THE RATIONAL OPERATION OF THE URBAN TRANSPORT LINE NETWORK BY MINIMISATION OF THE NEEDED VEHICLES

Jan Daněk, Miroslav Plevný, Dušan Teichmann

Introduction

The construction of the urban transport line network is one of the fundamental problems in the traffic practice. The problem of this network construction is more urgent at present, when the individual automobile traffic in the Czech towns leads to congestions. There are two ways to solve this problem - one of them is rather restrictive, and the second is preventive. The restrictive way has to do with paying toll in some section of the towns which is not welcomed by the drivers. The matter of the preventive way consists in subsidizing the public mass transportation from the public funds. The level of the travel standard in the public mass transportation can be increased by means of purchasing new vehicles and modernizing the older ones, the integration of the town and suburban transport, unification of the transport and tariff terms, time and space co-ordination of the public mass transport lines, improving the travellers´ waiting conditions, building the Park and Ride systems etc. The implementation of these actions requires substantial costs. Therefore it is necessary to solve the balance between the requirements of the inhabitants and the system economy. The operation of the public mass transportation must be rational. An example of the effectiveness quantification of traffic company is presented in [7], some other approaches are published in [1], [2] or [6]. One of the possible ways how to rationalise the public mass transportation is to minimise the number of vehicles in operation complying with the requirements of travellers. Alternative ways of solving this problem can be found e.g. in the article [3].

The linear programming is an effective theoretical instrument to solve the above problem [10]. Some methods of the linear programming to find the optimal solution in organisation of the public mass transportation have been applied in several Czech and Slovak towns up to now. Professor Jan Černý is regarded a founder of an efficient method of the public mass transportation lines organisation on the basis of linear programming in the Czech and Slovak Republic. His work [4] is presented as a frontier script in the utilization of the methods of linear programming in organisation of the public mass transportation. The original model (named PRIVOL) dealing with seats distribution within some selected lines is presented in this work. The minimum of the relative residue among the number of seats offered and the number of seats required is applied as a criterion. The minimal value of the relative residue during the optimisation will be maximized. The variables used in the modelling process are non-negative only. The reason for this requirement lies in the capacities of the computer techniques at that time.

The above mentioned mathematical model has been used many times during the period of its existence. In several papers (e.g. [8] and [9]) some significant records of the model PRIVOL application in designing the public mass transportation lines under the specific conditions of towns in Czech and Slovak Republic are mentioned. Furthermore, the article [9] shows several approaches applied abroad and highlights the advantage of combining the solution of the above mentioned problem with the vehicles cycle generation problem. Details about the PRIVOL-model can be found in [4] as well as in [11]. Except the detailed analysis of the original model in these articles some modification were mentioned consisting above all in the fact that instead of the particular seats just the vehicles are assigned to the lines and their number is minimized. This approach (inclusive the requirement of the integer variables) is possible mainly because of that there are much better and powerful computer techniques available at present in comparison with the time of the PRIVOL-model origin.

1. Mathematical Models of Assigning Vehicles to the Particular Lines

The four mathematical models of assigning vehicles to lines will be discussed now. These models differ primarily by the variety of the rolling--stock the transport company has at disposal. The traffic network is modeled by a net $S[V, H]$ where the symbol V is a set of knots representing all the important points of the original traffic network from the point of view of its branching or a possible line termination, and the symbol H is a set of arcs representing all the traffic ways between the pairs of the knots. The common feature of the above mentioned models is the independence of the number of the vehicle cycles on a single line from the type of the vehicle operating on this line. If this is the case the above mentioned models are relatively easily convertible to the form respecting the reality. It is assumed that the passengers demand is distributed relatively equally within the monitored period. The cumulated hourly traveller intensity on a single arc of the net is equal to the intensity of the most loaded section between two stops.

In the first model presented below we will suppose that the rolling-stock (fleet of vehicles) of a transport company includes only one vehicle type. This assumption will be extended in model 2, where we suppose a fleet consisting of more types of one kind of vehicle. Finally, in the third and fourth model we assume a heterogeneous fleet with more types of each kind of vehicle, whereas the model 3 did not suppose any predefined assignment of any kind of vehicle to a particular traffic line, and on the contrary, model 4 will suppose this assignment (e.g. in case of the existing traffic network).

All the four above mentioned models assume a wider set of lines L_0 in the net $S[V, H]$. A number of the vehicle cycles N_l implemented by a vehicle in a given time period is quantified for every line $l \in L_0$. Furthermore, the demand on transfer q_h is defined for each arc $h \in H$.

Model 1 – a homogenous fleet

Firstly, we suppose a linear model where all vehicles have the same capacity [10]. The fleet of vehicles includes only **one vehicle type** with the capacity k (all the vehicles have the same capacity). The task is to decide how many vehicles will be assigned to each line $l \in L_0$. We define the non-negative integer variables xl representing a number of vehicles assigned to the line $l \in L_0$. Symbol L_h denotes the set of lines operating at the arc $h \in H$, symbol Z^+ denotes the set of non--negative integer numbers in all the mathematical models below.

The mathematical model of the first problem will have the following form:

$$\text{minimize } f(x) = \sum_{l \in L_0} xl \tag{1}$$

$$\text{subject to} \sum_{l \in L_k} k\, N_l\, x_l \geq q_h, \quad h \in H \tag{2}$$

$$x_l \in Z^+, \qquad l \in L_0 \tag{3}$$

The criterion (1) of the optimizing problem represents the necessary number of all vehicles needed to operate the given net. The conditions (2) will ensure that on every arc of the net there will be available at least as many seats as is the passenger‘s request on this arc, and the conditions (3) are obligatory.

As the requirement that all the vehicles have the same number of seats has not always been fulfilled in practice (e.g. larger towns), next we will present some ways how it is possible to include the necessity of calculation with more vehicle types to the mathematical model.

Model 2 – a homogenous fleet with more vehicle types

The fleet of vehicles consists of the vehicles of one kind where however more types of this kind of vehicle are available (the vehicle type is unrelated to technical parameters of the vehicle but to the capacity of the vehicle). The set of vehicle types is denoted as J, and the capacity of the vehicle of the type $j \in J$ is denoted as k_j. We have to decide how many vehicles of type $j \in J$ will be assigned to each line $l \in L_0$. We define the non-negative integer variables x_{lj} representing the number of vehicles of the type $j \in J$ assigned to the line $l \in L_0$. As in the previous case Lh denotes the set of lines operating at the arc $h \in H$.

The mathematical model of the problem can be written as follows:

$$\text{minimize } f(x) = \sum_{l \in L_0} \sum_{j \in J} x_{lj} \tag{4}$$

$$\text{subject to} \sum_{l \in L_h} \sum_{j \in J} k_j\, N_l\, x_{lj} \geq q_h, \quad h \in H \tag{5}$$

$$x_{lj} \in Z^+, l \in L_0, \qquad j \in J \tag{6}$$

The criterion (4) of the optimizing problem represents the total number of all types of the necessary vehicles needed to operate the given net. The conditions (5) will ensure that for every arc of the net there will be available at least as many seats as is the passengers´ request on this arc, and the conditions (6) are obligatory.

The following two models represent a modification of the previous linear models where a heterogeneous fleet with more types of each of several kinds of vehicles is assumed to be put into service. These two models differ by the fact that there is no predefined assignment of any kind of vehicle to a concrete traffic line in the first case (It is possible to assign any kind of the vehicle to the line), and by the fixed prescription of this assignment in the second model. The first of these models is then applicable for example in the situations, where the network of lines has to be created, and the second one, when the existing network has to be reorganized.

Model 3 – a heterogeneous fleet without assigning any kind of vehicle to the particular lines

The fleet of vehicles contains several different kinds of vehicles where more types of each vehicle are available. We have to decide how many vehicles will be assigned to each line $l \in L_0$. Although the assignment of the vehicle kind to lines is not predefined, for practical reasons we will suppose that each line in operation can be operated by one kind of vehicle only (e.g. either trolleys or bus), which corresponds to the real situation. The transport company can use all the vehicle types available to operate the line $l \in L_0$. Symbol L_h denotes the set of lines operating at the arc $h \in H$, and symbol I denotes the set of the kinds of vehicles.

In the conditions of a heterogeneous fleet where the assignment of the kind of vehicle to the particular lines is not required the problem model can be constructed in the following way. If the line can be operated by more kinds of vehicles then it is necessary to define sets J_i of the vehicle types of i-th kind of vehicle. Let k_{ij} be the capacity of the vehicle of i-th kind and j-th type. Next, we have to define the non-negative integer variables x_{lij} representing the number of vehicles of i-th kind and j-th type assigned to line $l \in L_0$. It is required in this task that one line (if in operation) can be operated by a single vehicle kind only. In order to complete this we define a bivalent variable z_{li} where $z_{li} = 1$, if line $l \in L_0$ is operated by the i-th kind of vehicle, and $z_{li} = 0$ if it is the other way round. It will be necessary in the model to create a relation between variable x_{lij} and z_{li} expressing that for each $l \in L_0$ and $i \in I$ variables xlij can be positive only if the variable $z_{li} = 1$. The symbol T presents a prohibitive constant of the sufficient size.

The mathematical model of this problem can be written as follows:

$$\text{minimize } f(x) = \sum_{l \in L_0} \sum_{i \in I} \sum_{j \in J} x_{lij} \tag{7}$$

$$\text{subject to} \sum_{l \in L_h} \sum_{i \in I} \sum_{j \in J_i} N_l k_{ij} x_{lij} \geq q_h, \quad h \in H \tag{8}$$

$$\sum_{i \in I} z_{li} \leq 1, \qquad l \in L_0 \tag{9}$$

$$\sum_{j \in J_i} x_{lij} \leq z_{li} T, \qquad l \in L_0,\ i \in I \tag{10}$$

$$x_{lij} \in Z^+, \quad l \in L_0,\ j \in J,\ i \in I \tag{11}$$

$$z_{li} \in \{0;1\}, \qquad l \in L_0,\ i \in I \tag{12}$$

The criterion (7) of the optimizing problem represents the total number of all types of vehicles of any kind needed to operate the given net. The conditions (8) will ensure that the vehicles will be assigned to the lines in such a way that on every arc of the net there will be available at least as many seats as is the passenger‘s request on this arc. The conditions (9) along with the conditions (10) ensure that each line will be operated by maximum of one kind of vehicle with the appropriate number of vehicle types. The conditions (11) and (12) are obligatory.

Model 4 – a heterogeneous fleet with predefined assignment of a kind of vehicle to line

As in the previous Model 3 the fleet of vehicles contains several different kinds of vehicles where more types of each vehicle are available, however the assignment of a particular kind of vehicle to the particular line is predefined. We have to decide how many vehicles will be assigned to each line $l \in L_0$. To construct the model of this problem it is necessary for each line $l \in L_0$ to define sets J_l of all possible types of the given kind of vehicle that can operate this line. Symbol L_h denotes the set of lines operating at the arc $h \in H$. We define the variable x_{lj} representing the number of vehicles of type $j \in J_l$ assigned to the line $l \in L_0$. The model does not need to contain the restriction conditions ensuring that a line will be operated by maximum of one kind of vehicle because the assignment of the vehicle kind to the line is predefined.

We can write the mathematical model of this problem as follows

$$\text{minimize } f(x) = \sum_{l \in L_0} \sum_{j \in J_l} x_{lj} \quad (13)$$

$$\text{subject to } \sum_{l \in L_h} \sum_{j \in J_l} N_l \, k_j \, x_{lj} \geq q_h, \quad h \in H \quad (14)$$

$$x_{lj} \in Z^+, \quad l \in L_0, \, j \in J \quad (15)$$

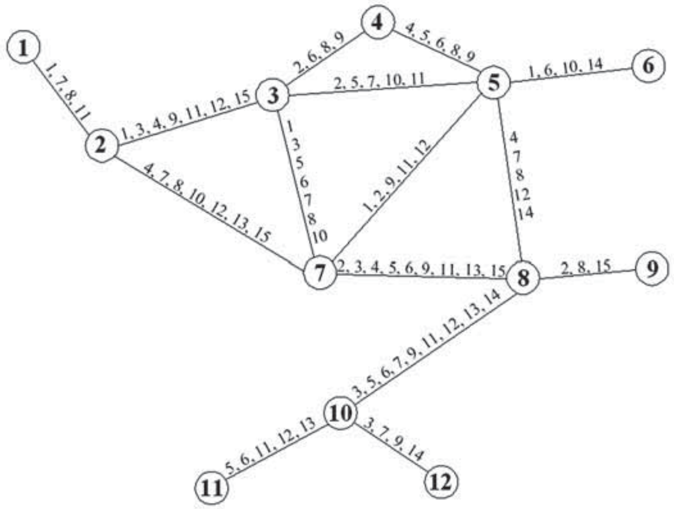
The criterion (13) of the optimizing problem represents the total number of all types of vehicles of any kind needed to operate the given net. The conditions (14) will ensure that vehicles will be assigned to the lines in such a way that on every arc of the net there will be available at least as many seats as is the passenger‘s request on this arc. The conditions (15) are obligatory. It is obvious that model (13), (14)-(15) is formally identical to model (4), (5)-(6) (i.e. to Model2) except that for every line a specific set of types of vehicles is predefined J_l.

2. Numerical Experiments

Numerical experiments were performed using the traffic network illustrated in Fig. 1, and they were focused on the functionality verification of the designed models. On the whole 15 lines (a wider set of lines) are designed in this net. Furthermore, we suppose that none of the vehicles can pass among the lines.

Tab. 1 contains information on the routes in the given net. From this table it follows that all

Fig. 1: The net of the public mass transportation lines



Source: own

Tab. 1: The lines routing in the net

Line	Route	Line	Route	Line	Route
1	1-2-3-7-5-6	6	6-5-4-3-7-8-10-11	11	1-2-3-5-7-8-10-11
2	4-3-5-7-8-9	7	1-2-7-3-5-8-10-12	12	3-2-7-5-8-10-11
3	2-3-7-8-10-12	8	1-2-7-3-4-5-8-9	13	2-7-8-10-11
4	3-2-7-8-5-4	9	2-3-4-5-7-8-10-12	14	6-5-8-10-12
5	4-5-3-7-8-10-11	10	2-7-3-5-6	15	3-2-7-8-9

Source: own

the lines are shuttle lines. For simplification and without a detriment to the commonness of the model we suppose that the travelling time in one direction in every arc of the net takes 10 minutes, and the vehicle turning time at the final stop of every line takes additional 10 minutes. The number of circulation N_l the vehicle carries out on line $l \in L_0$ in an hour, can be calculated according to the formula:

$$N_l = 60 \,/\, t_l \tag{16}$$

where:

N_l ... is the number of vehicle cycles carried out by the vehicle on the line $l \in L_0$ per hour [h^{-1}];

t_l ... is the time of the vehicle cycle on the line $l \in L_0$ [min], which can be calculated for the presented network as follows:

$$t_l = t'_l + t''_l + 2\,t_0 \tag{17}$$

where:

t'_l, t''_l ... are the travelling times of the vehicle on the line $l \in L_0$ in both directions [min];

t_0 ... is the vehicle turning time at the final stop of the line [min].

The basic time unit taken in calculations was 1 hour. The numbers of the vehicle cycles on every line per hour calculated according to formulas (16) and (17) are presented in Tab. 2.

The cumulated hourly passenger intensities on every arc of the net are presented in Tab. 3.

The mathematical solving of the above models was performed using the optimization software Xpress–IVE produced by Dash Optimization company [5]. Numerical experiments (NE) were carried out using the program version available to academic workers for education and research purposes.

The numerical experiment – Model 1 (NE1)

The vehicle capacity of 100 seats was used in the first numerical experiment by solving the Model 1 (NE1). The solution of this model, i.e. the numbers of vehicles assigned to lines, is presented in the following Tab. 4.

The computing time of this numerical experiment using the academic version of the optimization software Xpress-IVE took 0.2 s.

The numerical experiment – Model 2 (NE2)

In the second numerical experiment and in solving the Model 2 (NE2) we suppose that the transport company can use two types of vehicles with capacity of 100 respectively 130 seats to serve the lines. Furthermore, we consider that both types of vehicles can be used on any line. The solution of the Model 2, i.e. the numbers of vehicles assigned to lines, is presented in the Tab. 5.

Tab. 2: The numbers of vehicle cycles on lines

Line number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Number of vehicle cycles per hour	1/2	1/2	1/2	1/2	3/7	3/8	3/8	3/8
Line number	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**—**
Number of vehicle cycles per hour	3/8	3/5	3/8	3/7	3/5	3/5	3/5	

Source: own

Tab. 3: The cumulated hourly intensities of passengers

Arc of the net	**1-2**	**2-3**	**2-7**	**3-4**	**3-5**	**3-7**	**4-5**	**5-6**
Intensity [pass . hour^{-1}]	200	550	585	290	205	760	390	125
Arc of the net	**5-7**	**5-8**	**7-8**	**8-9**	**8-10**	**10-11**	**10-12**	**—**
Intensity [pass . hour^{-1}]	690	185	430	200	355	260	175	

Source: own

Tab. 4: The numbers of vehicles assigned to lines - NE1

Line	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**12**	**14**	**15**	**Σ**
Number of vehicles	4	3	-	-	3	-	-	2	5	6	-	4	-	-	-	27

Source: own

Tab. 5: The numbers of vehicles assigned to lines - NE2

Line	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Σ
Number of vehicles	1/5	1	-	-	1	-	-	3	4	3	-	1	2	-	-	21

Source: own

The solution of the model verified the logical expectations that the vehicles with higher capacities are preferred to be assigned to lines with the exception of line No 1, where one vehicle with the capacity of 100 seats and 5 vehicles with the capacity of 130 seats were assigned. What is interesting here is the fact that the optimal solution of the Model 2 differs from the previous case not only by the varied number of vehicles assigned to the lines but also by the insertion of a new line (line No13). The computing time of this second numerical experiment using the academic version of the optimization software Xpress–IVE took 0.1 s.

The numerical experiment – Model 3 (NE3)

There were two kinds of vehicles (trolleys and buses) available in the numerical experiment by solving the Model 3. In the fleet of trolleys there are two types of vehicles with capacities 100 and 120 seats, and in the fleet of buses there are three types of vehicles with capacities 80, 100 and 120 seats at disposal to serve the lines. Any kind of vehicle could be assigned to any line. One line can be served by one kind of vehicle only, but any type of this kind of vehicle could be assigned to this line.

The achieved solution of the Model 3, i.e. the numbers of vehicles assigned to lines, is presented in Tab. 6. The line number 5 is proposed to be operated by trolleys; other lines will be operated by buses. The table No. 6 contains the results of two experiments done with Model 3. The numerical experiment 3.1 was computed using the maximal vehicles capacity of 120 seats of both

Tab. 6: The numbers of vehicles assigned to lines - NE3

Line	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Σ
Number of vehicles NE 3.1	7	1	-	-	1	-	-	4	4	2	-	1	3	-	-	23
Number of vehicles NE 3.2	4	2	-	-	2	-	-	2	4	4	-	2	1	-	-	21

Source: own

Tab. 7: The assignment of vehicle kinds to lines

Line	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Kind of vehicle [T/A]	T	A	T	A	A	T	A	A	T	A	A	A	A	A	A

Source: own

Tab. 8: The numbers of vehicles assigned to lines - NE4

Line	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Σ
Number of trolleys - cap. 100	1	*	-	*	*	-	*	*	-	*	*	*	*	*	*	1
Number of trolleys - cap. 120	6	*	-	*	*	-	*	*	4	*	*	*	*	*	*	10
Total number of trolleys	**7**	*****	**0**	*****	*****	**0**	*****	*****	**4**	*****	*****	*****	*****	*****	*****	**11**
Number of buses - cap. 80	*	-	*	-	-	*	-	-	*	-	-	-	-	-	-	0
Number of buses - cap. 100	*	-	*	-	-	*	-	-	*	-	-	-	-	-	-	0
Number of buses - cap. 120	*	1	*	-	1	*	-	4	*	2	-	1	3	-	-	12
Total Number of buses	*****	**1**	*****	**0**	**1**	*****	**0**	**4**	*****	**2**	**-**	**1**	**3**	**-**	**-**	**12**

Source: own

trolleys and buses. In the solution there were only buses with maximum capacity of 120 seats assigned to all the lines. In the numerical experiment 3.2 the maximum vehicle capacity of both trolleys and buses was increased from 120 seats to 130 seats. The solution in this case showed the same number of 21 needed vehicles as in the numerical experiment 2.

The computing time using the academic version of optimization software Xpress–IVE took 0,2 s in the case NE 3.1, and 0,3 s in the case NE 3.2.

The numerical experiment – Model 4 (NE4)

Similar to the NE3 there were two kinds of vehicles (trolleys and buses) available in the numerical experiment by solving the Model 4 (NE4). The kinds of vehicles were assigned to the particular lines as indicated in Tab. 7, where T means trolley and A means bus.

In the fleet of trolleys there are two types of vehicles with capacities of 100 and 120 seats, and in the fleet of buses there are three types of vehicles with capacities 80, 100 and 120 seats at disposal to serve the lines. Any vehicle type of the assigned kind of vehicle can be used on the line. The result of the numeric experiment, i.e. the needed numbers of vehicles to operate the lines, is presented in the Tab. 8.

The computing time of this numerical experiment using the academic version of optimization software XpressIVE took 0.1 s.

Conclusion

In this paper we proposed and verified a mathematical model intended to minimise the number of the needed vehicles to operate the lines in the public mass transportation network. The formulated models emerged from the mathematical model PRIVOL [4], however they are modified to solve the defined problem. The optimizing criterion represents the total number of all the vehicles needed to operate the given net that is to be minimised.

The numerical experiments proved the functionality of the designed models. These implemented experiments showed that the optimization software Xpress-IVE did not have significant difficulties to solve the given problem corresponding to the size of the public mass transportation network of a medium-sized town. It is realistic to presuppose that the optimal solution of the above mentioned models would be possible to find even for the existing and larger public mass transportation networks.

The presented mathematical model modifications have significant importance especially from the point of view of the practical interpretation of the achieved results. When solving the original model presented in [4], [8], [9] we obtain the optimal number of seats needed on every line. Therefore it is necessary to solve another problem, where we have to decide how many individual vehicles must be assigned to each particular line, or whether the line has to be in operation at all (e.g. in situations, when the variable representing the number of the needed seats take a very low positive value). Using the models presented in this paper, the decision whether or not the line will be in operation is very simple. If some variables representing the number of vehicles assigned to the line acquire a positive value then the line will be in operation, and vice versa, if this variable equals zero then the line will not be in operation, because no vehicle will be assigned to this line.

The above mentioned models can be simply extended by adding the operation costs of the individual kinds of vehicles and their types. That can be done by adding appropriate coefficients expressing individual vehicle costs to the corresponding variables in the criterion of the Model 1 as well as Model 2, 3 and 4. The computation complexity of these modified models would be not significantly different.

References

[1] BORDŐRFER, R., GRŐTCHEL, M., PFETSCH, M. E. A column-Generation Approach to Line Planning in Public Transport. *Transportation Science*, 2007, Vol. 41, No. 1, pp. 123 – 132, ISSN 0041-1655.

[2] CHEN, H. Stochastic Optimization in Computing Multiple Headways for a Single Bus Line. *Journal of the Chinese Institute of Industrial Engineers*, 2007, Vol. 24, No. 5, pp. 351 – 359. ISSN 1017-0669.

[3] CONSTANTIN, I., FLORIAN, M. Optimizing Frequencies in a Transit Network. *International Transactions in Operational Research*. 1995, No. 2, pp. 149 – 164. ISSN 0969-6016.

[4] ČERNÝ, J., KLUVÁNEK, P. *Základy matematickej teórie dopravy*. Bratislava: VEDA, 1990. ISBN 80-224-0099-8.

[5] *Dash Optimization* [online]. See at: <http://www.dashoptimization.com>.
[6] ERLANDER, S., SCHÉELE, S. A Mathematical Programming Model for Bus Traffic in a Network. *Transportation and Traffic Tudory*. Sydney: REED, 1974, pp. 581 – 605.
[7] KLIEŠTIK, T. Quantification Effectiveness Activities Traffic Company by the Rules of Data Envelopment Analysis. *E+M Ekonomie a Management*, 2009, Vol. 12, No. 1, pp. 133 - 145. ISSN 1212-3609.
[8] PALÚCH, S., MAJER, T. K optimalizácii mestskej a prímestskej pravidelnej osobnej dopravy. In: *III. mezinárodní konference Vysoká škola jako facilitátor rozvoje společnosti a regionu 2007*. Kunovice: Evropský polytechnický institut Kunovice. 279 s. ss. 245 – 248. ISBN 80-7314-107-8.
[9] PEŠKO, Š. Podpora metód operačního výskumu při navrhovaní systému liniek. In: *Dopravná infraštruktúra v mestách*, 6. medzinárodná konferencia, 22.10.-23.10.2008, Zborník prispevkov na CD, Žilina: Žilinská univerzita, str. 29. ISBN 978-80-8070-913-6.
[10] PLEVNÝ, M. Application of Quantitative Optimization Techniques to Small Businesses. In *Mathematical Methods in Economics 2004*. Brno: Masaryk University Brno, 2004, pp. 256-260. ISBN 80-210-3496-3.
[11] TEICHMANN, D. O několika modifikacích matematického modelu přidělování vozidel linkám v městské hromadné dopravě. *Nová železniční technika*. č. 1, 2009. s. 20 – 23. ISSN 1210-3942.

prof. Ing. Jan Daněk, CSc.
VŠB - Technical University of Ostrava
Institute of Transport
jan.danek@vsb.cz

doc. Dr. Ing. Miroslav Plevný
University of West Bohemia in Pilsen
Faculty of Economics
plevny@kso.zcu.cz

Ing. Dušan Teichmann, Ph.D.
VŠB - Technical University of Ostrava
Institute of Transport
dusan.teichmann@vsb.cz

Doručeno redakci: 2. 10. 2009
Recenzováno: 2. 3. 2010; 5. 4. 2010
Schváleno k publikování: 12. 4. 2010

ABSTRACT

THE RATIONAL OPERATION OF THE URBAN TRANSPORT LINE NETWORK BY MINIMISATION OF THE NEEDED VEHICLES

Jan Daněk, Miroslav Plevný, Dušan Teichmann

The construction of the urban transport line network is one of the fundamental problems in the traffic practice. Efficient functioning of the public mass transportation supported from the public sources in the towns is more urgent at present, when the individual automobile traffic leads to congestions in central parts of the cities. The demand to increase the culture of travelling in the public mass transportation requires, however, substantial costs. Therefore it is necessary to solve the balance between the inhabitants´ requirements and the economy of the transportation system. The operation of the public mass transportation must be rational; the vehicles must not show unreasonable waiting times, etc. One of the possible ways how to rationalise the public mass transportation is to minimise the number of vehicles in operation complying with the requirements of the travellers.

The paper deals with the problem of minimising the number of vehicles needed to operate the lines of the public mass transportation network. To solve this problem the methods of linear programming are used. There are four mathematical models described in which the vehicles are assigned to the individual lines. The models arise from the mathematical model PRIVOL [4], and they vary from this model especially by the variety of the rolling-stock the transport company has at disposal.

By means of the constructed models a number of numerical experiments concerning the network corresponding to a medium-sized town were performed using the four constructed models. These numerical experiments demonstrated the functionality of the designed models.

The main contribution of the presented modifications of the original model PRIVOL consists in the fact, that instead of the optimal number of seats the new models are able to directly find an optimal number of vehicles needed to operate the particular lines to meet the expected demand of the passengers. That way the proposed approach allows to objectify the decision making of the transport company concerning the allocation of vehicles to the network lines.

Key Words: *traffic network, linear programming, PRIVOL, vehicles allocation, public mass transportation.*

JEL Classification: *C61, H40, L92.*