AN INTUITIONISTIC FUZZY DATA ENVELOPMENT ANALYSIS FOR EFFICIENCY EVALUATION UNDER UNCERTAINTY: CASE OF A FINANCE AND CREDIT INSTITUTION

Seyed Hossein Razavi Hajiagha, Hadi Akrami,
Edmundas Kazimieras Zavadskas, Shide Sadat Hashemi

Introduction

Performance evaluation is a necessary step on the path to organizational success that provides the possibility of measuring movement of organization in the direction of their goals and missions. Performance evaluation helps managers guide their organization toward achieving excellence and leadership and impressive results. Simons [1] defines performance evaluation and control systems as formal procedures based on the information which managers use to maintain or improve their organizational pattern. Based on this definition, performance evaluation systems will have the following four features:

1. The goal of each system is to control and evaluate the data performance;
2. Performance evaluation and control systems are based on formal procedures;
3. Performance evaluation and control systems are designed in specific forms to be used by managers;
4. Managers use performance evaluation and control systems to maintain or change their organizational activity pattern.

Data envelopment analysis is a non-parametric method to appraise the relative efficiency of a set of congruent units. Charnes, Cooper, and Rhodes first proposed this method in 1978 which the name of the basic model is known as CCR due to their names [2]. Data envelopment analysis has all the four features of these systems as a performance evaluation method. This method transmits the information in an effective way, its structure has been defined and its applications are effective in improving organizational units.

Over more than 30 years of development of this method, its idea has grown steadily and has been strengthened in several ways. Emrouznejad et al. [3], Cook and Seiford [4] and Liu et al. [5] have reported over a thousand different projects and applications using this technique. Data envelopment analysis is used to appraise the relative efficiency of a set of *n* congruent Decision Making Unit (DMU) that use *m* congruent input to produce *s* congruent output.

Like any other framework, data envelopment analysis has also been the subject of evolution. One of the important developments in this field related to circumstances that inputs and outputs are defined and measured under conditions of uncertainty. In fact, one of the assumptions of classic data envelopment models is their crispness of data. However, in situations where uncertainty is an inevitable feature of a real environment, the assumption of crispness of data and observations seems questionable. Also, most management decisions are not made based on known calculations and there is a lot of uncertainty and ambiguity in decision-making problems [6]. Zadeh [7] believes “As the complexity of system increases, our ability to express specific propositions about the behavior decreases”. In response to the need to present a formal framework to deal with uncertainty, various ideas in probability and statistics, fuzzy logic and the Grey systems theory is presented [8]. In data envelopment analysis issues, outputs like customer satisfaction, social responsibility, customer satisfaction and etc. are examples of variables which are subjective in nature and it is difficult to measure them accurately. In this regard, development of many models based on data envelopment analysis under uncertainty is being studied by researchers. Zhu and Cook [9] studied the wide variety of uncertainty in DEA models.

Fuzzy logic is presented by Zadeh [10] as a generalization of classical set theory. A fuzzy set is a set of pairs like (x, $\mu_{\tilde{A}}(x)$) in universe X which assign to each element $x \in X$ a membership function (degree) equal to $\mu_{\tilde{A}}(x)$. Sengupta [11], Kao and Liu [12], Guo and Tanaka [13], Lertworasirikul [14], Saati-Mohtadi et al. [15], Lertworasirikul et al. [16], Zerafat-Angiz et al. [17], Tavana et al. [18], Mirhedayatian [19], Ghapanchi et al. [20], Sadeghi et al. [21] and Houshyar et al. [22] are examples of works done on DEA models with use of fuzzy data.

Grattan-Guinness [23] and Gau and Buehrer [24] believe representing a verbal statement in the form of fuzzy sets is not enough. In fact, decision makers do not have sufficient information to assign a determined membership value to each element. On this basis, Atanassov [25] presented Intuitionistic Fuzzy Sets (IFS) which assign a membership and non-membership function to each element. Intuitionistic Fuzzy Sets are widely used on decision making issues. Li et al. [26], [27], Boran et al. [28], Wei et al. [29] and Zhang, Liu [30] are examples of applications.

In this paper, data envelopment analysis model is developed in a way that input and output data presented in the form of intuitionistic fuzzy sets. This model tries to solve the model based on the aggregation operator. In the second and third section of the paper the intuitionistic fuzzy sets and data envelopment analysis are briefly reviewed. The proposed intuitionistic fuzzy data envelopment analysis model is presented in forth section. In the fifth section an example of the application of this model in a problem of performance evaluation is studied and finally discussion and conclusion are provided in sixth section.

1. Intuitionistic Fuzzy Sets

Atanassov [25] introduced a generalize type of fuzzy sets as intuitionistic fuzzy sets.

Definition1. Assume that X is a universe. Intuitionistic fuzzy set A in X is a subset of elements like $x \in X$ in which:

$$A = \{\langle x, \mu_A(x), v_A(x)\rangle \mid x \in X\} \quad (1)$$

In Eq. (1), $\mu_A: X \to [0,1]$ and $v_A: X \to [0,1]$ shows membership and non-membership functions of the element $x \in X$ in A and for each $x \in X$,

$$0 \le \mu_A(x) + v_A(x) \le 1 \quad (2)$$

Xu [31] introduced $\alpha = \langle \mu_\alpha, v_\alpha \rangle$ as an intuitionistic fuzzy number for simplicity of presentation and computation.

Definition2. If $\alpha = \langle \mu_\alpha, v_\alpha \rangle$, $\alpha_1 = \langle \mu_{\alpha_1}, v_{\alpha_1} \rangle$ and $\alpha_2 = \langle \mu_{\alpha_2}, v_{\alpha_2} \rangle$ be some intuitionistic fuzzy numbers, then the following operational rules will be hold [31], [32].

$$\alpha_1 \oplus \alpha_2 = \langle \mu_{\alpha_1} + \mu_{\alpha_2} - \mu_{\alpha_1}\mu_{\alpha_2}, v_{\alpha_1} v_{\alpha_2} \rangle \quad (3)$$

$$\alpha_1 \otimes \alpha_2 = \langle \mu_{\alpha_1}\mu_{\alpha_2}, v_{\alpha_1} + v_{\alpha_2} - v_{\alpha_1} v_{\alpha_2} \rangle \quad (4)$$

$$\lambda\alpha = \langle 1 - (1 - \mu_\alpha)^\lambda, v_\alpha^\lambda \rangle, \lambda \succ 0; \quad (5)$$

$$\alpha^\lambda = \langle \mu_\alpha^\lambda, 1 - (1 - v_\alpha)^\lambda \rangle, \lambda \succ 0; \quad (6)$$

Definition3. If $\alpha = \langle \mu_\alpha, v_\alpha \rangle$ and $\beta = \langle \mu_\beta, v_\beta \rangle$ be two intuitionistic fuzzy numbers, then $S(\alpha) = \mu_\alpha - v_\alpha$ and $S(\beta) = \mu_\beta - v_\beta$ are the score functions [33] and $h(\alpha) = \mu_\alpha + v_\alpha$ and $h(\beta) = \mu_\beta + v_\beta$ are the accuracy functions [26] of intuitionistic fuzzy numbers. Hence, sequential relationship between these two numbers is defined as follows [32].

1. If $S(\alpha) \prec S(\beta)$ then $\alpha \prec \beta$.
2. If $S(\alpha) \prec S(\beta)$ then:
   a. If $h(\alpha) = h(\beta)$ then $\alpha = \beta$.
   b. If $h(\alpha) \prec h(\beta)$ then $\alpha \prec \beta$.

Simply, if $\mu_\alpha \prec \mu_\beta$ and $v_\alpha \succ v_\beta$, then $\alpha \prec \beta$ and if $\mu_\alpha = \mu_\beta$ and $v_\alpha = v_\beta$, then $\alpha = \beta$.

Definition4. Let $\alpha_j = \langle \mu_{\alpha_j}, v_{\alpha_j} \rangle (j = 1,2,\ldots,n)$ is a set of intuitionistic fuzzy numbers and GIFWA: $V^n \to V$ that V is the set of all intuitionistic fuzzy numbers. If

$$GIFWA_w(\alpha_1, \alpha_2, \ldots, \alpha_n) = (w_1\alpha_1^\lambda \oplus w_2\alpha_2^\lambda \oplus \ldots \oplus w_n\alpha_n^\lambda)^{1/\lambda} \quad (7)$$

Then GIFWA is called the generalized intuitionistic fuzzy weighted average operator which $\lambda \succ 0$ and w = (w_1, w_2, ..., w_n)^t is weighted vector of this operator that for $j = 1,2,\ldots,n$, $\sum_{i=1}^n w_j = 1$ and $w_j \ge 0$ [35]. According to the operational rules (3) – (6) between intuitionistic fuzzy numbers, Zhao et al. [35] showed that:

$$GIFWA_w(\alpha_1,\alpha_2,\ldots,\alpha_n)=\left\langle\left(1-\prod_{j=1}^{n}\left(1-\mu_{\alpha_j}^{\lambda}\right)^{w_j}\right)^{1/\lambda},\; 1-\left(1-\prod_{j=1}^{n}\left(1-\left(1-v_{\alpha_j}\right)^{\lambda}\right)^{w_j}\right)^{1/\lambda}\right\rangle \quad (8)$$

If $\lambda = 1$ then GIFWA operator decreased to IFWA operator.

2. Data Envelopment Analysis

Data envelopment analysis used to appraise the relative efficiency of a set of congruent decision making units that use some inputs for producing a set of outputs. This model was first introduced by Charnes, Cooper and Rhodes [2] and after the 1980 recognized as one of the main tools for modeling and analyzing the performance. In fact, this is a multi-factor productivity analysis model for measuring the relative efficiency of a set of congruent units. Performance is defined respect to multiple inputs and outputs as follows [36]:

$$\text{Performance} = \frac{\text{Weighted sum of outputs}}{\text{Weighted sum of inputs}} \quad (9)$$

The multiple form of CCR basic model could be shown as follow:

$$Max\left\{u^t y_0 \middle| v^t x_0 = 1, u^t y_j - v^t x_j \le 0, u^t, v^t \ge 0,\right\} \quad (10)$$

In this model, $j = 1, \ldots n$ shows the number of decision making units which a unit like j, use m dimensional input vector $x_j = [x_{1j}, \ldots, x_{mj}]$ to produce s dimensional output vector $y_j = [y_{1j}, \ldots, y_{sj}]$. The m dimensional vector $u^t = [u_1, \ldots, u_m]$ shows input variables weights and s dimensional vector $v^t = [v_1, \ldots, v_s]$ shows output variables weights and these weights are used to determine the efficiency of the unit under evaluation. The data envelopment analysis model runs for all decision making units and calculate the optimal value of and in a way that units under evaluation gain the maximum possible performance. This model is called input-oriented CCR model with constant returns to scale. The envelopment form of model (10) is shown as follow:

$$Min\{\theta | \theta x_0 \ge \lambda X, y_0 \le \lambda Y, \lambda \ge 0\} \quad (11)$$

This model is developed under assumptions like variable returns to scale, being input or output oriented, being additive and etc. Various sources have evaluated and disseminated models of data envelopment analysis like Charnes et al. [37], Ray [38] and Cooper et al. [39]. A comprehensive review on the applications of DEA is reviewed in Emrouznejad et al. [40], Cook and Seiford [4] and Liu et al. [5].

3. Data Envelopment Analysis with Intuitionistic Fuzzy Inputs and Outputs

Suppose that there are n decision-making units that each unit, $DMU_j (j = 1,2, \ldots, n)$, use the input vector $X_j = (x_{1j}, x_{2j}, \ldots, x_{mj})$ to produce the output vector $Y_j = (y_{1j}, y_{2j}, \ldots, y_{sj})$. Assume the input matrix X partition to two subset of intuitionistic fuzzy inputs $IFV_x = (x_{1j}, x_{2j}, \ldots, x_{pj})$ and crisp inputs $E_x = (x_{p+1j}, x_{p+2j}, \ldots, x_{mj})$. Similarly output matrix Y partition to $IFV_y = (y_{1j}, y_{2j}, \ldots, y_{lj})$ and $E_y = (y_{l+1j}, y_{l+2j}, \ldots, y_{sj})$. Now the input oriented variable returns to scale model BCC, originally introduced by Banker, Charnes, Cooper [41], is written as follows. It is clear that the model (12) is different from CCR model only at constraint $\sum_{j=1}^{n}\lambda_j = 1$ which is related to variable returns to scale assumption.

$$\begin{aligned} & Min\,\theta && (12)\\ & S.T. \sum_{j=1}^{n}\lambda_j x_{ij} \le \theta x_{i0} \quad i \in IFV_x && (i)\\ & \sum_{j=1}^{n}\lambda_j x_{ij} \le \theta x_{i0} \quad i \in E_x && (ii)\\ & \sum_{j=1}^{n}\lambda_j y_{ij} \ge y_{r0} \quad r \in IFV_y && (iii)\\ & \sum_{j=1}^{n}\lambda_j y_{ij} \ge y_{r0} \quad r \in E_y && (iv)\\ & \sum_{j=1}^{n}\lambda_j = 1 \\ & \lambda_j \ge 0,\; j = 1,2,\ldots,n \\ & \theta{:}unrestricted \end{aligned}$$

The constraints sets *(ii)* and *(iv)* are like the usual CCR model and do not need any special modification. Now consider the constraints set *(i)*. For $i \in IFV_x$, the right side of the constraint can be replaced by IFWA operator and the left side with section 3 of the definition 2.

$$\left\langle\left(1-\prod_{j=1}^{n}\left(1-\mu_{x_{ij}}\right)^{\lambda_j}\right), \prod_{j=1}^{n} v_{x_{ij}}^{\lambda_j}\right\rangle \le \left\langle 1-\left(1-\mu_{x_{i0}}\right)^{\theta}, v_{x_{ij}}^{\theta}\right\rangle \quad (13)$$

Similarly, the below relationship is hold for $r \in IFV_y$.

$$\left\langle \left(1-\prod_{j=1}^{n}\left(1-\mu_{y_{rj}}\right)^{\lambda_j}\right), \prod_{j=1}^{n} v_{y_{rj}}^{\lambda_j} \right\rangle \geq \left\langle \mu_{y_{r0}}, v_{y_{r0}} \right\rangle \tag{14}$$

Since IFWA operator is an intuitionistic fuzzy number, Eqs. (13) and (14) are corresponding to relations in constraints *(ii)* and *(iv)*. Based on the expressed sequential relationships between intuitionistic fuzzy numbers in definition 3, Eq. (13) is transformed to following two equations.

$$\begin{aligned} &\prod_{j=1}^{n}\left(1-\mu_{x_{ij}}\right)^{\lambda_j} \geq \left(1-\mu_{x_{i0}}\right)^{\theta} \\ &\prod_{j=1}^{n} v_{x_{ij}}^{\lambda_j} \geq v_{x_{ij}}^{\theta} \end{aligned} \tag{15}$$

A similar transformation could be done for relation (14)

$$\begin{aligned} &\prod_{j=1}^{n}\left(1-\mu_{y_{rj}}\right)^{\lambda_j} \leq 1-\mu_{y_{r0}} \\ &\prod_{j=1}^{n} v_{y_{rj}}^{\lambda_j} \leq v_{y_{r0}} \end{aligned} \tag{16}$$

The relations (15) and (16) are nonlinear relations that could be transformed to linear form by natural logarithm. Eqs. (17) and (18) shows this transformation on constraints sets (15) and (16).

$$\begin{aligned} &\sum_{j=1}^{n}\lambda_j \cdot ln\left(1-\mu_{x_{ij}}\right) \geq \theta \cdot ln\left(1-\mu_{x_{i0}}\right) \\ &\sum_{j=1}^{n}\lambda_j \cdot ln\left(v_{x_{ij}}\right) \geq \theta \cdot ln\left(v_{x_{i0}}\right) \end{aligned} \tag{17}$$

And

$$\begin{aligned} &\sum_{j=1}^{n}\lambda_j \cdot ln\left(1-\mu_{y_{r0}}\right) \leq ln\left(1-\mu_{x_{i0}}\right) \\ &\sum_{j=1}^{n}\lambda_j \cdot ln\left(v_{y_{r0}}\right) \leq ln\left(v_{y_{r0}}\right) \end{aligned} \tag{18}$$

Finally, the intuitionistic fuzzy BCC model (IF-BCC) is obtained as follow.

$$\begin{aligned} &Min\theta \\ &S.T.\ \sum_{j=1}^{n}\lambda_j \cdot \ln\left(1-\mu_{x_{ij}}\right) \geq \theta \cdot \ln\left(1-\mu_{x_{i0}}\right) && i \in IFV_x \\ &\sum_{j=1}^{n}\lambda_j \cdot \ln\left(v_{x_{ij}}\right) \geq \theta \cdot \ln\left(v_{x_{i0}}\right) && i \in IFV_x \\ &\sum_{j=1}^{n}\lambda_j x_{ij} \leq \theta x_{i0} && i \in E_x \\ &\sum_{j=1}^{n}\lambda_j \cdot \ln\left(1-\mu_{y_{r0}}\right) \leq \ln\left(1-\mu_{y_{r0}}\right) && r \in IFV_y \\ &\sum_{j=1}^{n}\lambda_j \cdot \ln\left(v_{y_{r0}}\right) \leq \ln\left(v_{y_{r0}}\right) && r \in IFV_y \\ &\sum_{j=1}^{n}\lambda_j y_{ij} \geq y_{r0} && r \in E_y \\ &\sum_{j=1}^{n}\lambda_j = 1 \\ &\lambda_j \geq 0,\ j = 1,2,\ldots,n \\ &\theta\text{:unrestricted} \end{aligned} \tag{19}$$

Note that the return to scale condition $\sum_{j=1}^{n}\lambda_j = 1$, meet the condition mentioned in IFWA operator that total weights should be equal to 1. Therefore the data envelopment analysis BCC model is a good option for solving performance evaluation problems with intuitionistic fuzzy data. Also, the definition of IF-BCC performance is quite like the definition of BCC performance which is presented in Cooper et al. [39]. This definition can be presented as follows.

Definition 5. A decision making unit is called IF-BCC efficient if and only if $\theta_0 = 1$ and all the slack variables are zero.

This model also has the following features:

1. $\theta_0 = 1$, $\lambda_0 = 1$, $\lambda_j = 0$, j = 1,2, ..., n, j $\neq$ 0 is a feasible for the model (19) and so the above model always has a feasible answer and the performance is less than one.
2. According to existence of the feasible region respect to the last characteristic θ is a real number in interval (0, 1].
3. $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n) \neq 0$ in model (19).

4. Application of IF-BCC model

In this section, an example of the use of IF-BCC model will illustrate. The example is devoted to the evaluation of 20 branches of a finance and credit institution. For this purpose, the use of data envelopment analysis model is proposed. Management of the organization considered sets of two inputs include the number of employees and an annual budget and three outputs include net profit, social responsibility and customer satisfaction, through studies and analysis of expert opinions to assess units.

On this basis, the first output variable is an objective measure that is calculated based on the company's financial data. But the second and third output variables are subjective indicators. Customer satisfaction index is evaluated through a series of questionnaires designed. The questionnaires were distributed among a random sample of customers and based on their views customer satisfaction index is calculated for each unit.

On the other hand, evaluation of the social responsibility index is done based on criteria and indicators defined in the EFQM excellence model. Accordingly, a group of assessors review the functional records of each unit and rate them in respective index. However, considering these scores as deterministic and exact values, decrease the semantic of them and ignore the ambiguity and uncertainty in these estimates. Thus, the scores per unit of this index are divided to a range from "very very low" to "very very high". Then each of these verbal expressions converts into an intuitionistic fuzzy number. Table 1 shows the scale used for this conversion.

Tab. 1: Intuitionistic Fuzzy Values for Customer Satisfaction and Social Responsibility Index

Intuitionistic fuzzy number	Verbal Phrase
⟨0.05,0.9⟩	Very Very Low (VVL)
⟨0.1,0.75⟩	Very Low (VL)
⟨0.25,0.6⟩	Low (L)
⟨0.4,0.5⟩	Below Average (BA)
⟨0.5,0.4⟩	Average (A)
⟨0.6,0.3⟩	Above Average (AA)
⟨0.7,0.20⟩	High (H)
⟨0.8,0.1⟩	Very High (VH)
⟨0.85,0.1⟩	Very Very High (VVH)
⟨0.9,0.05⟩	Excellent (E)

Source: own

Tab. 2: DMUs Inputs and Outputs

Units	Inputs		Outputs			IF–BCC Performance
	Budget	Staff	Profit	Customer satisfaction	Social responsibility	
1	13883	206	434.12	A	H	0.43
2	15370	227	250.01	VVH	L	0.67
3	7577	119	67	AA	AA	0.49
4	19000	254	246	L	BA	0.36
5	7116	120	59.6	VVH	H	0.95
6	10400	149	1142.2	E	BA	0.91
7	35929	601	1312.6	VVL	VH	1
8	6800	98	0.5	AA	VL	0.57
9	6216	70	276.3	AA	BA	0.75
10	18390	303	683.47	AA	E	0.55
11	3253	51	26.672	A	VVH	1
12	19200	282	2357.07	A	BA	0.59
13	5326	100	29.72	VH	VVL	0.96
14	5150	92	I.33	VH	VVH	1
15	9821	133	133.17	VL	BA	0.28
16	20756	206	50.8	E	VH	0.85
17	6798	274	185.89	VVL	BA	0.316
18	3370	81	77.46	H	A	1
19	6385	61	251.17	AA	H	1
20	12024	116	539.4	VL	L	0.68

Source: own

Table 2 shows the input and output of the 20 units under assessment. This data is proposed and evaluated based on the IF-BCC model. The model results are shown in the last column of Table 2. Based on the IF-BCC model, five units of 7, 11, 14, 18 and 19 are known as IF-BCC efficient. Another 15 units are known as radial inefficient. IF-BCC model also shows the combinatory efficacy based on the values of slack variables. Table 3 shows the values of slack variables.

According to Table 3, some decision-making units in addition to the radial inefficiency have mix inefficiency. The interpretation of shortcomings associated with intuitionistic non-fuzzy variables is performed such as the fundamentals of classical DEA. But providing accurate and quantitative interpretation about slack values of intuitionistic fuzzy variables is not easy. In this regard, we adopted two different views: First, the values of the degrees of membership can be considered a continuous spectrum and the slacks are interpreted carefully like the slacks in the exact variables. Second, according to dissociation of intuitionistic fuzzy data, deficiencies can be regarded as a signal to increase the output variables or decrease the input intuitionistic fuzzy varialble to a better level. According to this approach, for example, 0.0028 and 0.059 as membership and non-membership deficiencies of the first units means that this unit should increase its activities related to social responsibility and simultaneously consider maintaining the level of customer satisfaction. These analysis and interpretations are the main advantages of data envelopment analysis to evaluate the performance.

Tab. 3: Slack Variables of Decision Making Units Based on Intuitionistic Fuzzy BCC model

Unit	Additional input		Output slacks				
				Social responsibility		Customer satisfaction	
	Budget	Staff	Profit	Membership	Non-membership	Membership	Non-membership
1	0	0	0	0	0.0028	0.059	0
2	2636	0	0	0	0.143	0.922	1.1
3	0	0	0	0	0.048	0.028	0
4	1172.052	0	0	0.094	0	0	0.0045
5	647.9	0	0	0	0.288	0.588	0.62
6	1731.78	0	629.8	0	0.14	1.182	2.19
7	0	0	0	0	0	0	0
8	1061.03	0	0	0	0.076	0.778	0.805
9	1765.5	0	0	0	0.103	0.547	0.593
10	0	0.013	0	0.516	0.06	0.078	0
11	0	0	0	0	0	0	0
12	2217.304	0	0	0	0.017	0.205	0.288
13	0	4.7	310	0	0	1.84	2.19
14	1	0	0	0	0	0	0
15	282.5	0	0	0.11	0	0.04	0
16	7174	0	0	0.062	0	0.114	0
17	0	37	0	0.68	0.87	0.025	0
18	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0
20	4303.5	0	0	0.106	0	0.08	0

Source: own

Conclusion

Unknown data and vagueness in knowledge about systems create the challenge of uncertainty for decision makers. Performance evaluation using data envelopment analysis also faced such a challenge. Researchers have been proposed various frameworks to develop DEA models under uncertainty that stochastic, fuzzy and interval data could be noted. However, DEA is not extended under IFS environments. In this paper a form of data envelopment analysis model was presented that ambiguity and uncertainty of data are shown by intuitionistic fuzzy sets. Considering a membership and non-membership function (degree) for each element in the intuitionistic fuzzy sets help analysts to obtain a better picture of the uncertainty in data. In this regard, this research presented an approach for analyzing DEA model with variable returns to scale, BCC, in the circumstances that data uncertainty are displayed in the form of intuitionistic fuzzy sets. The presented model which is called IF-BCC, measure the efficiency of decision making units with intuitionistic fuzzy data. Efficiency scores of the proposed model have a similar meaning and interpretation with original BCC model. Furthermore, the model provided the possibility of analyzing the slack variables and determination of improvement projection for inefficient units.

This model is appropriate in situations where some inputs or outputs do not have an exact quantitative value. The proposed model is developed based on the weighted aggregation operator of intuitionistic fuzzy data. The condition of the operator is that the sum of the weights should be equal to 1 and therefore BCC model is appropriate. The application of the proposed model is examined in a real world case study of a finance and credit institution. Future researches can be concentrated on the development of other types of DEA models based on the notion of intuitionistic fuzzy (α, β)-cuts. While developing data envelopment analysis models based on interval valued intuitionistic fuzzy data is another area for further studies.

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Dr. Seyed Hossein Razavi Hajiagha
Institute for Trade Studies and Research
Tehran, Iran
s.hossein.r@gmail.com

Mr. Hadi Akrami
Institute for Trade Studies and Research
Tehran, Iran
hadi.akrami@gmail.com

Prof. Edmundas Kazimieras Zavadskas
Vilnius Gediminas Technical University
Faculty of Civil Engineering
Vilnius, Lithuania
edmundas.zavadskas@vgtu.lt

Mrs. Shide Sadat Hashemi
Allameh Tabatabaei University
Management and Accounting Faculty
Tehran, Iran
shide_hashemi@yahoo.com

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Abstract

AN INTUITIONISTIC FUZZY DATA ENVELOPMENT ANALYSIS FOR EFFICIENCY EVALUATION UNDER UNCERTAINTY: CASE OF A FINANCE AND CREDIT INSTITUTION

Seyed Hossein Razavi Hajiagha, Hadi Akrami, Edmundas Kazimieras Zavadskas, Shide Sadat Hashemi

Performance evaluation is a challenging issue for managers. Data envelopment analysis is a non parametric and linear programming based approach to appraise the relative efficiency of a set of congruent units. One of the shortcomings of classic data envelopment model is their crispness of data. In this paper, a data envelopment model is extended in which inputs and outputs are ambiguous and are expressed in the form of intuitionistic fuzzy sets. The proposed method is extended based on a weighted aggregation operator which is defined for intuitionistic fuzzy data. This model applied the advantages of intuitionistic fuzzy data in capturing the uncertainty. The main advantages of the proposed method are its simplicity and consistency with classic models. The proposed method is applied in a real instance and its results are examined.

Key Words: *performance evaluation; data envelopment analysis; BCC model; intuitionistic fuzzy sets; aggregation operator.*

JEL Classification: *D61, D80, G21.*