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Modelling and modal properties of the railway vehicle bogie with two individual wheelset drives

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Abstract

The paper deals with mathematical modelling of vibration and modal analysis of two-axled bogie of a railway vehicle. In comparison with recent publications introducing mathematical models of an individual wheelset drive, this paper is focused on modelling of complex bogie vibration. The bogie frame is linked by primary suspension to the two wheelset drives with hollow shafts and by secondary suspension to the car body. The method is based on the system decomposition into three subsystems – two individual wheelset drives including the mass of the rail and the bogie frame coupled with a half of the car body – and on modelling of couplings among subsystems. The eigenvalues of a linearized autonomous model and stability conditions are investigated in dependence on longitudinal creepage and forward velocity of the railway vehicle. The nonlinear model will be used for investigating the dynamic loading of bogie components caused by different types of excitation.

Keywords: railway vehicle bogie, two-axled bogie, bogie vibration, eigenvalues, stability

1. Introduction

Dynamic properties of individual wheelset drives of railway vehicles are usually investigated using torsional models, as it was shown e.g. in [4], [6] and for drives with a hollow shaft in [8]. These models however do not enable investigation of spatial vibration of drives components caused by the track irregularities, wheelsets unbalance and by polygonalized running surface of the wheels. Hence, new and complex models of railway vehicles or of their components, presented e.g. in books [3], [7], in the latest works [2], [5] and there cited papers, were developed. None of mentioned works contains detailed models of wheelset drive components and of couplings among them e.g. gearing, clutches, elastic supports of engine stators and of gear housings to the bogie frame etc. From this point of view, individual wheelset drives with a hollow shaft embracing the wheelset axle (fig. 1) indicate some specific features. Their dynamic properties were investigated in [10] and the extended model including bending vibration of the wheelset supported by elastic ballast is studied in [1]. The excitation caused by track irregularities and wheel running surface is transmitted from both wheelsets through the primary and secondary suspension elements to the car body and to the bogie frame, whose vibration retroactively influences the motion of both individual wheelsets. The influence of visco-elastic couplings among mentioned subsystems on modal properties of wheelset drives was not yet investigated.

The aim of this article is to develop an original mathematical model of the whole bogie including two individual wheelset drives with a hollow shaft and to parametrize the model for

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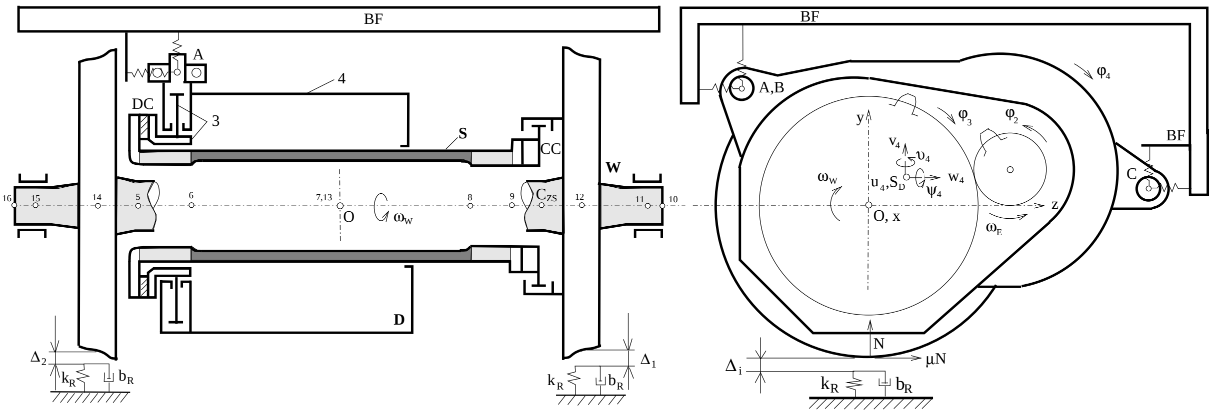

Fig. 1. Scheme of the individual wheelset drive.

particular bogie of the electric locomotive 109E which is being developed for speeds about $200\ km/h$ by the company ŠKODA TRANSPORTATION s. r. o. In this contribution, the linearized model is derived and modal properties are investigated in dependence on the longitudinal creepage of wheels and on forward velocity of the locomotive. Further, such operational conditions are studied, when the system becomes unstable and the antislip protection has to be activated.

2. The methodology of creation of complex mathematical model of the bogie

To develop the complex mathematical model of the bogie (fig. 2) it is efficient to disassemble the bogie into three subsystems - *individual drives* (ID1 and ID2), that include couplings among wheels, rails and ballast and are placed central symmetrical in the bogie, and further into a *bogie frame* linked by secondary suspension and dampers with a half of car body (BFCB) (fig. 3).

In the *first phase*, the conservative mathematical model of mutually isolated subsystems in their local configuration spaces defined by generalized coordinates $\mathbf{q}_{ID1}, \mathbf{q}_{BFCB}, \mathbf{q}_{ID2}$ is created. After defining global vector of generalized coordinates of the system, couplings among subsystems are modelled and are supposed to be ideally elastic. Especially, the support of engine stators and drives housings of both individual drives are concerned. The individual drives are linked with the bogie frame by rubber silent blocks placed at points A_1, B_1, C_1 and A_2, B_2, C_2 representing centers of elasticity. The primary suspension between journal boxes and bogie frame is placed at points P_5, P_6, P_9, P_{10}.

The validity of physical structure is examined by eigenmode calculation using conservative models of subsystems and of the whole system linked by elastic couplings. Eigenmodes corresponding to zero eigenfrequencies are characterized by a motion with no couplings and components deformation.

In the *second phase* of modelling, the conservative model is completed with the damping influence of internal couplings of subsystems (gearing damping, clutch damping, ballast damping and damping of secondary suspensions linked at points $T_1 - T_6$) and with damping of couplings among subsystems (silent blocks, primary suspension, among journal boxes and bogie frame at point $T_7 - T_{10}$).

In the last, *third phase* of modelling, the creep forces in the wheel-rail contact, drive torques of engines, static load given by gravitational forces and the kinematic excitation representing

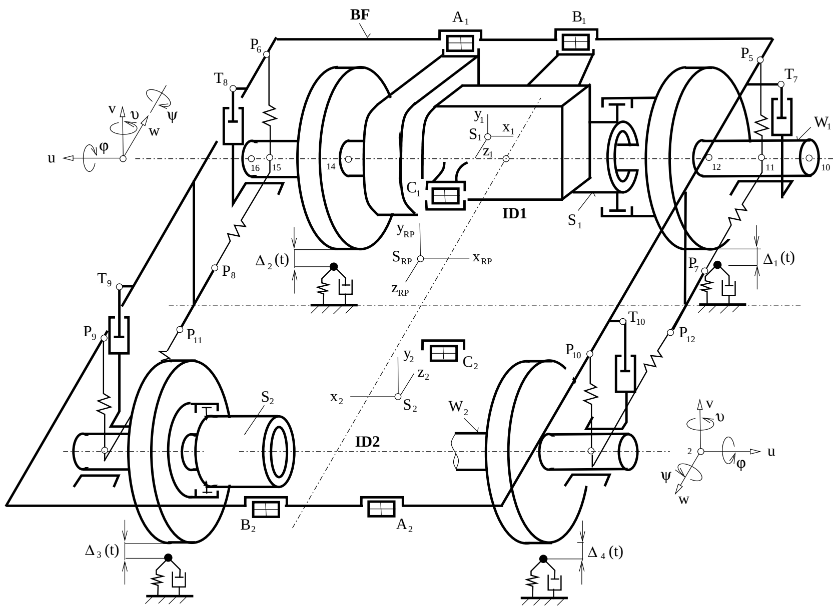

Fig. 2. Scheme of the bogie.

track irregularities and wheel running surface are defined and the mathematical model is completed by their influence.

2.1. Conservative mathematical model of individual uncoupled wheelset drive

The conservative model of individual drive (fig. 1) is expressed under the assumption of absolutely smooth rails in the configuration space

$$\mathbf{q}_{ID} = [\mathbf{q}_D^T \ \mathbf{q}_S^T \ \mathbf{q}_W^T]^T, \tag{1}$$

where the vectors of generalized coordinates

$$\begin{aligned} \mathbf{q}_D &= [\boldsymbol{\varphi}^T \ \mathbf{q}_4^T]^T \in \mathbb{R}^9, \quad \boldsymbol{\varphi}^T = [\varphi_1 \ \varphi_2 \ \varphi_3], \\ \mathbf{q}_S &= [\mathbf{q}_5^T \ \mathbf{q}_6^T \ \ldots \ \mathbf{q}_9^T]^T \in \mathbb{R}^{28}, \quad \mathbf{q}_9^T = [u_9 \ \varphi_9 \ \vartheta_9 \ \psi_9], \\ \mathbf{q}_W &= [\mathbf{q}_{10}^T \ \mathbf{q}_{11}^T \ \ldots \ \mathbf{q}_{16}^T]^T \in \mathbb{R}^{42}, \end{aligned} \tag{2}$$

are assigned to single drive (D), which is assembled from components 1 (engine rotor), 2 (drive pinion), 3 (gear with the catch driver of the driven part of the disc clutch) and 4 (engine stator linked with the gear housing), to composite hollow shaft (S) with the driven part of the disc clutch (DC) and with the driving part of the claw clutch (CC) and to the wheelset (W) including the coupling among wheels, rails and ballast, respectively. The composite hollow shaft and the wheelset are modelled as spatial vibrating one-dimensional continua discretized by finite element method at nodal points 5 – 9 (S), 10 – 16 (W) with rigid discs mounted at nodes 5 (DC), 9 (CC), 11, 15 (journals) and 12, 14 (wheels), respectively. The vectors of generalizes coordinates $\mathbf{q}_i$ in nodal points $i = 4, \ldots, 8, 10, \ldots, 16$ have the form $\mathbf{q}_i = [u_i \ v_i \ w_i \ \varphi_i \ \theta_i \ \psi_i]^T$, where u_i, v_i, w_i are translational deflections in the coordinate axes x, y, z and φ_i, θ_i, ψ_i are rotational deflections around these axes shifted to corresponding node. Transversal displacements v_9 and w_9 at the node 9 of the composite shaft are due to torque transmission by the claw clutch coupled with displacements of the wheel centre at the node 12 and therefore are not independent. The components 1, 2, 3 rotate within the spatially vibrating engine stator, which is wired in the gearbox housing whose displacements are described by u_4, v_4 to ψ_4 outspread to the mass centre S_D (fig. 1). Angular speeds of the engine rotor $\dot{\varphi}_1 = \omega_E$ and of the wheelset $\dot{\varphi}_i = \omega_W$, $i = 10$ to 16, correspond to pure rolling of the wheelset by operational speed of the vehicle v. In the above described configuration space, the conservative model of individual uncoupled wheelset drive is described by symmetrical mass and stiffness matrices [1]

$$\begin{aligned} \mathbf{M}_{ID} &= \text{diag}(\mathbf{M}_D\ \mathbf{M}_S\ \mathbf{M}_W) + \mathbf{M}_{CC}, \\ \mathbf{K}_{ID} &= \text{diag}(\mathbf{K}_D\ \mathbf{K}_S\ \mathbf{K}_W) + \mathbf{K}_{CC} + \mathbf{K}_{DC}, \end{aligned} \tag{3}$$

of order $n_{ID} = 79$. The matrix indices correspond to before mentioned designation of system components. The matrix $\mathbf{K}_D$ displays the influence of discrete couplings compliance – driven shaft, gearing and ballast – and matrices $\mathbf{K}_{DC}$ and $\mathbf{K}_{CC}$ involve compliance of disc and claw clutch. The validity of physical structure of the uncoupled drive is examined by eigenmode calculation. Eight of zero eigenfrequencies have to correspond to eigenmodes which are characterized by a motion with no couplings and components deformation.

2.2. Conservative mathematical model of bogie frame linked with car body

Vibration of this subsystem is modelled under the assumption of spatial vibration of rigid bogie frame (BF), which is described by the vector $\mathbf{q}_{BF} = [u_{BF}\ v_{BF}\ w_{BF}\ \varphi_{BF}\ \vartheta_{BF}\ \psi_{BF}]^T$ and linked by the secondary suspension (fig. 3) with a half of car body (CB). We assume, the car body moves in the vertical direction only. This subsystem is now displayed in the configuration space

$$\mathbf{q}_{BFCB} = [\mathbf{r}_{BF}^T\ \boldsymbol{\varphi}_{BF}^T\ \mathbf{r}_{CB}^T]^T \in \mathbb{R}^9, \tag{4}$$

where the coordinates of vectors $\mathbf{r}_{BF}$ and $\mathbf{r}_{CB}$ express lateral, vertical and longitudinal displacements of mass centres of corresponding bodies and the coordinates of the vector $\boldsymbol{\varphi}_{BF}$ describe angle displacements of the bogie frame (fig. 3).

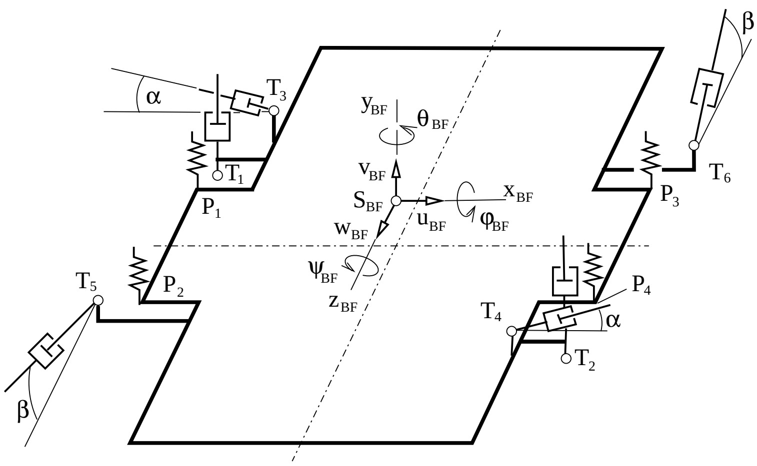

Fig. 3. Scheme of the bogie frame with secondary suspension and dampers.

Kinetic and potential energy of the subsystem is then given by following terms

$$\begin{aligned} E_k &= \frac{1}{2} m_{BF} \dot{\mathbf{r}}_{BF}^T \dot{\mathbf{r}}_{BF} + \frac{1}{2} \dot{\boldsymbol{\varphi}}_{BF}^T \mathbf{I}_{BF} \dot{\boldsymbol{\varphi}}_{BF} + \frac{1}{2} m_{CB} \dot{\mathbf{r}}_{CB}^T \dot{\mathbf{r}}_{CB}, \\ E_p &= \sum_{i=1}^{4} \frac{1}{2} \left(\mathbf{r}_{BF}^T - \mathbf{r}_{CB}^T + \boldsymbol{\varphi}_{BF}^T \mathbf{R}_{P_i} \right) \mathbf{K}_P \left(\mathbf{r}_{BF} - \mathbf{r}_{CB} + \mathbf{R}_{P_i}^T \boldsymbol{\varphi}_{BF} \right), \end{aligned} \tag{5}$$

where m_{BF}, m_{CB} are masses of components, $\mathbf{I}_{BF}$ is inertia matrix of the bogie frame expressed in the coordinate system x_{BF}, y_{BF}, z_{BF} whose origin is identical to the centre of mass S_{BF} (fig. 3), $\mathbf{K}_P$ is diagonal matrix of secondary suspension springs in their local parallel coordinate systems with the origin placed at points P_i and antisymmetrical matrices $\mathbf{R}_{P_i}$ correspond to radiusvectors of cross product corresponding to the points P_i.

Substituting the term (5) to Lagrange's equations we obtain conservative model of the subsystem described by symmetrical mass and stiffness matrices $\mathbf{M}_{BFCB}$ and $\mathbf{K}_{BFCB}$ of order 9. Eliminating the 7. and 9. row and column, the model describes vertical vibration of the car body only. The subsystem has one eigenmode with no springs deformation corresponding to zero eigenfrequency.

2.3. Modelling of couplings among subsystems

To model couplings among subsystems, it is efficient to define the global vector of generalized coordinates of the whole system in the block form (tab. 1). Subvectors containing one

$$\mathbf{q} = [\ \overbrace{\overline{\boldsymbol{\varphi}}^T\ \ \overline{\mathbf{q}}_4^T\ \ \overline{\mathbf{q}}_S^T\ \ \overline{\mathbf{q}}_W^T}^{\mathbf{q}_{ID1}^T}\ \ \overbrace{\mathbf{q}_{BF}^T\ \ v_{CB}}^{\mathbf{q}_{BFCB}^T}\ \ \overbrace{\overline{\overline{\boldsymbol{\varphi}}}^T\ \ \overline{\overline{\mathbf{q}}}_4^T\ \ \overline{\overline{\mathbf{q}}}_S^T\ \ \overline{\overline{\mathbf{q}}}_W^T}^{\mathbf{q}_{ID1}^T}\ ]^T$$

1 2 3 4

$$1\ \ldots\ \mathbf{K}_{W_1,BF},\ 2\ \ldots\ \mathbf{K}_{D_1,BF},\ 3\ \ldots\ \mathbf{K}_{D_2,BF},\ 4\ \ldots\ \mathbf{K}_{W_1,BF}$$

Tab. 1. Table of generalized coordinates.

(two) bars are assigned to individual drive ID1 (ID2) and were defined in (2).

The matrix $\mathbf{K}_{D,BF} = \mathbf{K}_{D_1,BF} + \mathbf{K}_{D_2,BF}$ describes the support of engine stators with gear housings to the bogie frame at silent blocks A_1, B_1, C_1 for ID1 and A_2, B_2, C_2 for ID2 (fig. 2) and is derived by the methodology presented in [9]. The stiffness of primary suspension at points P_5, P_6, P_9, P_{10} and longitudinal wheelset guide of both whelsets between journal boxes and bogie frame at points P_7, P_8, P_{11}, P_{12} (fig. 2) is modelled by the matrix $\mathbf{K}_{W,BF} = \mathbf{K}_{W_1,BF} + \mathbf{K}_{W_2,BF}$. All coupling stiffness matrices are symmetrical of order 165 and their nonzero elements correspond to coupling displacements of linked components according to their position in the global vector of generalized coordinates, as mentioned in tab. 1.

2.4. Conservative model of the railway vehicle bogie

In the configuration space

$$\mathbf{q} = [\mathbf{q}_{ID1}^T\ \mathbf{q}_{BFCB}^T\ \mathbf{q}_{ID2}^T]^T \in \mathbb{R}^{165}, \tag{6}$$

which is in detail described in tab. 1, the conservative model of the bogie (system) is defined by symmetrical mass and stiffness matrices of order 165, having form

$$\begin{aligned} \mathbf{M} &= \mathrm{diag}(\mathbf{M}_{ID}\ \mathbf{M}_{BFCB}\ \mathbf{M}_{ID}), \\ \mathbf{K} &= \mathrm{diag}(\mathbf{K}_{ID}\ \mathbf{K}_{BFCB}\ \mathbf{K}_{ID}) + \mathbf{K}_{D,BF} + \mathbf{K}_{W,BF}. \end{aligned} \tag{7}$$

Chosen eigenfrequencies of conservative models of uncoupled individual drive and of the whole bogie together with the characteristic of eigenmode vibration are presented in tab. 2. According to analysis of eigenfrequencies, a number of higher frequencies from the 22. for ID

Individual drive		Bogie		
ν	f_ν [Hz]	ν	f_ν [Hz]	eigenmode characterization
$1 \div 8$	0	1,2	0	torsion of ID1 or ID2 with no deformation
9	0.459	3	0.990	vertical of CB in phase with BF and ID
10	0.615	4	1.59	lateral of BF in phase with ID
11	4.31	5	1.60	longitudinal of BF in phase with ID
12	12.16	6	1.75	yaw of BF in phase with ID and lateral of ID1 in opposite phase with ID2
13	13.77	7	3.88	torsion of IDs in phase, in opposite phase with Ws
14	29.20	8	3.96	torsion of IDs in opposite phase and in opposite phase with Ws
15	48.97	9	6.01	tilting of BF about lateral axis in phase with stators
$43,44^\star$	1604	10	6.72	vertical of BF in phase with ID
$76 \div 79^\star$	44132	11	9.65	longitudinal of Ws in opposite phase
—	—	12	10.95	tilting of BF around longitudinal axis
—	—	13	11.01	yaw of Ws and of stators in phase with BF
—	—	14	12.11	yaw of Ws in opposite phase
—	—	15	12.84	tilting of BF around longitudinal axis
—	—	$28,29^\star$	48.97	torsional twisting of Ws
—	—	$92 \div 95^\star$	1604	bending of Ws between wheels
—	—	$158 \div 165^\star$	44132	bending of wheelset ends

* double number of the bogie natural frequencies compared to the individual wheelset drive

Tab. 2. Natural frequencies of the individual wheelset drives and of the bogie.

and couples of eigenfrequencies from the couple of 43. and 44. eigenfrequency for the ID repeat in the mathematical model of the bogie. This property is given by the wheelset symmetry and by two identical individual drives in the bogie. Because of the linkage of the drives to the bogie, such single eigenmodes exist, which correspond to lateral, longitudinal, roll or yaw motion of the bogie (e.g. 4. ,5., 6., 9., 12. and 15. eigenmode).

3. Complex mathematical model of the bogie

3.1. Modelling of damping

According to the methodology of bogie model creation, influences of internal coupling damping are appended to models of subsystems. The structure of damping matrix of individual drive has a similar structure as the stiffness matrix described in (3)

$$\mathbf{B}_{ID} = \mathrm{diag}(\mathbf{B}_D\ \mathbf{B}_S\ \mathbf{B}_W) + \mathbf{B}_{DC} + \mathbf{B}_{CC}, \tag{8}$$

whereas the matrix $\mathbf{B}_D$ includes the gearing damping b_z. Further, we suppose the material damping of the composite shaft, disk and claw clutches to be proportional to corresponding stiffness matrices

$$\mathbf{B}_S = \beta_S \mathbf{K}_S, \quad \mathbf{B}_{DC} = \beta_{DC} \mathbf{K}_{DC}, \quad \mathbf{B}_{CC} = \beta_{CC} \mathbf{K}_{CC}. \tag{9}$$

The damping matrix of the wheelset $\mathbf{B}_W$ includes damping influence of the rail ballast and is diagonal with nonzero elements $b_{14,14} = b_{26,26} = b_R$. Particular form of matrix $\mathbf{B}_{BFCB}$ is similar to stiffness matrix $\mathbf{K}_{BFCB}$, which is derived using Lagrange's equations. The matrix respects damping coefficients of each damper of the secondary suspension mounted at points T_1 to T_6 (fig. 3).

Damping matrix of the bogie has a structure which is similar to the structure of stiffness matrix defined in (7)

$$\mathbf{B} = \mathrm{diag}(\mathbf{B}_{ID}\ \mathbf{B}_{BFCB}\ \mathbf{B}_{ID}) + \mathbf{B}_{D,BF} + \mathbf{B}_{W,BF}. \tag{10}$$

Proportional damping matrix $\mathbf{B}_{D,BF} = \beta\mathbf{K}_{D,BF}$ expresses the damping influence of silent blocks which support engine stators and their housings to bogie frame. Matrix $\mathbf{B}_{W,BF}$ describes damping of primary suspension among journal boxes and bogie frame at points T_7 to T_{10} (fig. 2).

3.2. External and adhesion (creep) forces acting on the bogie

To analyze the modal properties of the bogie we neglect track and wheel irregularities which are source of kinematic excitation ($\Delta_i(t) \equiv 0, i = 1, 2, 3, 4$). Let us suppose an operational state of the railway vehicle running on the straight track which is given by the longitudinal creepage s_0 of all wheels, by forward velocity v of the vehicle and by vertical wheel force N_0. To all mentioned operational parameters correspond engine torques, drawing force of the bogie and longitudinal creep forces at the contact between rails and wheels given by

$$M(s_0, v) = 2\mu_0 N_0 r\frac{1}{p}, \quad F_0 = 4\mu_0 N_0, \quad T_0 = \mu_0 N_0, \tag{11}$$

where $\mu_0 = \mu(s_0, v)$ is longitudinal creep coefficient [10, 1], $p = \omega_E/\omega_W$ is speed ratio and r is the wheel radius.

If the static equilibrium is disturbed by any of possible excitation sources, the bogie vibrates and the vector of generalized coordinates can be expressed as a sum of static and dynamic displacements

$$\mathbf{q}(t) = \mathbf{q}_0 + \Delta\mathbf{q}(t), \tag{12}$$

where before the disturbance, the velocity vector $\dot{\mathbf{q}}_0$ has nonzero coordinates corresponding to rotation of system components with forward velocity v. Therefore other coordinates of vector $\Delta\dot{\mathbf{q}}(t)$ are identical with $\dot{\mathbf{q}}(t)$, that is why we delete the designation $\Delta\dot{\mathbf{q}}_i$ in there.

Longitudinal $T_{i\,ad}$, lateral $A_{i\,ad}$ creep forces and spin torque $M_{i\,ad}$ act at the contact patches between rails and wheels and their magnitude can be expressed in following way (index i corresponds to nodes designation to which wheels are fixed on the axles)

$$\begin{aligned} T_{i\,ad} &= \mu(s_i, v)N_i, \\ A_{i\,ad} &= b_{22}(\dot{u}_i + r\dot{\psi}_i) + b_{23}\dot{\vartheta}_i, \\ M_{i\,ad} &= -b_{23}(\dot{u}_i + r\dot{\psi}_i) + b_{33}\dot{\vartheta}_i. \end{aligned} \tag{13}$$

In the term concerning longitudinal creep forces, longitudinal adhesion coefficient μ was introduced [6], [8], which depends on longitudinal creepage defined by

$$s_i = s_0 + \frac{\pm\dot{w}_i \mp r\Delta\dot{\varphi}_i}{v}, \quad s_0 = \frac{r\omega_W}{v}. \tag{14}$$

Upper signs correspond to wheelset W_1 and lower signs to wheelset W_2, which rotate with angular velocity ω_W before the system disturbance. Coefficients b_{ij} agree with Kalker's coefficients [3] computed for constant wheel force N_0.

To analyze modal properties and stability conditions of the bogie, torque characteristics of engines and creep characteristics are linearized in the neighbourhood of the state before the disturbance. We obtain

$$M = M(s_0, v) - b_M \Delta\dot{\varphi}_1,$$
$$\mu(s_i, v) = \mu_0 + \left[\frac{\partial \mu}{\partial s_i}\right]_{s_i = s_0} (s_i - s_0)\,. \tag{15}$$

Linearized longitudinal creep forces can be then expressed for $N_i = N_0$ in the form

$$T_{i\,ad} = \mu_0 N_0 + b_{11}(\pm \dot{w}_i \mp r\Delta\dot{\varphi}_i) \tag{16}$$

and according Kalker's theory we have defined the coefficient of linearized longitudinal damping at the contact patch

$$b_{11} = \frac{N_0}{v}\left[\frac{\partial \mu}{\partial s_i}\right]_{s_i = s_0}. \tag{17}$$

After expressing the engine torque according (15) and creep forces according (13) and (16), the vector of external and creep forces can be written in following form

$$\mathbf{f}(t) = \mathbf{f}_0 + \Delta\mathbf{f}, \tag{18}$$

where $\mathbf{f}_0$ is vector of static force effects before the disturbance, defined in (11), including gravitational forces. The disturbance vector of linearized engine torques and creep force effects have form

$$\Delta\mathbf{f} = -\left[\mathbf{B}_M + \mathbf{B}_{ad}(s_0, v)\right]\Delta\dot{\mathbf{q}}, \tag{19}$$

where the structure of mentioned matrices results from the definition of the vector of generalized coordinates in tab. 1.

3.3. Linearized model of the bogie

By completion of linearized model derived in chap. 2.4 with the influence of damping (chap. 3.1) and with external and creep forces (chap. 3.2) we obtain full linearized model of the bogie. It has the from

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{B}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{f}(t), \tag{20}$$

where matrices are given by terms (7) and (10). When we express vector of generalized coordinates $\mathbf{q}(t)$ in the form (12) and vector of external and creep forces $\mathbf{f}(t)$ according to (18) and (19) with respect to the static equilibrium condition $\mathbf{K}\mathbf{q}_0 = \mathbf{f}_0$, we obtain

$$\mathbf{M}\Delta\ddot{\mathbf{q}}(t) + [\mathbf{B} + \mathbf{B}_M + \mathbf{B}_{ad}(s_0, v)]\Delta\dot{\mathbf{q}}(t) + \mathbf{K}\Delta\mathbf{q}(t) = \mathbf{0}. \tag{21}$$

The matrix $\mathbf{B}_M = \mathrm{diag}(b_M\ 0\ \ldots\ 0\ b_M\ 0\ \ldots)$ is diagonal with nonzero elements on positions 1,1 and 87,87, matrix

$$\mathbf{B}_{ad}(s_0, v) = \mathrm{diag}(\ldots\ \overline{\mathbf{B}}_{ad}\ \ldots\ \overline{\mathbf{B}}_{ad}\ \ldots\ \overline{\mathbf{B}}_{ad}\ \ldots\ \overline{\mathbf{B}}_{ad}) \tag{22}$$

is block diagonal with nonzero nonsymmetrical blocks

$$\overline{\mathbf{B}}_{ad} = \begin{bmatrix} b_{22} & 0 & 0 & 0 & b_{23} & rb_{22} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_{11} & -rb_{11} & 0 & 0 \\ 0 & 0 & -rb_{11} & r^2 b_{11} & 0 & 0 \\ -b_{23} & 0 & 0 & 0 & b_{33} & -rb_{23} \\ rb_{22} & 0 & 0 & 0 & rb_{23} & r^2 b_{22} \end{bmatrix} \tag{23}$$

localized at positions corresponding to displacements of nodal points $i = 12, 14$ for both wheelsets (fig. 2).

4. Spectral properties and stability conditions of the bogie

It is efficient to investigate spectral properties in dependence on operational parameters s_0, v, N_0. The adhesion coefficient μ_0 is expressed according terms (28) and (29) in [10] in dependence on longitudinal creepage s_0 and on forward velocity v. All coefficients b_{ij} in matrix $\overline{\mathbf{B}}_{ad}$ depend further on vertical wheel force N_0. To perform the analysis of spectral properties, the coefficients were evaluated for standard wheel-rail contact conditions, see [8], [10], and for $N_0 = 1.055 \cdot 10^5$ [N].

Eigenvalues of linearized model of the bogie (21) are defined by eigenvalue problem solution

$$[\lambda_\nu \mathbf{N}(s_0, v) + \mathbf{P}]\mathbf{u}_\nu = \mathbf{0} \tag{24}$$

in the state space $\mathbf{u} = [\Delta\dot{\mathbf{q}}^T \;\; \Delta\mathbf{q}^T]^T$, defined by matrices

$$\mathbf{N}(s_0, v) = \begin{bmatrix} \mathbf{0} & \mathbf{M} \\ \mathbf{M} & \mathbf{B} + \mathbf{B}_M + \mathbf{B}_{ad}(s_0, v) \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} -\mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{bmatrix}. \tag{25}$$

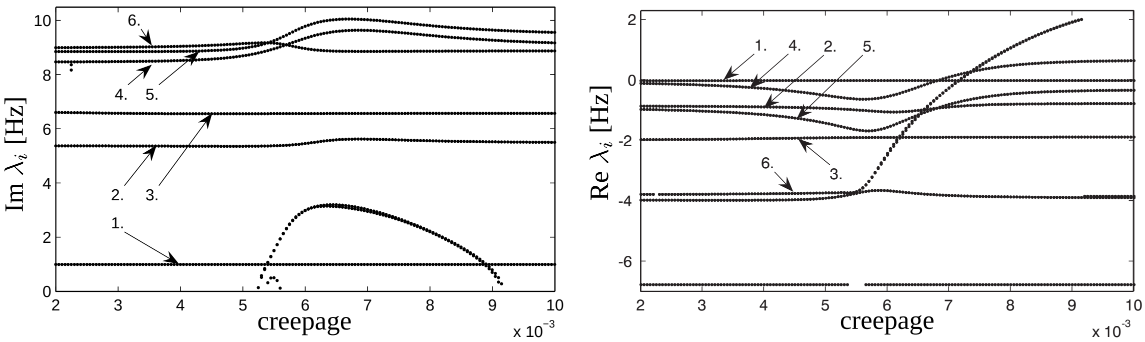

Fig. 4. Dependence of real and imaginary parts of eigenvalues on longitudinal creepage s_0.

For an illustration, in fig. 4 we present the dependence of real and imaginary parts of eight lowest eigenvalues on the longitudinal creepage s_0 which are calculated for the forward velocity $v = 200\ kmph$. According to the analysis, we can conclude that the stability is determined by a pair of complex conjugate eigenvalues. The stability boundary is defined by the creepage $s_0 = 0.0086$ for $v = 100\ kmph$ and by the creepage $s_0 = 0.0067$ for $v = 200\ kmph$. This type of unstability is known as flutter and for higher values of creepage the type of system unstability changes to divergence stability.

5. Conclusion

This paper presents an original methodology of mathematical modelling of the railway bogie including two individual drives with hollow shafts embracing wheelset axles. The methodology is based on the system decomposition to three subsystems – individual drives and the bogie frame linked with the car body – and on modelling of couplings among subsystems. Models of individual drives described in mutually revolved coordinate systems are identical. From the analysis of modal properties ensues, that the elastic support of engine stators and gear drives housings to spatial vibrating bogie frame influences dynamical properties of drives. Therefore, it is necessary to investigate dynamic loading of drives components caused by excitation sources (track irregularities, ballast properties, unbalance of wheels and their ovality), which cause spatial vibration of the bogie frame, using bogie model that represents a linked system.

The linearized model expressed in perturbance coordinates with respect to operational state of static equilibrium before the perturbation in dependence on operational parameters is used to analyze eigenvalues which are further used to detect resonant states with periodical excitation sources or stability conditions of the system. In a close future, the nonlinear model of the railway bogie completed with excitation will be used for simulation of vibration and of dynamical loading of drives components caused by different excitation sources.

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