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Fracture mechanics parameters of multilayer pipes

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Abstract

Multilayer pipes consisting of different materials are frequently used in praxis because of partial improvement of the properties of pipe systems. To estimate lifetime of these pipes the basic fracture parameters have to be determined. In this work finite element calculations are applied in order to estimate the stress intensity factor *K* and T-stress values for a new type of non-homogenous C-shape specimen. The application of calculated *K* and *T* values to laboratory estimation of fracture toughness and its transferability to real pipe system is discussed.

Keywords: multilayer pipes, *K*-calibration, T-stress, finite element method

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1. Introduction

Polyethylene (HDPE) and polypropylene (PP) materials substitute nowadays traditional materials for production of pipes (steel, cast-iron), because they can be considered modern and ecologic. Multilayer pipes composed of these materials can improve the properties of a pipes system and therefore they are frequently used in praxis.

In this paper the fracture mechanics behaviour of a multilayer pipe composed from three layers, see fig. 1, is analysed. The studied system is formed by two protective layers (inside and outside) made of extremely durable PE material (XSC 50) and a middle jointing part of PE 100 material. For the lifetime estimation of the pipe the damage of the middle layer is decisive. The relevant parameter for resistant evaluation of the pipe material against slow crack growth is its fracture toughness. To measure the fracture toughness of the jointing middle part the special non-homogeneous C-type specimen machined directly from the three-layer pipe has been suggested [11], see fig. 2. For fracture toughness estimation in laboratory, basic fracture mechanics parameters of specimens have to be known. Fracture parameters of homogeneous testing specimens are usually known and their determination is well documented in the literature, see e.g. [8]. In the case of non-homogeneous specimens the problem of fracture parameters determination is more complicated and needs additional numerical simulations.

The aim of the present contribution is to estimate the values of the stress intensity factor *K* and the T-stress for a crack initiated from the inner surface of the non-homogeneous C-type specimen, see fig. 3. The special case of the crack with its tip at the bi-material interface between inner and middle parts of the pipe is also considered. The application of the results obtained to laboratory measurements is discussed.

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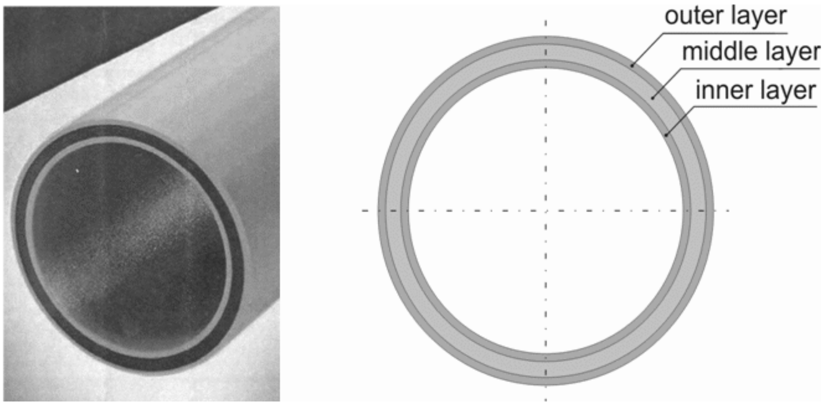

Fig. 1. Three-layer pipe studied in the contribution.

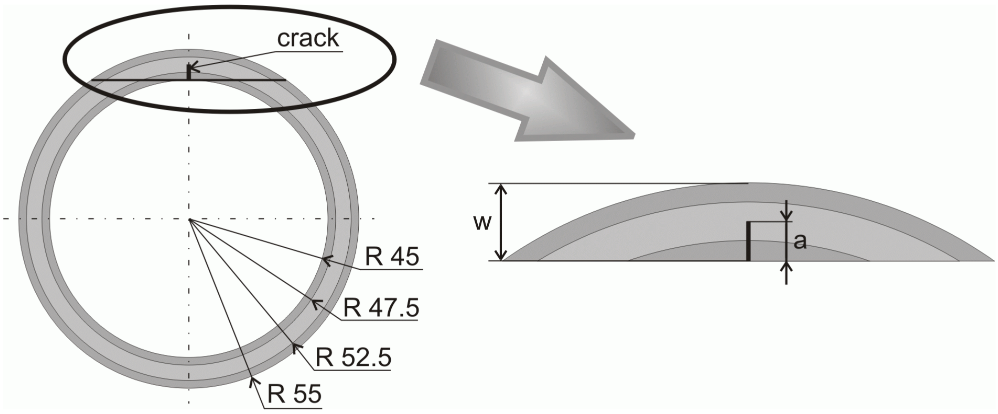

Fig. 2. Non-homogeneous C-type test specimen machined directly from the pipe. *W* is the width of the specimen, *a* is the corresponding crack length. The dimensions of the pipe are given in mm.

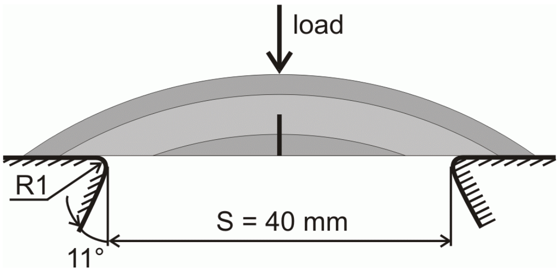

Fig. 3. Experimental set-up used for measurements of fracture toughness using the C-type specimen.

2. Numerical model

The assumptions of linear elastic fracture mechanics (LEFM) are supposed. The model geometry and boundary conditions follow the experimental set-up used for measurements of fracture toughness, see fig. 3. It is assumed further that the material interfaces are of welded type (ideal adhesion). The both used materials are homogenous, isotropic and linear elastic. The mechanical properties of the pipe layer materials are characterised by values of the Young´s modulus E_i (inner), E_o (outer) and E_m (middle). The values of Young's modulus E of the individual layers were determined from standard tensile tests [12]. It is typical for these materials that the value of their Young´s modulus depends on the temperature. Using experimental data following values were obtained: $E_i = E_o = 827$ MPa, $E_m = 1213$ MPa for room temperature (23 °C) and $E_i = E_o = 2740$ MPa, $E_m = 3391$ MPa for temperature -60 °C. The values of Poisson´s ratios were in all cases the same, i.e. $\nu_i = \nu_o = \nu_m = \nu = 0.35$. It has to be mentioned that Poisson's ratio has not so significant influence on solved problem [3], [10]. For numerical simulations the C-type specimen model was loaded by force $F = 100$ N, see fig. 3. The dimensions of the specimen were: the width $W = 10$ mm and the thickness $B = 10$ mm.

The numerical calculations were performed by finite element method (FEM) using system ANSYS 10.0. Plane strain conditions were applied. Because of the symmetry, only one half of the specimen needs to be considered. Stress and strain distribution near the crack tip was studied. The corresponding values of stress intensity factors K were estimated using the standard KCALC procedure as implemented in ANSYS. Values of the T-stress were evaluated using direct method derived directly from T-stress definition [2].

The finite element mesh used in this study and the boundary conditions applied on the model are shown in fig. 4. The mesh around the crack tip has to be refined because of high stress concentration. Special crack tip finite elements with shifted mid-nodes were used to model a crack tip stress singularity. The boundary conditions and the applied load correspond to normal mode of loading, i.e. $K = K_I$.

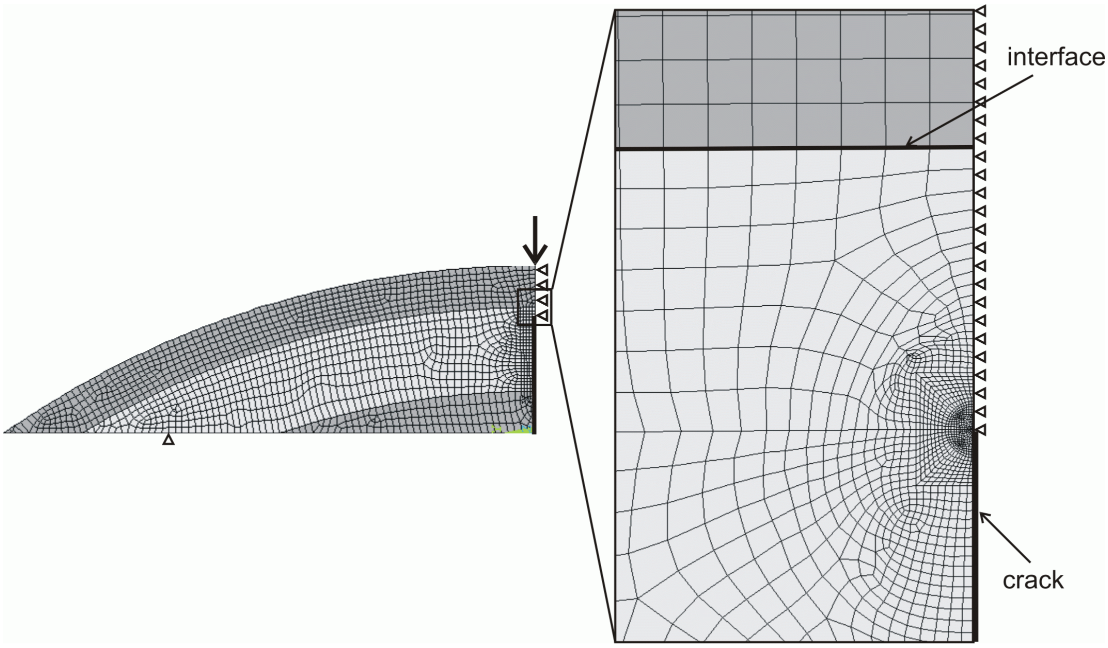

Fig. 4. The finite elements mesh and employed boundary conditions.

3. Results

First, the K-calibration curve, i.e. dependence of the stress intensity factor on crack length, was estimated. K values are given by following equation:

$$K = \frac{F \cdot S}{B \cdot W^{3/2}} \cdot f(a/W) \, , \tag{1}$$

where a is the crack length, the meaning of W and S is shown in figs. 2 and 3, B is the thickness of the specimen and F is the applied load.

In the fig. 5 correction functions *f*(*a/W*) for various studied cases are shown. Correction function *f*(*a/W*) for three point bending specimen was obtained according to equation (2), see [8] .

$$f(a/W) = 2.9(a/W)^{1/2} - 4.6(a/W)^{3/2} + 21.8(a/W)^{5/2} - 37.6(a/W)^{7/2} + 38.7(a/W)^{9/2} . \quad (2)$$

In the other cases (e.g. homogenous C-type specimen and two non-homogenous C-type specimens) the corresponding expressions were estimated from numerically obtained *K* values.

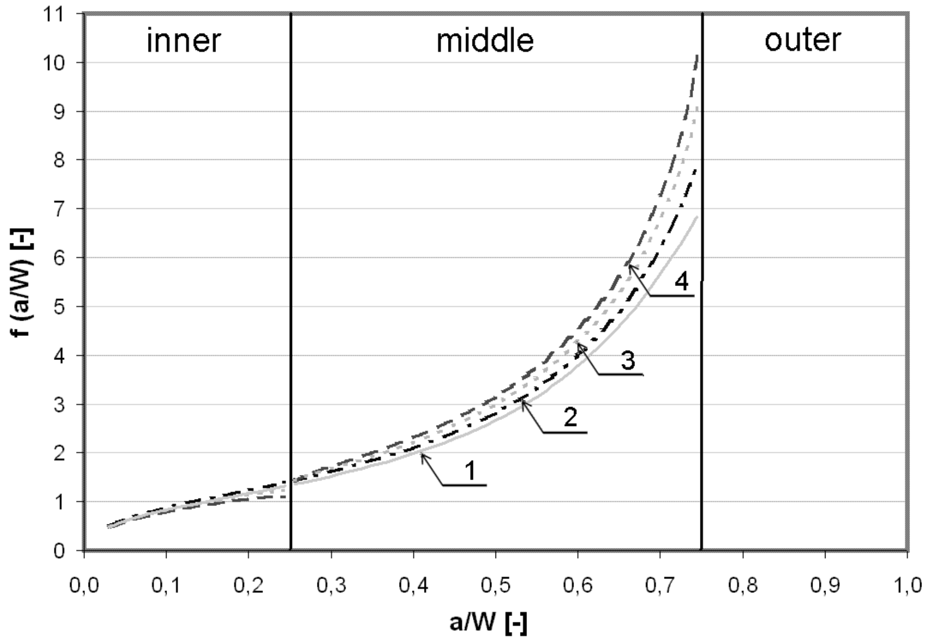

Fig. 5. Correction function *f*(*a/W*) for various cases. 1-standard three-point bending specimen, 2-homogeneous C-type specimen, non-homogenous C-type specimen: 3-temperature -60 °C, 4-temperature 23 °C.

Polynomial form of the correction function *f*(*a/W*) obtained from the numerical results using finite element method are presented in figs. 6 and 7.

The constraint effect (effect of the structure geometry) was estimated using the elastic T-stress. Recently many papers proposed *K-T* description of the stress field in the case of brittle fracture [1]. In this contribution the direct method was used for T-stress estimation [2]. Values of T-stress were obtained using relation:

$$T = \sigma_x - \sigma_y \quad \text{for} \quad \theta = 0 \ , \qquad (3)$$

where σ_x and σ_y are stress components in the direction of crack extension ($\theta = 0$).

The calculated T-stress values were converted to dimensionless biaxiality factor *B* following relation [5]:

$$B = \frac{T\sqrt{\pi a}}{K} \ , \qquad (4)$$

where *T* and *K* are values of the T-stress and the stress intensity factor *K* numerically calculated for the given crack length *a*. The behaviours of the biaxiality factor *B* for standard three point bend specimen, homogenous C-type specimen and two non-homogenous C-type specimens for various temperatures (23 °C and -60 °C) are shown in fig. 8.

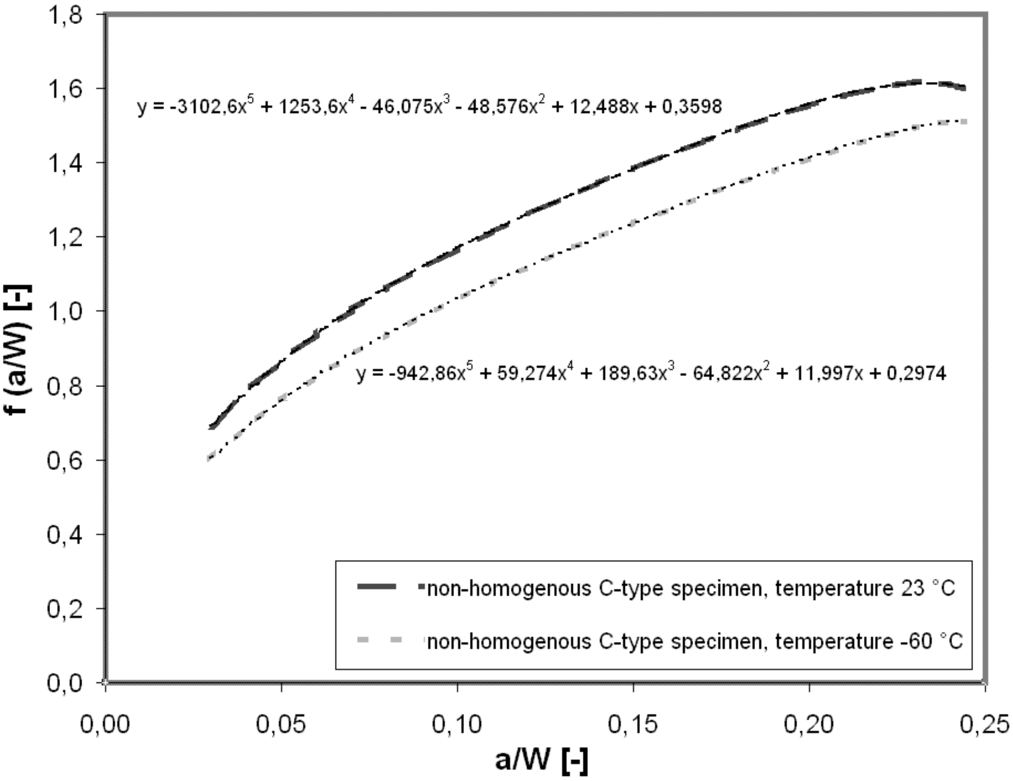

Fig. 6. Numerically estimated curves and analytical form of correction function *f*(*a/W*) in inner layer for temperatures -60 °C and 23 °C.

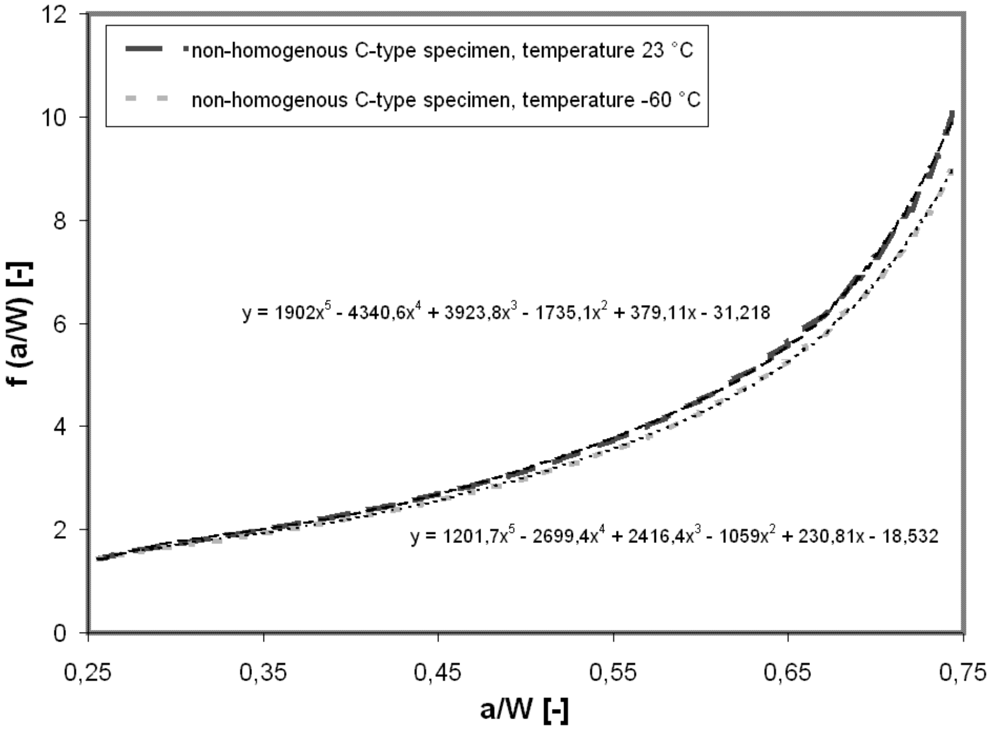

Fig. 7. Numerically estimated curves and analytical form of correction function *f*(*a/W*) in middle layer for temperatures -60 °C and 23 °C.

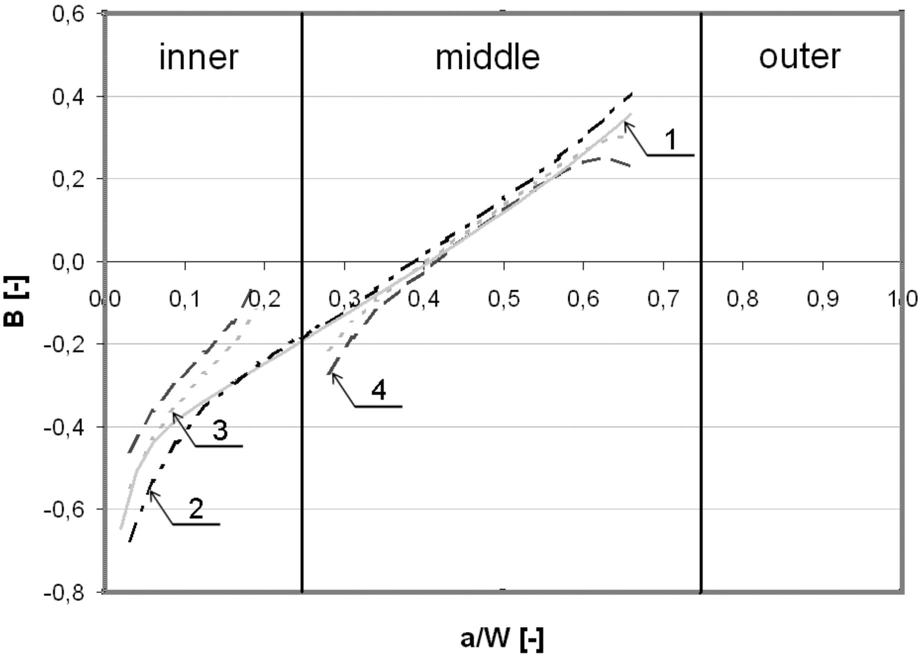

Fig. 8. Behaviours of biaxiality factor *B* for various cases. 1-standard three-point bending specimen, 2-homogeneous C-type specimen, non-homogenous C-type specimen: 3-temperature -60 °C, 4-temperature 23 °C.

As a second problem the crack with its tip at the material interface was solved. First, stress singularity exponent has to be determined. For the perpendicular crack with its tip at the material interface the analytical solution exists in the literature, e. g. [6], [7], [9], [13]. The values of the corresponding stress singularity exponents are given in tab. 1 together with corresponding values of the generalized stress intensity factors as obtained from numerical calculations. The critical force necessary for penetration of the interface was estimated using the criteria based on the mean opening stress value [9], see equation (5).

$$\frac{\sigma_{crit}}{\sigma_{appl}} = \frac{F_{crit}}{F_{appl}} = \frac{K_C}{H(F_{appl})} \cdot \frac{2d^{\,p-1/2}}{2-p+g_R} \,. \tag{5}$$

In this relation σ_{crit}, F_{crit} are values of critical load, σ_{appl}, F_{appl} are values of applied load, K_C is the fracture toughness of the material beyond the interface, H (F_{appl}) is generalized stress intensity factor established for given applied load. The parameter d relates to microstructural characteristic (in this case d = lamellar thickness), see [4], p is stress singularity exponent and g_R is known function of material properties along the relation (6).

$$g_R(\lambda) = \lambda - \cos\lambda\pi - \frac{\beta\left[\alpha + 2\lambda - \left(1 + 2\alpha - 4\alpha\lambda^2\right)\cos\lambda\pi + \left(1+\alpha\right)\cos 2\lambda\pi\right]}{1 + 2\alpha + 2\alpha^2 - 2\left(\alpha + \alpha^2\right)\cos\lambda\pi - 4\alpha^2\lambda^2} \,. \tag{6}$$

In equation (6) α and β are Dunder's parameters depending on material properties, i.e. Young's modulus and Poisson's ratio of both materials [9].

Effective value of stress intensity factor K_{eff} was established from the critical value of applied load (above this load the crack will start to propagate through the material interface into the second material) using the relation (1) for homogenous C-type specimen, see tab. 1.

	temperature [°C]	p [-]	H [MPa.m^p]	F_{crit} [N]	K_{eff} [MPa.m$^{1/2}$]
inner interface	23	0,46454	0,7467	759	4,3
	-60	0,47972	0,6550	527	3,0
outer interface	23	0,53914	2,3360	73	2,4
	-60	0,52144	2,6830	76	2,5

Tab. 1. Parameters characterizing the behaviour of the crack with its tip at the interface between the inner and the middle layer of the pipe.

4. Conclusion

Finite element method was used for the determination of stress intensity factors *K* and T-stress for a non-homogenous C-type specimen. The computational model used for *K* estimation is based on KCALC procedure implemented in system ANSYS. The density of the employed mesh was sufficient for most crack lengths and the further refinements of the mesh had no influence on the results. However, the present model cannot describe the stress field correctly if the crack tip is situated closer to the interface between inner and middle layers (or middle and outer layers). In the case of the inner interface ($E_i < E_m$) the stress intensity factor decreases moderately as the crack tip approaches the interface, while it relatively steeply increases behind the interface. In the case of outer interface ($E_m > E_o$) the stress intensity factor grows before the crack penetrates the interface. Decreasing of the *K* values is expected beyond the second material interface (this case was not studied because of its small significance). To describe behaviour of the crack with its tip at the interface a modification of LEFM has been used.

The second parameter used in LEFM for the description of the fracture behaviour is T-stress (alternatively dimensionless biaxiality factor *B*). Fig. 8 shows that the biaxiality parameter is close to zero in the middle layer which plays key role for lifetime estimation of the three-layer pipe. It can be also concluded that for crack lengths in inner and middle part where $a/W < 0.5$, the values of the measured fracture toughness can be safely transferred to real pipe systems. The corresponding fracture stress will then be conservative. On the other hand fracture toughness values determined for crack lengths outside this interval can give non-conservative results of critical fracture stresses.

An important question regarding a crack with the tip at the interface of two materials is whether it will stop or continue growing in the second material. Therefore a crack with its tip at the material interface was modelled. Using the mean stress criterion the critical value of applied load was established. Under this load the crack with the tip at the material interface will penetrated the interface into the second material. In the studied cases the elastic mismatch between materials of inner and middle parts is small enough and the interface does not play a significant role.

Acknowledgements

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