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A computational investigation of vibration of stationary and rotating structures submerged in a liquid

J. Zapoměl[a,*], L. Čermák[b], F. Pochylý[b]

[a] Institute of Thermomechanics, ASCR, branch at VSB-TU of Ostrava, Ostrava, Czech Republic
[b] Faculty of Mechanical Engineering, Brno University of Technology, Brno, Czech Republic

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Abstract

To realize some technological processes components of a number of stationary and rotating machines work submerged in various liquids. Therefore their vibration is significantly influenced by their interaction with the medium in the surrounding space. An important tool for investigation of their behavior is a computer modelling method. In the computational models it is assumed that the vessel and the submerged bodies are absolutely rigid, the liquid is perfect and incompressible, amplitude of the vibration is small, and the flow and the oscillations are 2D. On these assumptions the pressure distribution in the liquid is described by a Laplace equation. For its solution a finite element or a finite difference methods can be used. The region filled with the liquid is, in general, of irregular shape. To describe its geometry or to perform its discretization the Bézier surfaces can be utilized. In the cases when the region filled with the liquid changes its boundaries there is a possibility to transform solution of the governing equation from the primary region into the unit square domain making use of the dimensionless coordinates. The advantage is that this approach does not require to change the discretization even if the primary region changes its shape. On the other hand the form of the transformed equation is more complicated than the form of the original one.

Keywords: submerged structures, vibration, computational investigation

1. Introduction

Components of a number of stationary or rotating machines work submerged in various liquids to realize the technological process or during the undesirable operating conditions. Their vibration is then significantly influenced by their interaction with the medium in the surrounding space. An important tool for investigation of their behaviour is a computer modelling method.

To set up the equation of motion of a submerged body it is necessary to express the force by which the liquid acts on it. The force components are obtained by integration of the pressure distribution over the surface of the body.

If amplitude of vibration of the body is small, the influence of the liquid on its motion is usually taken into account by additional masses. They express inertia effects of the liquid whose flow is induced by oscillation of the body. In general the additional masses depend on its position relative to the wall of the vessel and determination of their magnitudes starts from calculation of the pressure distribution in the liquid.

The basic governing equations describing the flow and the pressure in liquids and the principal approaches to their solution can be found in a number of publications, e.g. in [1]. The

*Corresponding author. Tel.: +420 59 732 3267, e-mail: jaroslav.zapomel@vsb.cz.

mutual interaction between the vibrating body and the liquid is described in [3]. For the analysis a finite element method is suggested. Properties of the used finite elements are discussed in [2]. Application of a finite difference method in combination with the Bézier surfaces can be found in [6]. Vibration of a rotating rotor having a disc submerged in a perfect liquid is analyzed in [5].

2. Governing equations

An important tool for investigation of behavior of solids submerged in liquids is a computer modelling method. In the computational model it is assumed that (i) the vessel and the bodies are absolutely rigid, (ii) the liquid is perfect and incompressible, (iii) amplitude of the vibration is small, and (iv) the flow and the oscillations are 2D.

Vibration of the body is governed by its equation of motion and the isothermal 2D flow of incompressible Newtonian liquid by the Navier-Stokes equations (1)–(2) and the equation of continuity (3).

$$\frac{\partial p}{\partial x_1} + \rho\left(\frac{\partial v_1}{\partial t} + v_1\frac{\partial v_1}{\partial x_1} + v_2\frac{\partial v_1}{\partial x_2}\right) - \eta\left(\frac{\partial^2 v_1}{\partial x_1^2} + \frac{\partial^2 v_1}{\partial x_2^2}\right) = 0 \tag{1}$$

$$\frac{\partial p}{\partial x_2} + \rho\left(\frac{\partial v_2}{\partial t} + v_1\frac{\partial v_2}{\partial x_1} + v_2\frac{\partial v_2}{\partial x_2}\right) - \eta\left(\frac{\partial^2 v_2}{\partial x_1^2} + \frac{\partial^2 v_2}{\partial x_2^2}\right) = 0 \tag{2}$$

$$\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} = 0 \tag{3}$$

x_1, x_2 — cartesian coordinates,
v_1, v_2 — velocity components in the cartesian frame of reference,
p — pressure,
t — time,
ρ, η — density, dynamical viscosity of the liquid.

Vibration of the body induces the flow and the pressure in the liquid. The pressure produces the hydraulic force that acts back on the body and effects its motion. Therefore the equations governing the oscillation of the body and the flow are mutually coupled via the state parameters.

As it is assumed that amplitude of vibration of the body is small, the Reynolds number of the induced flow is low and therefore the convective terms in (1) and (2) can be neglected. Because the liquid is considered as perfect, the viscous terms are zero. After these modifications the Navier-Stokes equations take a simple form

$$\frac{\partial p}{\partial x_1} + \rho\frac{\partial v_1}{\partial t} = 0 \tag{4}$$

$$\frac{\partial p}{\partial x_2} + \rho\frac{\partial v_2}{\partial t} = 0 \tag{5}$$

Consequently they are differenciated with respect to the cartesian coordinates x_1 and x_2 and added

$$\frac{\partial^2 p}{\partial x_1^2} + \frac{\partial^2 p}{\partial x_2^2} + \rho\frac{\partial}{\partial t}\left(\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2}\right) = 0 \tag{6}$$

Taking into account the equation of continuity (3), the third term on the left hand side of equation (6) is zero and therefore the pressure distribution in the liquid is described by a Laplace equation

$$\frac{\partial^2 p}{\partial x_1^2} + \frac{\partial^2 p}{\partial x_2^2} = 0 \tag{7}$$

To perform its solution the boundary condition must be added. As the liquid is perfect, its particles touching the surface of the body or of the vessel do not adhere to it. Therefore their movement in the normal direction depends on the motion of the body or of the vessel but in the tangential direction their motion is not constrained. The boundary condition then follows from the Navier-Stokes equation transformed to the normal direction

$$\frac{\partial p}{\partial x_n} = -\rho a_n \tag{8}$$

x_n — cartesian coordinate in the normal direction,
a_n — acceleration of the sliding motion of the point on the boundary in the normal direction.

Transformation of (8) into the original cartesian frame of reference gives

$$\frac{\partial p}{\partial x_1} \cos \alpha_n + \frac{\partial p}{\partial x_2} \sin \alpha_n = -\rho \left(a_{x1} \cos \alpha_n + a_{x2} \sin \alpha_n \right) \tag{9}$$

a_{x1}, a_{x2} — accelerations of the sliding motion of the point on the boundary in the x_1, x_2 directions,
α_n — directional angle of the outer normal of the boundary (orientation into the liquid).

3. Bézier surfaces

In general Bézier surfaces are polynomial functions that assign a value of some quantity (geometric or physical) to the points situated in a square domain whose positions are specified by dimensionless coordinates. They are defined by the control points and by the Bernstein polynomials

$$s = \sum_{i=0}^{M} \sum_{j=0}^{N} s_{ij} B_{1i}(u_1) B_{2j}(u_2) \tag{10}$$

where

$$B_{1i}(u_1) = \binom{M}{i} u_1^i (1-u_1)^{M-i} \tag{11}$$

$$B_{2j}(u_2) = \binom{N}{i} u_2^j (1-u_2)^{N-j} \tag{12}$$

s — geometric of physical quantity,
M, N — nonnegative integer numbers defining the number of control points,
s_{ij} — control points,
u_1, u_2 — dimensionless coordinates ($0 \le u_1 \le 1$, $0 \le u_2 \le 1$),
B_{1i}, B_{2j} — Bernstein polynomials.

From the geometric point of view they represent a transformation between the cartesian coordinates of the points situated in the primary region of a general quadrilateral shape and their images in the squre auxiliary domain

$$x_1 = \sum_{i=0}^{M}\sum_{j=0}^{N} x_{1ij}\binom{M}{i}u_1^i(1-u_1)^{M-i}\binom{N}{j}u_2^j(1-u_2)^{N-j} \tag{13}$$

$$x_2 = \sum_{i=0}^{M}\sum_{j=0}^{N} x_{2ij}\binom{M}{i}u_1^i(1-u_1)^{M-i}\binom{N}{j}u_2^j(1-u_2)^{N-j} \tag{14}$$

A suitable selection of control points x_{1ij}, x_{2ij} enables to achieve magnitudes of cartesian coordinates of the points forming the border of the primary region (to achieve the required shape of the border) or to achieve a required distribution of points inside the region. Therefore even if the points in the auxiliary region are distributed regularly, their images in the primary one can be arranged ununiformly. This can be utilized for the discretization to concentrate the nodes in the areas in the primary region where greater gradients of the physical quantities are expected.

4. Analysis of the effect of the fluid on vibration of the body utilizing a finite element method

A finite element method can be used for solving the Laplace's equation (7). As shown in [3] and [2], its solution with the boundary condition (8) is equivalent to minimizing the functional Ψ

$$\Psi = \frac{1}{2}\int_\Omega \left[\left(\frac{\partial p}{\partial x_1}\right)^2 + \left(\frac{\partial p}{\partial x_2}\right)^2\right] \mathrm{d}x_1\,\mathrm{d}x_2 - \int_{\Gamma_B} \rho a_n p\,\mathrm{d}s - \int_{\Gamma_V} \rho a_n p\,\mathrm{d}s \tag{15}$$

Ψ — functional,
Ω — investigated region (area filled with liquid),
Γ_B — interior boundary of the investigated region (outer surface of the body),
Γ_V — outer boundary of the investigated region (interior surface of the vessel).

This approach requires to divide the region filled with the liquid into the finite elements and to set up the Gram matrix and the right hand side vector of the system from the Gram matrices and the right hand side vectors of all elements [3]. These manipulations make possible to express the functional Ψ in the form

$$\Psi = \frac{1}{2}\mathbf{p}^T\mathbf{H}\mathbf{p} - \mathbf{p}^T(a_{B1}\mathbf{g}_{B1} + a_{B2}\mathbf{g}_{B2} + a_{V1}\mathbf{g}_{V1} + a_{V2}\mathbf{g}_{V2}) \tag{16}$$

$\mathbf{H}$ — coefficient matrix,
$\mathbf{p}$ — vector of the pressure,
$\mathbf{g}_{B1}, \mathbf{g}_{B2}$ — coefficient vectors,
$\mathbf{g}_{V1}, \mathbf{g}_{V1}$ — coefficient vectors,
a_{B1}, a_{B2} — x_1, x_2 components of the disc centre acceleration,
a_{V1}, a_{V2} — x_1, x_2 components of the vessel centre acceleration.

The procedure how to derive a triangular fluid element is described in [3]. To achieve the minimum of Ψ it must hold

$$\left[\frac{\partial \Psi}{\partial \mathbf{p}}\right] = \mathbf{o} \tag{17}$$

$\mathbf{o}$ — zero vector.

After performing this manipulation calculation of the pressure distribution in the liquid arrives at solving a set of linear algebraic equations

$$\mathbf{Hp} = a_{B1}\mathbf{g}_{B1} + a_{B2}\mathbf{g}_{B2} + a_{V1}\mathbf{g}_{V1} + a_{V2}\mathbf{g}_{V2} \tag{18}$$

Because of the right hand side of (18) the pressure at all nodes is given as a linear combination of four components. The coefficients of proportionality are acceleration components of the sliding motion of the interior and outer boundries

$$\mathbf{p} = a_{B1}\mathbf{p}^*_{B1} + a_{B2}\mathbf{p}^*_{B2} + a_{V1}\mathbf{p}^*_{V1} + a_{V2}\mathbf{p}^*_{V2} \tag{19}$$

$\mathbf{p}^*_{B1}, \mathbf{p}^*_{B2}$ — vectors of the pressure in all nodes produced by the unit acceleration of the body in the x_1, x_2-directions,
$\mathbf{p}^*_{V1}, \mathbf{p}^*_{V2}$ — vectors of the pressure in all nodes produced by the unit acceleration of the vessel in the x_1, x_2-directions.

Because the liquid is perfect, no tangential forces acting on the submerged body or on the wall of the vessel are produced and the components of the resulting force acting on the body are obtained by integration of the pressure distribution around the circumference and along the height (thickness) of the submerged part of the body

$$\begin{aligned} F_{F1} &= -h_B a_{B1} \int_{\Gamma_B} p^*_{B1} \cos\alpha_n \,\mathrm{d}s - h_B a_{B2} \int_{\Gamma_B} p^*_{B2} \cos\alpha_n \,\mathrm{d}s - \\ &\quad - h_B a_{V1} \int_{\Gamma_V} p^*_{V1} \cos\alpha_n \,\mathrm{d}s - h_B a_{V2} \int_{\Gamma_V} p^*_{V2} \cos\alpha_n \,\mathrm{d}s \end{aligned} \tag{20}$$

$$\begin{aligned} F_{F2} &= -h_B a_{B1} \int_{\Gamma_B} p^*_{B1} \sin\alpha_n \,\mathrm{d}s - h_B a_{B2} \int_{\Gamma_B} p^*_{B2} \sin\alpha_n \,\mathrm{d}s - \\ &\quad - h_B a_{V1} \int_{\Gamma_B} p^*_{V1} \sin\alpha_n \,\mathrm{d}s - h_B a_{V2} \int_{\Gamma_B} p^*_{V2} \sin\alpha_n \,\mathrm{d}s \end{aligned} \tag{21}$$

F_{F1}, F_{F2} — x_1, x_2 components of the hydraulic force acting on the disc,
h_B — height (thickness) of the body (submerged part of the body),
p^*_{B1}, p^*_{B2} — pressure produced by the unit acceleration of the body in the x_1, x_2-directions,
p^*_{V1}, p^*_{V2} — pressure produced by the unit acceleration of the vessel in the x_1, x_2-directions.

The relalationships (20) and (21) can be also expressed in a matrix form

$$\begin{bmatrix} F_{Fy} \\ F_{Fz} \end{bmatrix} = -\mathbf{M}_{BF} \begin{bmatrix} a_{B1} \\ a_{B2} \end{bmatrix} - h_D \begin{bmatrix} \int_{\Gamma_B} p^*_{V1} \cos\alpha_n \,\mathrm{d}s + \int_{\Gamma_B} p^*_{V2} \cos\alpha_n \,\mathrm{d}s \\ \int_{\Gamma_B} p^*_{V1} \sin\alpha_n \,\mathrm{d}s + \int_{\Gamma_B} p^*_{V2} \sin\alpha_n \,\mathrm{d}s \end{bmatrix} \begin{bmatrix} a_{V1} \\ a_{V2} \end{bmatrix} \tag{22}$$

$\mathbf{M}_{BF}$ can be considered as the additional mass matrix of the submerged body. It expresses inertia properties of the liquid by which vibration of the body is effected. In general matrix $\mathbf{M}_{BF}$ is real, full, and not symmetric. Its elements depend on position of the body relative to the wall of the vessel. If the vessel does not vibrate, the second term on the right hand side of (22) takes a zero value.

5. Example 1

Rotor of the investigated rotor system (Fig. 1) consists of a shaft (SH) and of two discs (D1, D2). The shaft is coupled with a rigid frame (FR) by two hydrodynamic bearings (B1, B2). Each of them is equipped with one deep central circumferential groove into which the lubricating oil is supplied. Disc D1 mounted on the overhung end of the shaft is placed in a vessel (VS) filled with a liquid and is totally submerged. Disc D2 is coupled with a rigid rotor of an electric motor by a prestressed flexible belt. The rotor rotates at constant angular speed (300 rad/s) and is loaded by centrifugal forces caused by unbalances of both discs.

The task was to analyze the motion of the rotor after the initial transient component of its vibration is died out.

In the computational model the shaft was represented by a beam like body, both discs were considered as absolutely rigid, and the liquid as perfect and incompressible. For the purpose of the calculation the shaft was discretized into finite elements.

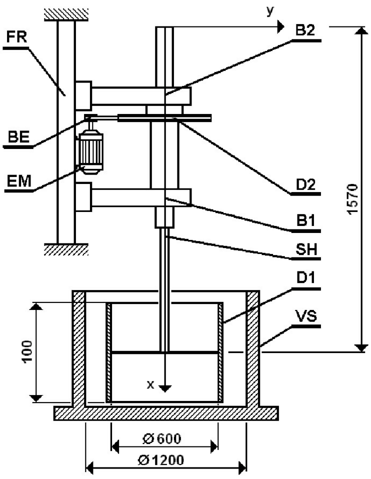

Fig. 1. Scheme of the rotor system

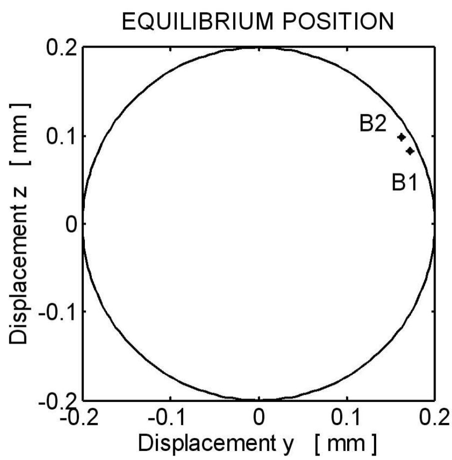

Fig. 2. Equilibrium positions of the journals in B1, B2

Lateral vibration of such rotor is governed by a nonlinear equation of motion

$$\mathbf{M}\ddot{\mathbf{x}} + (\mathbf{B} + \eta_V \mathbf{K}_{SH} + \omega \mathbf{G})\dot{\mathbf{x}} + (\mathbf{K} + \omega \mathbf{K}_C)\mathbf{x} = \mathbf{f}_A + \mathbf{f}_V + \mathbf{f}_B(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{f}_F(\ddot{\mathbf{x}}) \tag{23}$$

$\mathbf{M}, \mathbf{B}, \mathbf{K}, \mathbf{G}, \mathbf{K}_C$ — mass, damping, stiffness, gyroscopic, circulation matrices [8],
$\mathbf{K}_{SH}$ — stiffness matrix of the shaft [8],
$\mathbf{f}_A, \mathbf{f}_V, \mathbf{f}_B, \mathbf{f}_F$ — vectors of applied, constrained, bearing, fluid induced forces,
$\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}$ — vectors of the rotor general displacements, velocities, accelerations,
ω — angular speed of the rotor rotation,
η_V — coefficient of viscous damping of the shaft material.

Determination of the bearing forces is discussed e.g. in [4]. Taking into consideration relationship (22) the force by which the liquid acts on the disc can be expressed in the following manner

$$\mathbf{f}_F(\ddot{\mathbf{x}}) = -\mathbf{M}_F \ddot{\mathbf{x}} + \mathbf{f}_{FV} \tag{24}$$

$\mathbf{M}_F$ — additional mass matrix,
$\mathbf{f}_{FV}$ — vector of the fluid induced forces acting on the disc due to the movement of the vessel.

After a simple manipulation the equation of motion (23) can be modified into this form

$$(\mathbf{M} + \mathbf{M}_F)\ddot{\mathbf{x}} + (\mathbf{B} + \eta_V \mathbf{K}_{SH} + \omega \mathbf{G})\dot{\mathbf{x}} + (\mathbf{K} + \omega \mathbf{K}_C)\mathbf{x} = \mathbf{f}_A + \mathbf{f}_V + \mathbf{f}_B(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{f}_{FV} \tag{25}$$

For its solution a modified Newmark method has been used [7]. Some of the results are summarized in the following figures.

If the rotor were not excited by time varying forces, it would take the equilibrium position and the rotor journal centres in bearings B1 and B2 would be located relative to the bearing shell as drawn in Fig. 2. Trajectory of the disc D1 centre after the initial transient component of the vibration is died out is shown in Fig. 3. As rotation of the shaft at location where the disc is attached to it and because displacement of the disc relative to its diametre and to diametre of the vessel are small, the assumption of 2D flow of the liquid is acceptable. The Fourier transform of the time history of y-displacement of the disc centre is drawn in Fig. 4. It is evident that in the resulting response except the vibration component having the principal frequency (300 rad/s) a number of subharmonic and ultraharmonic ones are excited too.

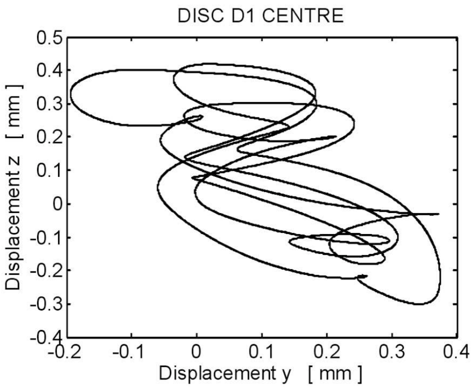

Fig. 3. Trajectory of the disc D1 centre

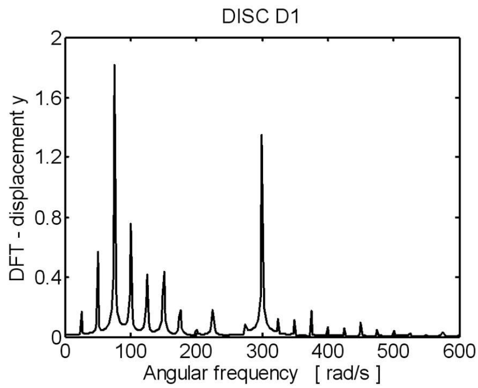

Fig. 4. Fourier transform of the x1-displacement of D1

6. Analysis of the effect of the fluid on vibration of the body utilizing a finite difference method

The region filled with a liquid has often an irregular shape. Therefore one of the ways how to apply a finite difference method is to transform the governing equation into the dimensionless coordinates and to perform its solution on a regular auxiliary domain. For this purpose the Bézier surfaces can be utilized.

This approach requires:

- to assign an auxiliary domain to the primary one,
- to establish a geometric transformation between the coordinates of corresponding points in the primary and auxiliary regions imploying Bézier surfaces, relations (13), (14),
- to convert the governing equation from cartesian into the dimensionless coordinates,
- to solve the transformed governing equation by a finite difference method,
- to transform the results from auxiliary to the primary region.

Utilizing the transformation relationships

$$\frac{\partial p}{\partial x_K} = \frac{\partial p}{\partial u_1}\frac{\partial u_1}{\partial x_K} + \frac{\partial p}{\partial u_2}\frac{\partial u_2}{\partial x_K} \tag{26}$$

$$\frac{\partial^2 p}{\partial x_K^2} = \frac{\partial}{\partial x_K}\left(\frac{\partial p}{\partial u_1}\right)\frac{\partial u_1}{\partial x_K} + \frac{\partial p}{\partial u_1}\frac{\partial^2 u_1}{\partial x_K} + \frac{\partial}{\partial x_K}\left(\frac{\partial p}{\partial u_2}\right)\frac{\partial u_2}{\partial x_K} + \frac{\partial p}{\partial u_2}\frac{\partial^2 u_2}{\partial x_K^2} \tag{27}$$

where $x_K \in \{x_1; x_2\}$ and

$$\frac{\partial}{\partial x_K}\left(\frac{\partial p}{\partial u_1}\right) = \frac{\partial^2 p}{\partial u_1^2}\frac{\partial u_1}{\partial x_K} + \frac{\partial^2 p}{\partial u_1 \partial u_2}\frac{\partial u_2}{\partial x_K} \tag{28}$$

$$\frac{\partial}{\partial x_K}\left(\frac{\partial p}{\partial u_2}\right) = \frac{\partial^2 p}{\partial u_1 \partial u_2}\frac{\partial u_1}{\partial x_K} + \frac{\partial^2 p}{\partial u_2^2}\frac{\partial u_2}{\partial x_K} \tag{29}$$

the Laplace equation (7) and the boundary condition (9) are transformed to the form

$$a_1\frac{\partial^2 p}{\partial u_1^2} + a_2\frac{\partial^2 p}{\partial u_1 \partial u_2} + a_3\frac{\partial^2 p}{\partial u_2^2} = 0 \tag{30}$$

$$b_1\frac{\partial p}{\partial u_1} + b_2\frac{\partial p}{\partial u_2} = -\rho(a_{x1}\cos\alpha_n + a_{x2}\sin\alpha_n) \tag{31}$$

a_1, a_2, a_3, b_1, b_2 — coefficients.

Application of this approach requires to form a rectangular mesh in the auxiliary domain, to relate equations (30) and (31) to its interior and border nodes respectively, and to replace their derivatives by corresponding differencies. These manipulations arrive at a set of linear algebraic equations having the form of (18). The unknowns are pressures in the individual nodes. Performing operations given by relations (20)–(22) leads to the additional mass matrix of the body submerged in the liquid.

7. Example 2

The investigated body is a disc placed in a cylindrical vessel filled with a liquid (Fig. 5). The disc is coupled with the surrounding space by two springs and two dampers and is excited by a rotating force of constant magnitude and angular frequency. The task was to analyze trajectory of the centre of the disc.

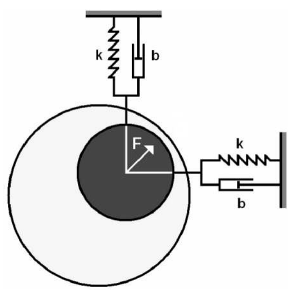

Fig. 5. Scheme of the investigated system

Vibration of the disc is governed by the equations of motion

$$m_D\ddot{x}_{D1} = -b\dot{x}_{D1} - kx_{D1} + F_A\cos\omega_B t + F_{F1} \tag{32}$$

$$m_D\ddot{x}_{D2} = -b\dot{x}_{D2} - kx_{D2} + F_A\sin\omega_B t + F_{F2} \tag{33}$$

m_D — mass of the disc,
b, k — coefficient of linear damping of the damper, stiffness of the spring,
$\mathbf{x}_{D1}, \dot{\mathbf{x}}_{D1}, \ddot{\mathbf{x}}_{D1}$ — displacement, velocity, acceleration of the disc centre in the x_1 direction,

$\mathbf{x}_{D2}, \dot{\mathbf{x}}_{D2}, \ddot{\mathbf{x}}_{D2}$ — displacement, velocity, acceleration of the disc centre in the x_2 direction,
F_A, F_{F1}, F_{F2} — amplitude of the rotating force, hydraulic force in the x_1, x_2 directions,
ω_B — angular frequency of the rotating force.

Before performing their solution components of the hydraulic force by which the liquid acts on the disc must expressed.

$$F_{F1} = -h_D \int_{\Gamma_D} p \cos \alpha_n \, \mathrm{d}s \tag{34}$$

$$F_{F2} = -h_D \int_{\Gamma_D} p \sin \alpha_n \, \mathrm{d}s \tag{35}$$

h_D — height (thickness) of the disc,
Γ_D — boundary of the investigated region (outer surface of the disc).

The pressure distribution in the liquid was obtained by solving the Laplace equation transformed into the dimensionless coordinates (30) by means of a finite difference method. The procedure resulted into solution of a set of linear algebraic equations having the form of (18). Application of relationships (20)–(22) enables to express the hydraulic forces by means of additional masses. Then their substitution in (32) and (33) and taking into account that the vessel does not move lead to the equations of motion of the disc in the resulting form

$$(m_D + m_{F11})\ddot{x}_{1D} + m_{F12}\ddot{x}_{2D} + b\dot{x}_{1D} + kx_{1D} = F_A \cos \omega t \tag{36}$$

$$m_{F21}\ddot{x}_{1D} + (m_D + m_{F22})\ddot{x}_{2D} + bx_{2D} + kx_{2D} = F_A \sin \omega t \tag{37}$$

$m_{F11}, m_{F12}, m_{F21}, m_{F22}$ — additional masses.

For their solution the implicit Newmark method has been applied. Even if amplitude of vibration of the disc was small, the change of geometry of the region filled with the liquid was taken into consideration and values of the additional masses were repeatedly calculated at each integration step. Because solution of the governing equation for the pressure distribution was performed on the auxiliary region, its discretization remained unchanged and the change of geometry resulted only into the change of the coefficients in equation (30) and in the relationship for the boundary condition (31).

Some results of the analysis can be seen in the following figures.

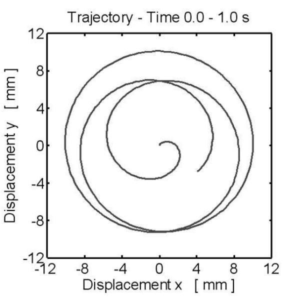

Fig. 6. Disc centre trajectory

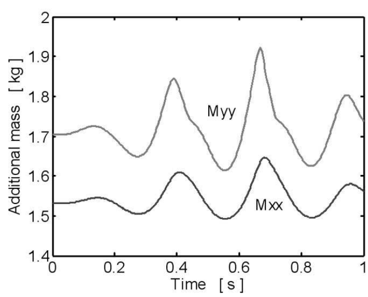

Fig. 7. Additional masses

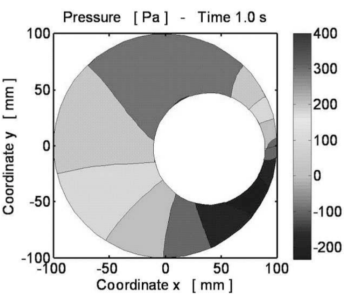

Fig. 8. Pressure distribution

Trajectory of the disc centre and time history of magnitudes of the additional masses during the investigated period are evident from Fig. 6 and 7. Because of vibration position of the disc relative to the wall of the vessel changes and it causes a slight variation of magnitudes of the additional masses. The pressure distribution in the liquid at the specified moment of time is drawn in Fig. 8.

8. Conclusion

The mentioned approaches represent two possibilities for analysis of vibration of stationary and rotating bodies submerged in liquids. In the computational model it is assumed that the bodies and the vessel are absolutely rigid and that the liquid is perfect and incompressible. Both mentioned approaches are based on assigning a square auxiliary domain to the primary region, in general of irregular shape. For geometric transformation between both domains the Bézier surfaces are used.

The first procedure applies a finite element method. The auxiliary domain is discretized, the nodes of the mesh are transformed from auxiliary into the primary region, and solution of the governing equation is performed in the cartesian coordinates. Properties of the Bézier surfaces enable to distribute the nodes in the primary region ununiformly even if the mesh in the auxiliary domain is regular. If the primary region changes its boundaries, the discretization of the auxiliary domain do not need to be changed. The change appears only in magnitudes of the control points of the Bézier surfaces.

The second procedure utilizes a finite difference method. The governing equation describing the pressure distribution is transformed from cartesian into the dimensionless coordinates and the solution is performed on the auxiliary domain. This approach has several advantages. Discretization of the auxiliary domain can be always rectangular even if the primary region is of irregular shape. In the case when the primary domain changes its boundaries the coefficients of the transformed governing equation are functions of time but the discretization of the auxiliary domain is not changed. This approach is suitable especially in cases when the shift of the boundaries during one integration step is larger than is the size of the finite elements or of the cells if the finite difference method is applied.

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