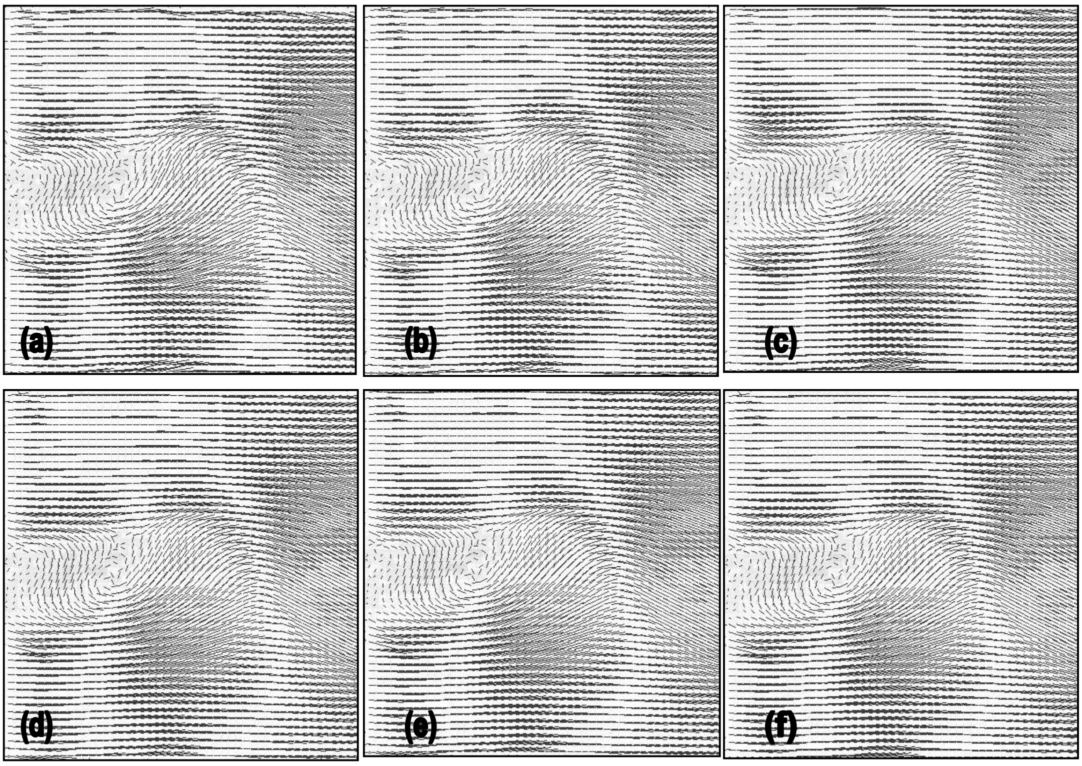

Figure 2 The results of applying the First Order method on the synthetic data. Shown here are the intermediate results at (a) 20 iterations, (b) 50 iterations, (c) 100 iterations, (d) 200 iterations, (e) 300 iterations and (f) 500 iterations.

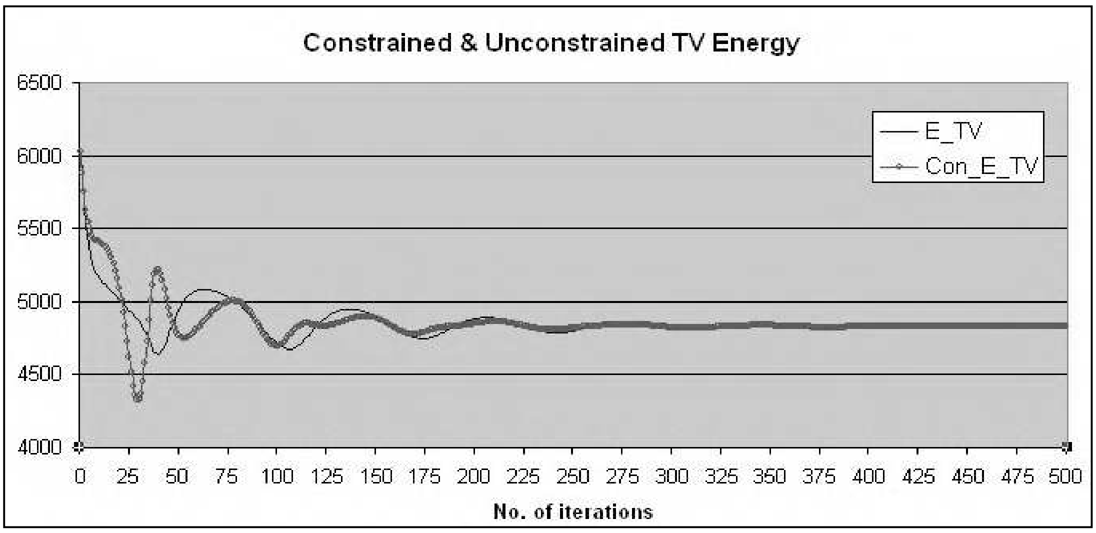

Figure 3 A plot of the constrained and unconstrained energy against no. of iterations for the synthetic dataset.

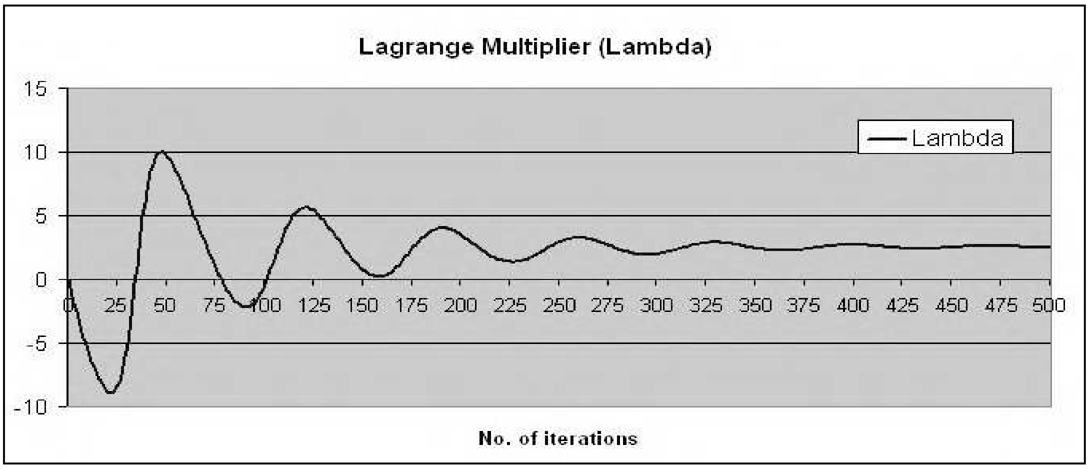

Figure 4 A plot of the value of lambda against the no. of iterations for the synthetic dataset.

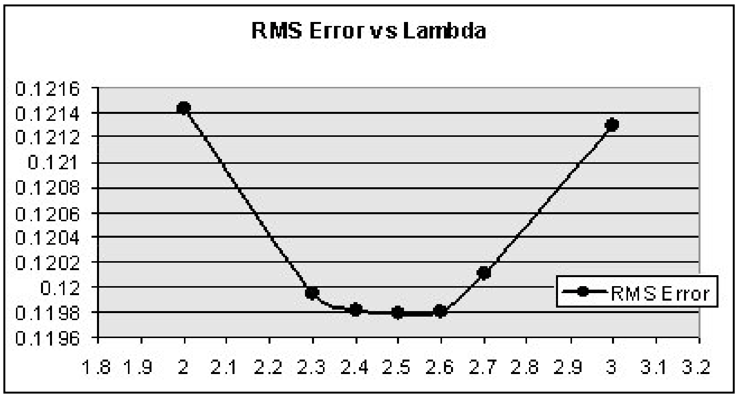

Figure 5 A plot of the RMS error against the value of lambda chosen for the variational method. The minimum point of this curve is at lambda = 2.5, which is the same value as found by the First Order method.

To verify that the value of regularization parameter found by the proposed method is the optimal value, experiments with a range of lambda values were used with RMS error of the result shown in Figure 5. The optimal value of lambda found empirically coincides with the value obtained by the First Order method, justifying the robustness of the proposed technique.

After validating our method using the synthetic dataset, the proposed method was applied to MR flow velocity data acquired from six patients with sequential examination following myocardial infarction. Clinical studies have shown that dilation of left ventricle may occur after myocardial infarction [Bra01] and this topological change of the left ventricle will significantly disturb the flow pattern in the chamber [Yan98b]. In a patient with dilated left ventricle, early diastolic inflow through the mitral valve was directed towards the posterior free wall rather than the apex of the ventricle. This is illustrated in the flow pattern reconstructed from MR velocity data as shown in Figure 6. The corresponding flow pattern reconstructed from MR velocity data restored by the proposed method is illustrated in Figure 7. It is obvious that the flow depicted in the restored flow pattern is more consistent and the restored flow pattern is easier to interpret visually than the unprocessed one. Hence, this restoration method can generate a better description of the flow for visual assessment in clinical flow studies.

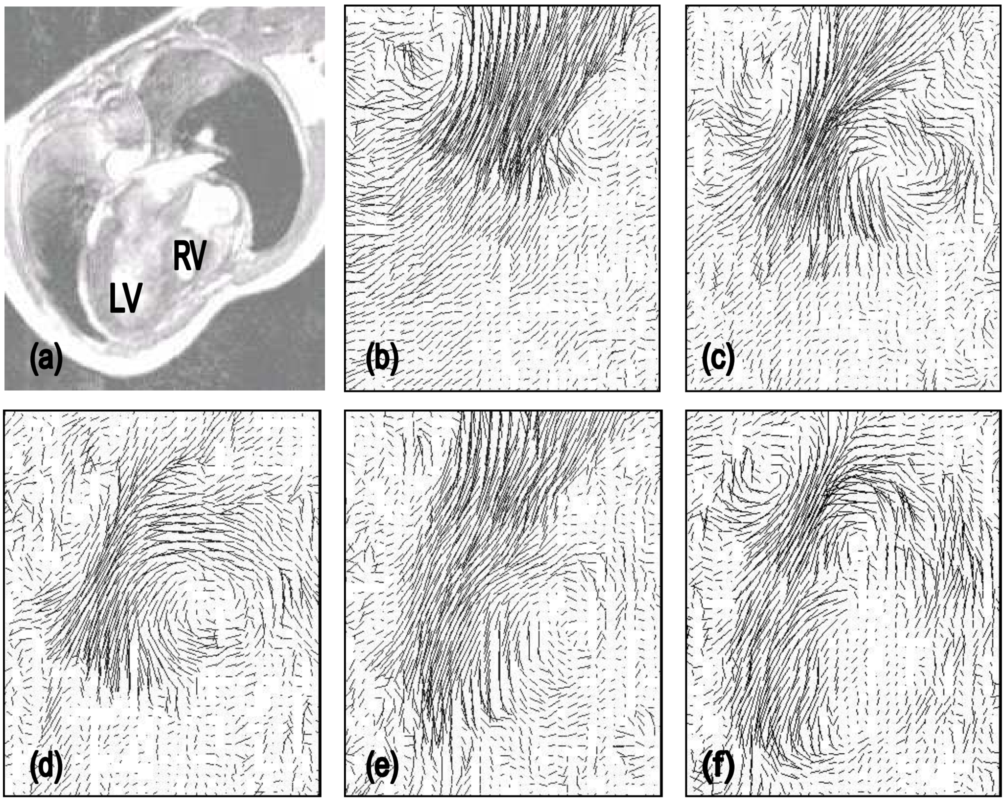

Figure 6 (a) A horizontal long axis MR image showing the left ventricle (LV) and right ventricle (RV) of the heart. (b-f) The flow pattern reconstructed from MR velocity data at different phases of the cardiac cycle.

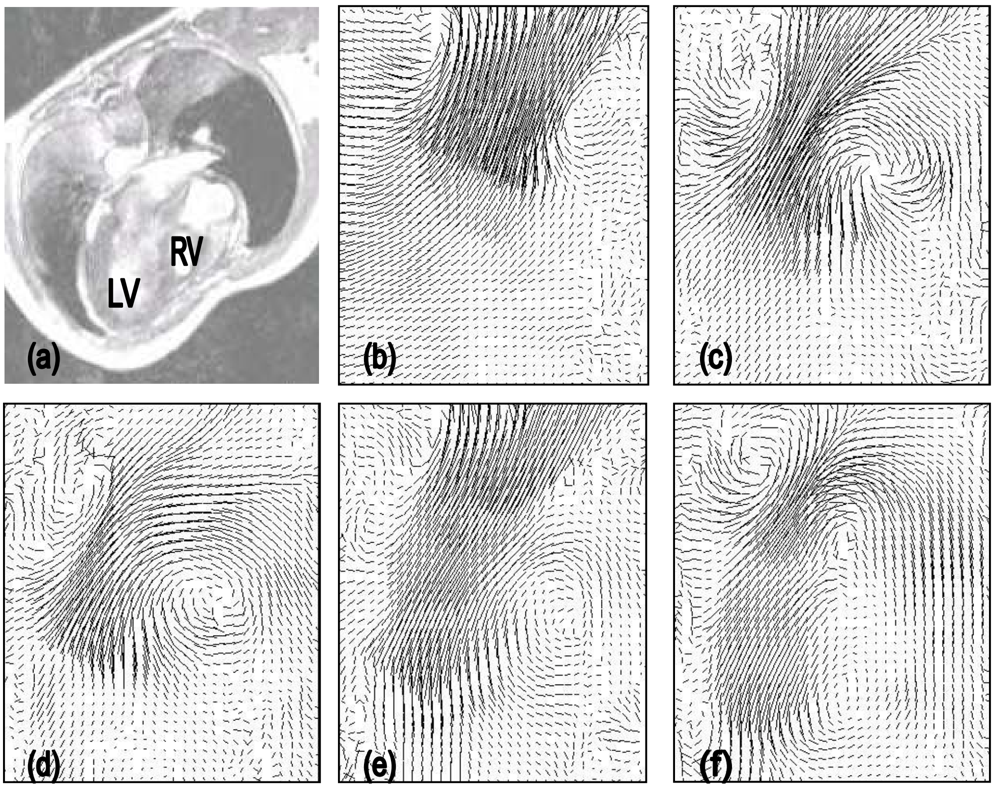

Figure 7 (a) A horizontal long axis MR image showing the left ventricle (LV) and right ventricle (RV) of the heart. (b-f) The flow pattern reconstructed from restored MR velocity data at different phases of the cardiac cycle.

5. CONCLUSIONS

We have demonstrated that the TV-based variational method is effective in restoring velocity field and that the choice of the regularization term greatly affects the restoration result. The proposed new computational scheme based on the First Order Lagrangian method is proven to be simple and effective. The main advantage of this method is that it converges to the optimal solution so that no explicit or empirically defined stopping criteria are needed, thus greatly enhances its practical applicability.

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