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Methods and tools for simplified dynamic simulations in real time based on expression approximation

M. Štefan[a], Z. Šika[a], M. Valášek[a,*]

[a] *Dept. of Mechanics, Faculty of Mechanical Engineering, CTU in Prague, Karlovo nám. 13, 121 35 Prague 2, Czech Republic*

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Abstract

The core of this paper is the methodology of the dynamical models' simplification for the real time simulation. The simplified simulation models are based on neuro-fuzzy modelling approach, which was originally designed for predictive control-oriented modelling of nonlinear dynamical systems. The two ways of the neuro-fuzzy modelling utilization are presented. First, the training of the predictive dynamical neuro-fuzzy model and, second, the training of the statical approximation of the right-hand side of the system's state space description. We demonstrate the results on the examples of nonlinear spring damper system and double pendulum.

Keywords: real time simulation, LOLIMOT, neuro-fuzzy model, identification of dynamic systems, state space

1. Introduction

For the real time simulation of complex dynamical systems one often needs to find simplifications of the original model. In practice, such simplifications are developed usually ad hoc and some robust unified strategy is missing. On the other hand, in the model predictive control branch the neuro-fuzzy models for short time simulation (i.e., prediction) are utilized. We investigate the possibilities of the adaptation of the neuro-fuzzy approach in the real time simulation.

To identify a dynamic system from measured data, we use the algorithm called LOLIMOT that builds the so called neuro-fuzzy model of the dynamic system under the consideration. So far, this methodology was used in such a way that the values of quantities have been considered in several consequent times to model the derivatives. We propose here a different utilization of the neuro-fuzzy identification methodology that allows also for the measured derivatives of the quantities and, mainly, builds statical neuro-fuzzy models of the right-hand side of the state space description of the system's dynamics.

2. Neuro-fuzzy models and LOLIMOT

We shall briefly present here the basic concepts and terminology used in the neuro-fuzzy identification as introduced by [1].

*Corresponding author. Tel.: +420 224 357 361, e-mail: michael.valasek@fs.cvut.cz.

2.1. Neuro-fuzzy model of a system

Roughly speaking, neuro-fuzzy model of a system, be it statical or dynamical, linear or nonlinear, is a such a model that is composed of linear approximations of the system's behavior in different areas of the measured data region. The parameters of these local linear models (LLMs) are estimated by, e.g., the least square method applied to the measured data set. The data sub-regions are specified by the so called validity functions.

The output $\hat{y}$ of the neuro-fuzzy model (*LOLI-model*) with p inputs $\mathbf{u} = (u_1 \dots u_j \dots u_p)$ is given as

$$\hat{y} = \sum_{i=1}^{M} \hat{y}_i \Phi_i(\mathbf{u}) = \sum_{i=1}^{M} (w_{i0} + w_{i1}u_1 + w_{i2}u_2 + \cdots + w_{ip}u_p)\Phi_i(\mathbf{u}) \tag{1}$$

where M is the number of the data sub-regions (see below) represented by the normalized orthogonal Gaussian validity functions Φ_i defined as

$$\Phi_i(\mathbf{u}) = \frac{\mu_i(\mathbf{u})}{\sum\limits_{j=1}^{M} \mu_j(\mathbf{u})}; \tag{2}$$

the quantity

$$\mu_i(\mathbf{u}) = exp\left(-\frac{1}{2}\left(\frac{(u_1 - c_{i1})^2}{\sigma_{i1}^2} + \frac{(u_2 - c_{i2})^2}{\sigma_{i2}^2} + \cdots + \frac{(u_p - c_{ip})^2}{\sigma_{ip}^2}\right)\right) \tag{3}$$

is the so-called membership function (MSF) with c_{ij} being the centers of these sub-regions. For each sub-region a local linear model $\hat{y} = (1\, u_1 \dots u_p)^T \mathbf{w} = \mathbf{x}^T \mathbf{w}$, specified by the parameters $\mathbf{w}_i = (\dots w_{ik} \dots)^T$, $i = 1..M$, $k = 0..p$, is constructed.

2.2. Estimation of the LLM's parameters

Ordering the measured data into a matrix

$$\mathbf{X} = \begin{pmatrix} 1 & u_{11} & u_{21} & \cdots & u_{p1} \\ 1 & u_{12} & u_{22} & \cdots & u_{p2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & u_{1N} & u_{2N} & \cdots & u_{pN} \end{pmatrix} = \begin{pmatrix} \vdots \\ \mathbf{x}_s \\ \vdots \end{pmatrix} \tag{4}$$

where each row represents single data sample $\mathbf{x}_s$ of all inputs, $s = 1..N$ (N being the total number of the measured samples), and forming the diagonal matrix $\mathbf{Q}_i$ of the validity function values Φ_i with respect to the ith LLM and corresponding to each measured data sample as $\mathbf{Q}_i = diag(\dots \Phi_i(\mathbf{u}_s) \dots)$, the ith LLM's parameters can be estimated according to

$$\mathbf{w}_i = \left(\mathbf{X}^T \mathbf{Q}_i \mathbf{X}\right)^{-1} \mathbf{X}^T \mathbf{Q}_i \mathbf{y} \tag{5}$$

where $\mathbf{y} = (\dots y_s \dots)$ is the vector of measured system outputs, the dependance of which on the measured inputs is the subject of the identification;[1] consult [1].

Note: The determination of the parameters $\mathbf{w}_i$ is also called the *training phase* of the LOLI-model; the calculation of the LOLI-model output $\hat{y}$ [see (1)] to given inputs $\mathbf{u}$ when the parameters $\mathbf{w}_i$ and the partitioning of the data region specified by $\Phi_i(\mathbf{u})$ are known is called the *simulation phase* of the LOLI-model.

[1]Provided that $N \geq p + 1$.

If, for some reason, some data samples are more important than others, introducing matrix $\mathbf{R} = diag(\ldots r_s \ldots)$, $s = 1..N$ of desired weights for each data sample the weighted least square method can be used:

$$\mathbf{w}_i = \left(\mathbf{X}^T\mathbf{Q}_i\mathbf{R}\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{Q}_i\mathbf{R}\mathbf{y}.^2 \tag{6}$$

2.3. LOLIMOT - the identification algorithm

Assume we have ordered the measured data of a system behavior, i.e., inputs and output(s), into the matrix **X**, each column of which (but the first one) corresponding to one of the input quantities u_j, $j = 1..p$. The limits of the measured values constitute a p-dimensional hyper-rectangle that is called the data region, on which the resulting LOLI-model is to approximate the system's behavior.

LOLIMOT[3], as introduced by O. Nelles [1], is an algorithm that in each iteration i splits the rectangular measured data region by the axis-orthogonal cuts into two halves (sub-regions) in jth dimension and constructs the validity functions Φ_i according to (2) and linear models determined by parameters $\mathbf{w}_i$ according to (6). Only the worst performing LLM, i.e., the one with the largest local error

$$I_i = \sum_{j=1}^{N}(\hat{y}_j - y_j)^2\Phi_i(\mathbf{u}_j), \tag{7}$$

is considered for splitting in each LOLIMOT iteration while the split in all dimensions is tried and only the best split, i.e., the one yilding the largest decrease of the total error

$$\epsilon_i = (\hat{\mathbf{y}}_i - \mathbf{y}_i)^2, \tag{8}$$

is adopted. The algorithm stops as soon as the termination criterion, e.g., the LOLI-model complexity is reached (maximum number M of LLMs) or the total LOLI-model error drops below a specified threshold, is met.

For more detailed description of the algorithm, please refer to [1] for systems with single output (MISO) or to [2] for systems with multiple outputs (MIMO).

3. Predictive vs. statical approximation of dynamical systems

Originally, to utilize the LOLIMOT approach to identification of dynamical system $y^{(m)} = f(\ldots, u_j, \dot{u}_j, \ddot{u}_j, \ldots, y, \dot{y}, \ldots y^{(m-1)})$, Nelles [1] suggests to adopt the input/output discrete description of the nonlinear system's behavior, i.e.,

$$y(k) = g(\mathbf{u}(k), \mathbf{u}(k-1), \ldots, y(k-1), \ldots, \Delta t); \tag{9}$$

k denoting the discrete time. Thus, the regression vector x may generally contain not only values of the measurable quantities but also a portion of their time history. For example, to identify the coefficients of the discrete difference equation of a nonlinear dynamical system $\ddot{y} = f(u_1, \dot{u}_1, \ddot{u}_2, y, \dot{y})$ whereas it is possible to measure only u_1, u_2 and y, the regression vector **x** must contain the elements $u_1(k)$, $u_1(k-1)$, $u_2(k)$, $u_2(k-1)$, $u_2(k-2)$, $y(k-1)$, $y(k-2)$ (allowing the direct coupling among u_1 and u_2, and y). The severe disadvatage of this strategy

[2]For nonweighted data, the **R** equals unitary matrix.

[3]The acronym of the words **lo***cal* **li***near* **mo***del* **t***ree*.

is that it strongly depends on the sampling time of the measured data that is implicitly contained in it.[4]

Nevertheless, the LOLIMOT algorithm can be utilized also for state space description

$$\dot{\chi} = \mathbf{F}(\chi, \upsilon) \tag{10}$$

of the system provided the positions and velocities can be measured. Than, the regression vector $\mathbf{x}$ contains only the measured quantities and, consequently, the measured data does not need to be sampled equidistantly, as the sampling does not matter. In other words, adopting the state space description of the identified system, the resulting neuro-fuzzy model is trained as statical function approximating the right-hand side of (10).

4. Applications

We have investigated two examples of dynamical systems: a) a nonlinear spring damper system with 1 degree of freedom (DOF), and b) a double pendulum.

In both cases, the Matlab/Simulink was used to create the dynamical model of the physical system, an external excitation was imposed on it and the positions, velocities, and accelerations were recorded; thus, a bundle of data was obtained that were used for training the LOLI-model.

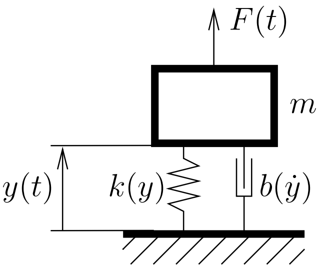

Fig. 1. Mechanical model of 1-DOF nonlinear spring damper system.

Two different types of LOLI-model were created: a) the *predictive one*, i.e. several consecutive values of positions only were utilized for training, and b) the *statical one*, i.e. the simultaneous values of positions and velocities were used for training (statical approximation of the rihgt-hand side of the state space model). Then, the same external excitation as that used for training was imposed on the created LOLI-model and the responses of the Simulink physical model and LOLI-model were compared.[5]

Finally, both the system and the LOLI-model were put through the testing excitation signal (different from the one used during the training phase) and, again, the responses were compared.

4.1. 1-DOF nonlinear spring damper system

Consider a nonlinear spring damper system with single degree of freedom according to fig. 1; mass $m = 2kg$. The damper is modelled by the following two forces

$$F_k = k_1 y + k_2 y^3 \quad , \quad F_b = b_1 y + b_2 y^3. \tag{11}$$

The signal used to train the LOLI-model is composed of a) 8s long chirp with starting frequency $f^0 = 0.5Hz$, target frequency $f^1 = 150Hz$ at time $T = 20s$, and amplitude of $A_c = 30N$ and b) 7s long random signal with sampling 0.5s and amplitude $A_s = 35N$.

[4] As we have already experienced, the rule of at least 10 times higher the sampling frequency than the highest frequency of the data cannot be applied here since the least square method tends to favour the higher frequencies and the quality of the resulting LOLI-model can be easily unacceptable for lower frequencies.

[5] Note the fact that in the case of the predictive LOLI-model, no integration is necessary to calculate the values of the inputs to the LOLI-model in the next time step from the LOLI-model output in the preceeding time step, unlike in the statical case where the integration is necessary, as the LOLI-model output is usually the acceleration—the derivative of the LOLI-model inputs containing positions and velocities.

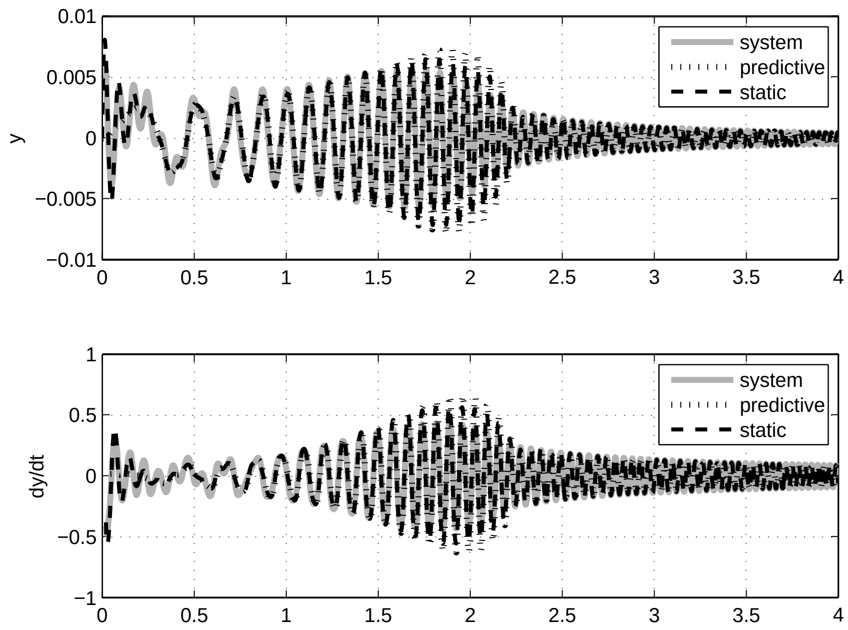

Fig. 2. Spring damper system: Responses to the training signal (position, velocity).

The matrix of recorded data had the following structure:

$$\bar{\mathbf{X}} = (k \;\; y \;\; \dot{y} \;\; \ddot{y} \;\; F); \tag{12}$$

k denoting the dicrete time.

Predictive LOLI-model The predictive LOLI-model for y was trained with 6 LLMs and the structure of the LOLI-model inputs (i.e., of the regression vector $\mathbf{x}$) was

output	*inputs*
$\hat{y}$	$(1 \; y(k-1) \; y(k-2) \; F(k-1))$

assumming no direct influence of $\hat{y}$ by F.

The responses of the system and LOLI-model to the training excitation are on fig 2 (only the first 4 seconds).

As the testing excitation force F the chirp signal with with starting frequency $f^0 = 0.1Hz$, target frequency $f^1 = 100Hz$ at $T = 10s$, and amplitude $A = 50N$ was used (simulation time 10s). The first 4 seconds of the corresponding responses are on fig 3.

Statical LOLI-model As the statical approximation of the right-hand side of the state space description, the LOLI-model with again 6 LLMs was trained for the acceleration $\ddot{y}$. The structure of the regression vector $\mathbf{x}$ was

output	*inputs*
$\hat{\ddot{y}}$	$(1 \; y(k) \; \dot{y}(k) \; F(k))$

The same testing excitations were used as for the predictive case. The responses of the system and both types of LOLI-models to the training and testing excitations are on figs. 2 and 3.

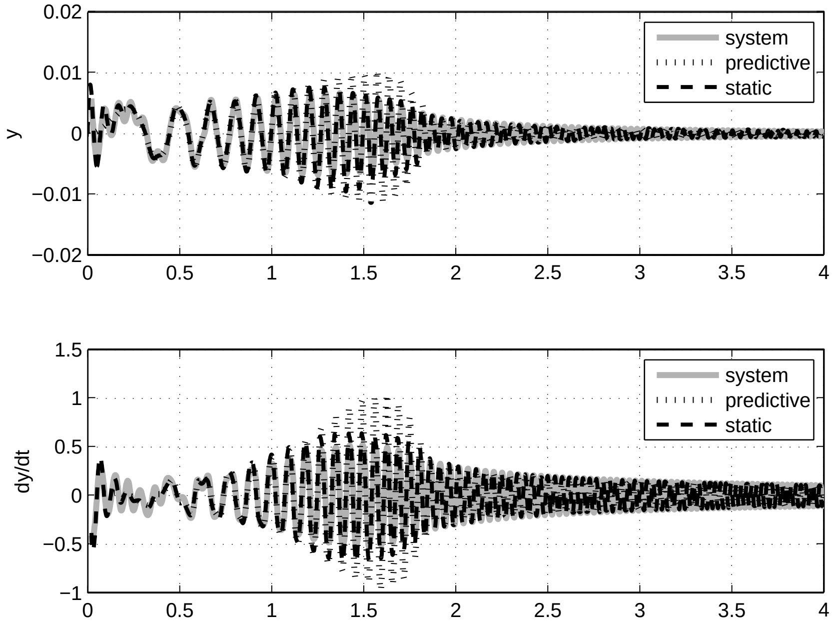

Fig. 3. Spring damper system: Responses to the testing signal (position, velocity).

4.2. Double pendulum

Consider a double pendulum according to fig. 4 made of two connected bars with the following characterists (body index 1 denoting frame): mass density $\rho = 7800kg/m^3$, gravitational acceleration $g = 9.81ms^{-2}$, cross sections of the bars $b_1 \times h_1 = b_2 \times h_2 = 0.05 \times 0.05m$, lengths $l_2 = 0.5m$, $l_3 = 0.8m$, torsional stiffness coefficients $k_{12} = 50Nm/rad$, $k_{12} = 20Nm/rad$, torsional damping coefficients $b_{12} = 5Nms/rad$, $b_{12} = 2Nms/rad$.

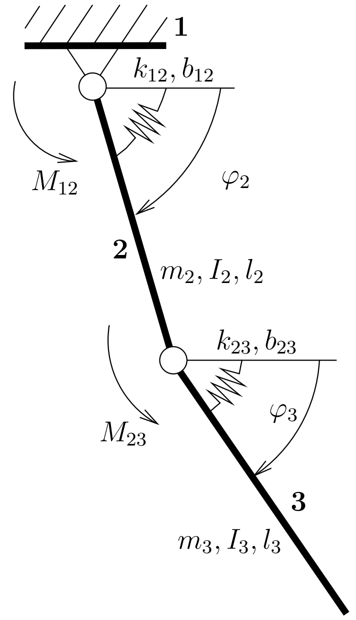

Fig. 4. Mechanical model of double pendulum.

To create both the predictive and statical LOLI-model of the double pendulum, the training signals of the driving torques M_{12}, M_{23} with amplitudes $A_{12} = 10Nm$, $A_{23} = 35Nm$ composed of a) 10s long amplitude modulated pseudo-random binary signal run through a system of first order with time constant 0.05s, b) 10s long chirp with starting frequencies $f^0_{12} = 0.0015Hz$, $f^0_{23} = 0.0025Hz$ and target frequencies $f^1_{12} = 2Hz$, $f^1_{12} = 3Hz$, and c) 10s long stochastic signal with sampling 0.1s were used.

The matrix of recorded data had the following structure:

$$\bar{\mathbf{X}} = (k \;\; \varphi_2 \;\; \dot{\varphi}_2 \;\; \ddot{\varphi}_2 \;\; \varphi_3 \;\; \dot{\varphi}_3 \;\; \ddot{\varphi}_3 \;\; M_{12} \;\; M_{23}); \qquad (13)$$

k denoting the dicrete time.

Predictive LOLI-model The predictive LOLI-models with 6 LLMs were trained for both system outputs φ_2, and φ_3. The structures of the regression vectors $\mathbf{x}$) for both LOLI-models were as follows

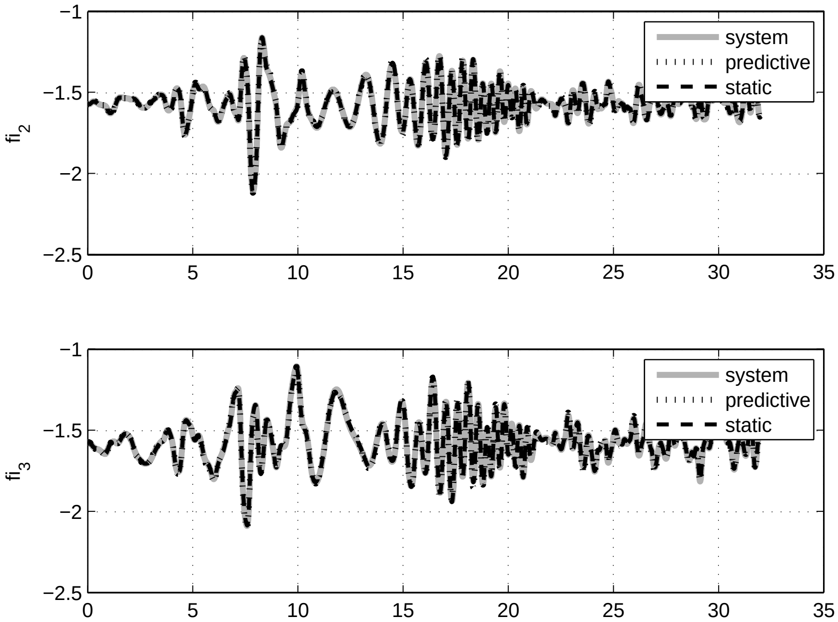

Fig. 5. Double pendulum: Comparison of the responses to the training excitation.

output	*inputs*
$\hat{\varphi}_2$	$(1\ \varphi_2(k-1)\ \varphi_2(k-2)\ \varphi_3(k)\ \varphi_3(k-1)\ \varphi_3(k-2)\ M_{12}(k-1)\ M_{23}(k-1))$
$\hat{\varphi}_3$	$(1\ \varphi_2(k)\ \varphi_2(k-1)\ \varphi_2(k-2)\ \varphi_3(k-1)\ \varphi_3(k-2)\ M_{12}(k-1)\ M_{23}(k-1))$

The responses of the system and LOLI-model to the training excitation are on fig. 5.

As the testing excitation torques M_{12}, M_{23} the chirp signals with starting frequencies $f_{12}^0 = 0.01Hz$, $f_{23}^0 = 0.08Hz$, target frequencies $f_{12}^1 = 8.5Hz$, $f_{12}^1 = 13Hz$, amplitudes $A_{12} = 5Nm$, $A_{23} = 15Nm$ were used (simulation time 11s). The corresponding responses are on fig. 6.

Statical LOLI-model As the statical approximation of the right-hand side of the state space description, the LOLI-models with 20 LLMs were trained for the accelerations $\ddot{\varphi}_2$ and $\ddot{\varphi}_3$. The structures of the LOLI-model inputs (i.e., of the regression vector $\mathbf{x}$) were the same for for both LOLI-models:

output	*inputs*
$\hat{\ddot{\varphi}}_2$	$(1\ \varphi_2(k)\ \dot{\varphi}_2(k)\ \varphi_3(k)\ \dot{\varphi}_3(k)\ M_{12}(k)\ M_{23}(k))$
$\hat{\ddot{\varphi}}_3$	$(1\ \varphi_2(k)\ \dot{\varphi}_2(k)\ \varphi_3(k)\ \dot{\varphi}_3(k)\ M_{12}(k)\ M_{23}(k))$

The responses of the system and LOLI-model to the training excitation are on fig. 5.

The same testing excitations were used as for the predictive case; the responses are in the fig. 6.

4.3. Discussion

Up to now results of these experiments indicate that the statical approximation of the right-hand side of the state space description gives better results in long time simulation than the predictive one. The necessary complexity of the statical LOLI-model (the number of LLMs) is still an open question.

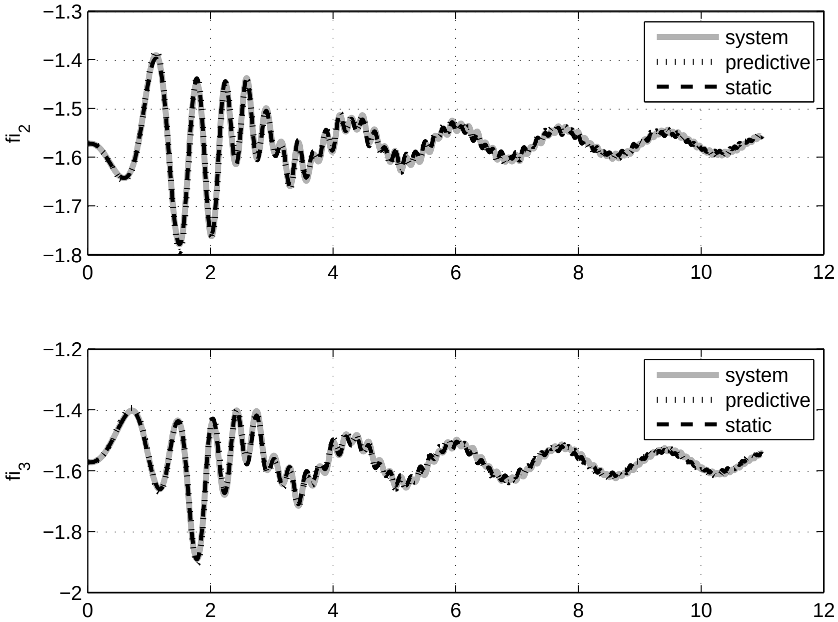

Fig. 6. Double pendulum: Comparison of the responses to the testing excitation.

5. Conclusion

The neuro-fuzzy based identification by the LOLIMOT algorithm has been used to approximate the right-hand side of the state space description of a nonlinear dynamical system's behavior. This approach has been tested on two examples—the nonlinear spring damper system and double pendulum.

Besides the statical approximation of the right-hand side of the state space description, the predictive LOLI-models have been trained and successfully used for long time simulation.

The results seem to prove the applicability of the LOLIMOT's approach in the areas of the real-time modelling of damped dynamical systems. The applicability of this framework for systems without damping brings severe problems with the stability of the resulting simulation model.

Acknowledgements

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