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Modelling of acoustic transmission through perforated layer

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Abstract

The paper deals with modeling the acoustic transmission through a perforated interface plane separating two halfspaces occupied by the acoustic medium. We considered the two-scale homogenization limit of the standard acoustic problem imposed in the layer with the perforated periodic structure embedded inside. The homogenized transmission conditions govern the interface discontinuity of the acoustic pressure associated with the two halfspaces and the magnitude of the fictitious transversal acoustic velocity. By numerical examples we illustrate this novel approach of modeling the acoustic impedance of perforated interfaces.

Keywords: acoustic transmission, perforated layer, homogenization technique

1. Introduction

The purpose of the paper is to demonstrate the homogenization approach applied to modelling of the acoustic transmission through perforated planar structure. We consider the acoustic medium occupying domain Ω which is subdivided by perforated plane Γ_0 in two disjoint subdomains Ω^+ and Ω^-, so that $\Omega = \Omega^+ \cup \Omega^- \cup \Gamma_0$, see Fig. 3. In the differential form the problem for unknown acoustic pressures p^+, p^- reads as follows:

$$\begin{aligned} c^2\nabla^2 p^+ + \omega^2 p^+ = 0 \quad & \text{in } \Omega^+, \\ c^2\nabla^2 p^- + \omega^2 p^- = 0 \quad & \text{in } \Omega^-, \\ + \text{ boundary conditions} \quad & \text{on } \partial\Omega, \end{aligned} \tag{1}$$

In a case of no convection flow the usual transmission conditions are given by

$$\frac{\partial p^+}{\partial n^+} = -\mathrm{i}\frac{\omega\rho}{Z}(p^+ - p^-), \quad \frac{\partial p^-}{\partial n^-} = -\mathrm{i}\frac{\omega\rho}{Z}(p^- - p^+), \tag{2}$$

where n^+ and n^- are the outward unit normals to Ω^+ and Ω^-, respectively, ω is the frequency, ρ is the density and Z is the *transmission impedance*; this complex number is characterized by features of the actual perforation considered and is determined using experiments in the acoustic laboratories, see e.g. [6].

We suggest a more refined mathematical treatment of such transmission problem, which results in constraints involving several *homogenized coefficients* computed directly for a specified shape of perforation. As an advantage, with such modelling approach one can think of *inverse problems* aimed at optimal design of the perforated structure to obtain a desired acoustic response, see e.g. [1, 8].

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2. Problem formulation

By indices $^\varepsilon$ we denote dependence of variables on scale parameter $\varepsilon > 0$; similar convention is adhered in the explicit reference to the layer thickness $\delta > 0$. By the Greek indices we refer to the coordinate index 1 or 2, so that $(x_\alpha, x_3) \in I\!R^3$.

2.1. Geometry

Let $\Omega_\delta \subset I\!R^3$ be an open domain shaped as a layer bounded by $\partial\Omega_\delta$ which is split as follows

$$\partial\Omega_\delta = \Gamma_\delta^+ \cup \Gamma_\delta^- \cup \partial\Omega_\delta^\infty, \tag{3}$$

where $\delta > 0$ is the layer thickness, see Fig. 1. The acoustic medium occupies domain $\Omega_\delta^\varepsilon \setminus S_\delta^\varepsilon$, where S_δ^ε is the solid obstacle which in a simple layout has a form of the periodically perforated sheet.

For homogenization technique, it is important to have a fixed domain, therefore the *dilatation* is considered, cf. [4]; let Γ_0 be the plane spanned by coordinates $1,2$ and containing the origin. Further let Γ_δ^+ and Γ_δ^- be equidistant to Γ_0 with the distance $\delta/2$ Therefore, $x_3 \in]-\delta/2, \delta/2[$ and we introduce the rescaling $x_3 = z\delta$.

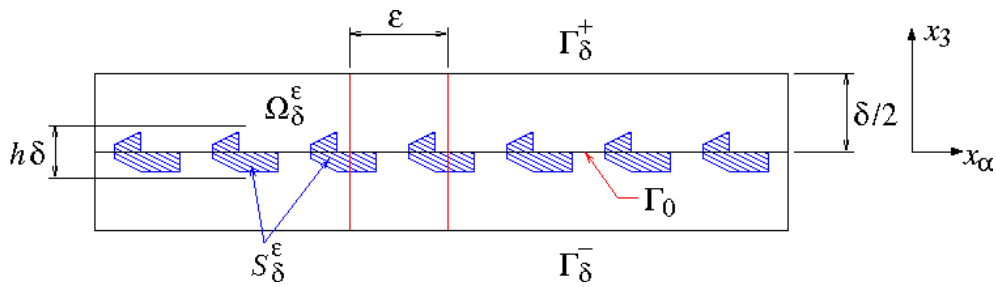


Fig. 1. The layer Ω_δ of the acoustic medium with periodic "solid perforations" S_δ^ε.

2.2. Boundary value problem in the transmission layer

The problem of acoustics is defined in $\Omega_\delta^\varepsilon$. We assume a monochrome stationary incident wave with the frequency ω and no convection velocity of the medium, so that

$$\begin{aligned} c^2\nabla^2 p^{\varepsilon\,\delta} + \omega^2 p^{\varepsilon\,\delta} &= 0 \quad \text{in } \Omega_\delta^\varepsilon, \\ c^2 \frac{\partial p^{\varepsilon\,\delta}}{\partial n^\delta} &= -\mathrm{i}\omega g^{\varepsilon\,\delta\pm} \quad \text{on } \Gamma_\delta^\pm, \\ \frac{\partial p^{\varepsilon\,\delta}}{\partial n^\delta} &= 0 \quad \text{on } \partial\Omega_\delta^\infty, \end{aligned} \tag{4}$$

where $c = \omega/k$ is the speed of sound propagation, $g^{\varepsilon\,\delta\pm}k^2$ is the interface normal acoustic velocity; by n^δ we denote the normal vector outward to Ω_δ.

3. Homogenization

For passing to the limit $\varepsilon \to 0$ we consider a proportional scaling between the period length and the thickness, so that $\delta = \varkappa\varepsilon$, for a fixed $\varkappa > 0$. Further, we need a convenient prepositions on the problem data involved in (4). Note that $g^{\varepsilon\,\delta\pm}$ is defined on Γ_0, which is equidistant to $\Gamma^{\pm}$; we assume

$$g^{\varepsilon\,\delta+} \rightharpoonup g^{0+}, \quad g^{\varepsilon\,\delta-} \rightharpoonup g^{0-}, \quad \frac{1}{\delta}\left(g^{\varepsilon\,\delta+} + g^{\varepsilon\,\delta-}\right) \rightharpoonup 0,$$

weakly in $L^2(\Gamma_0)$, which yields

$$g^{0\pm} \equiv g^{0+} = -g^{0-}. \tag{5}$$

The homogenized coefficients are introduced using so called corrector functions computed for the reference periodic cell $Y =]0,1[^2 \times] - 1/2, +1/2[\subset I\!R^3$ which is perforated by the solid (rigid) obstacle T, so that the acoustic medium occupies domain $Y^* = Y \setminus T$.We refer to the upper and lower boundaries of Y by $I_y^+ = \{y \in \partial Y : \ z = 1/2\}$ and $I_y^- = \{y \in \partial Y : \ z = -1/2\}$.

The limit (homogenized) problem is obtained by the *periodic unfolding* method, see e.g. [5], applied to the week formulation of (4).

3.1. Local microscopic problems

The microscopic and macroscopic problems are introduced by virtue of the following decomposition

$$p^1(x_\alpha, y) = \pi^\beta(y)\partial_\beta p^0(x_\alpha) + \mathrm{i}\omega\xi^{\pm}(y)g^{0\pm}(x_\alpha), \tag{6}$$

where $\pi^\beta, \xi^{\pm} \in H^1_{\#(1,2)}(Y)/I\!R$, $\beta = 1,2$ are solutions of the local microscopic problems:

$$\int_{Y^*}\left[\partial^y_\alpha \xi^{\pm}\,\partial^y_\alpha q + \frac{1}{\varkappa^2}\partial_z\xi^{\pm}\partial_z q\right] + \frac{|Y|}{c^2\varkappa}\left(\int_{I_y^+} q - \int_{I_y^-} q\right) = 0, \quad \forall q \in H^1_{\#(1,2)}(Y)/I\!R, \tag{7}$$

$$\int_{Y^*}\partial^y_\alpha(y^\beta + \pi^\beta)\,\partial^y_\alpha q + \frac{1}{\varkappa^2}\int_{Y^*}\partial_z\pi^\beta\partial_z q = 0, \quad \forall q \in H^1_{\#(1,2)}(Y)/I\!R,\ \beta = 1,2. \tag{8}$$

3.2. Macroscopic problem in transmission layer

Homogenized transmission behaviour is expressed in terms of *interface mean acoustic pressure* $p^0 \in H^1(\Gamma_0)$, and *fictitious acoustic velocity* $g^{0\pm}L^2(\Gamma_0)$ which satisfy the interface probe (to hold for all $q \in H^1(\Gamma_0)$ and $\psi \in L^2(\Gamma_0)$)

$$\begin{aligned} &\int_{\Gamma_0} A_{\alpha\beta}\partial^x_\beta p^0\partial^x_\alpha q - \frac{|Y^*|}{|Y|}\omega^2\int_{\Gamma_0} p^0 q = -\mathrm{i}\omega\int_{\Gamma_0} B_\alpha\partial^x_\alpha q\, g^{0\pm}, \\ &\int_{\Gamma_0}(p^+ - p^-)\psi - \int_{\Gamma_0} D_\beta\partial^x_\beta p^0\psi = \mathrm{i}\omega\int_{\Gamma_0} F^{\pm}g^{0\pm}\psi, \end{aligned} \tag{9}$$

where the homogenized equations are expressed in terms of the corrector functions π^β and $\xi^{\pm}$:

$$A_{\alpha\beta} = \frac{c^2}{|Y|}\int_{Y^*}\partial^y_\gamma(y^\beta + \pi^\beta)\,\partial^y_\gamma(y^\alpha + \pi^\alpha) + \frac{c^2}{|Y|\varkappa^2}\int_{Y^*}\partial_z\pi^\beta\partial_z\pi^\alpha, \tag{10}$$

$$B_\alpha = \frac{c^2}{|Y|}\int_{Y^*}\partial^y_\alpha\xi^{\pm}, \tag{11}$$

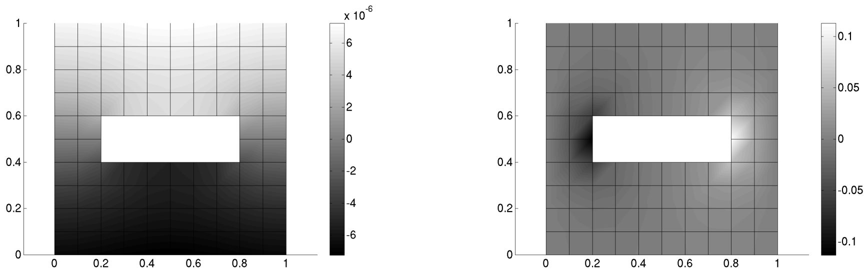
$$D_\alpha = \frac{1}{|I_y|}\left(\int_{I_y^+}\pi^\alpha - \int_{I_y^-}\pi^\alpha\right) = \varkappa B_\alpha, \tag{12}$$

$$F^{\pm} = \frac{1}{|I_y|}\left(\int_{I_y^+}\xi^{\pm} - \int_{I_y^-}\xi^{\pm}\right). \tag{13}$$

We remark that while $(9)_1$ results as the homogenization limit of (4), eq. $(9)_2$ is derived specially as the constraint of the interface discontinuity between p^+ and p^-.

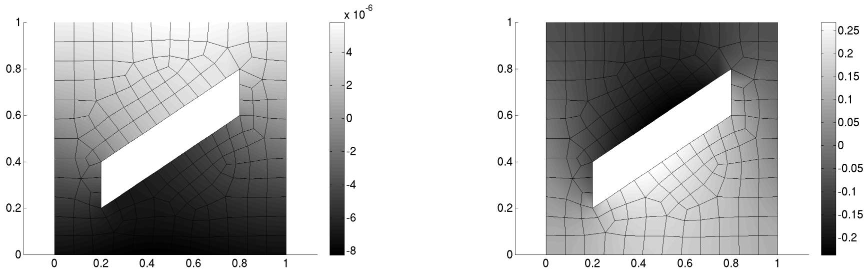
The transmission conditions on interface Γ_0 are defined in terms of p^0 and $g^{0\pm}$:

$$\begin{aligned} c^2 \frac{\partial p^+}{\partial n^+} &= \mathrm{i}\omega g^{0\pm} \text{ on } \Gamma_0, \\ c^2 \frac{\partial p^-}{\partial n^-} &= -\mathrm{i}\omega g^{0\pm} \text{ on } \Gamma_0. \end{aligned} \tag{14}$$



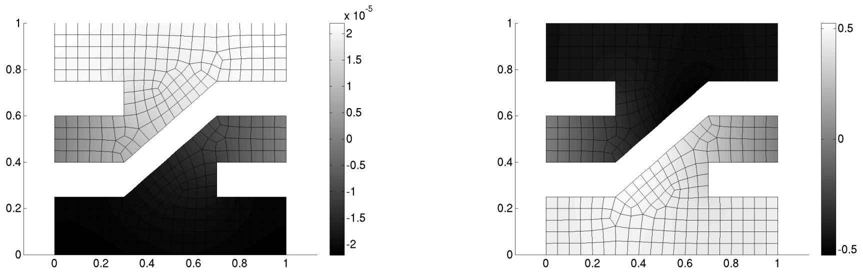
Mic. #1

$A = 1.1546 \cdot 10^5\ (\mathrm{m/s})^2$, $B = 0$, $D = 0$, $F = 1.3913 \cdot 10^{-5}\ \mathrm{s}^2$



Mic. #2

$A = 1.7035 \cdot 10^5\ (\mathrm{m/s})^2$, $B = -0.2509$ m, $D = -0.2509$ m, $F = 1.3237 \cdot 10^{-5}\ \mathrm{s}^2$



Mic. #3

$A = 2.1855 \cdot 10^5\ (\mathrm{m/s})^2$, $B = -0.8974$ m, $D = -0.8974$ m, $F = 4.2653 \cdot 10^{-5}\ \mathrm{s}^2$

Fig. 2. Distribution of $\xi^\pm$, π in Y^* and homogenized coefficients for three shapes of perforations.

For illustration, in Fig. 4 the local corrector functions $\xi^\pm$ and π are displayed for 2D examples of three different shapes of the perforations.

3.3. Modelling acoustic waveguide – influence of perforation type

The following numerical example shows the global response at the macroscopic scale. The homogenized model (1)+(9) is compared with the “standard” model with the transmission impedance (see [6])

$$Z = \rho \left(0.006c + \mathrm{i}\omega \left(t + 0.75d\right)\right)/\Phi, \tag{15}$$

where porosity $\Phi = 40$ %, thickness $t = 0.2$ mm and hole diameter $d = 0.4$ mm. A perforated structure with these parameters corresponds to the microstructure #1 in Fig. 4.

The geometry of the acoustic waveguide is depicted in Fig. 3 and the following boundary conditions are applied:

$$\begin{aligned} \mathrm{i}\omega\rho v + c\frac{\partial p}{\partial n} &= 0 \quad \text{on } \Gamma_{\mathrm{in}}, \\ \mathrm{i}\omega p + c\frac{\partial p}{\partial n} &= 0 \quad \text{on } \Gamma_{\mathrm{out}}. \end{aligned} \tag{16}$$

The first condition prescribes the velocity of the incident wave on the input ($v = 1$ m/s) and the second one ensures the anechoic output. The acoustic medium has the density $\rho = 1.55$ kg/m^3 and the speed of sound propagation is $c = 343$ m/s.

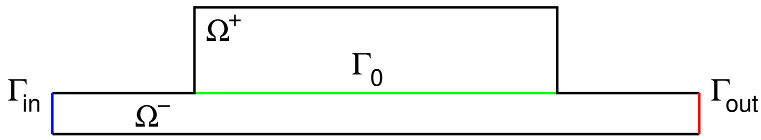


Fig. 3. Macroscopic domain Ω.

4. Conclusion

We derived the transmission conditions, see [7], involving homogenized parameters (10)-(13) which reflect specific features of the periodic perforation. The separating structures can be quite general, thus not only flat plates with holes may be considered. Moreover, even the "no-obstacle" situation is treated by the present model, when $Y = Y^*$ and $\varkappa \to +\infty$. Then $\pi^\beta = \xi^\pm \equiv 0$, therefore both $F^\pm$ and D_β vanish, so that $(9)_2$ yields continuity $p^+ = p^-$ on Γ_0.

Using the standard model we obtained the imaginary part of the transmission impedance (15) as $\mathrm{Im}(Z) = 6.1\ \mathrm{kg/(ms^2)}$. The relevant transmission impedance resulting from the homogenized model (Mic. #1) is $Z_{hom} = 8.0\ \mathrm{kg/(ms^2)}$. It is important to note that relation (15) was derived for the 3D structures while our homogenized model is just 2D.

The presented example is motivated by modelling of muffler type structures, see e.g. [2, 3].

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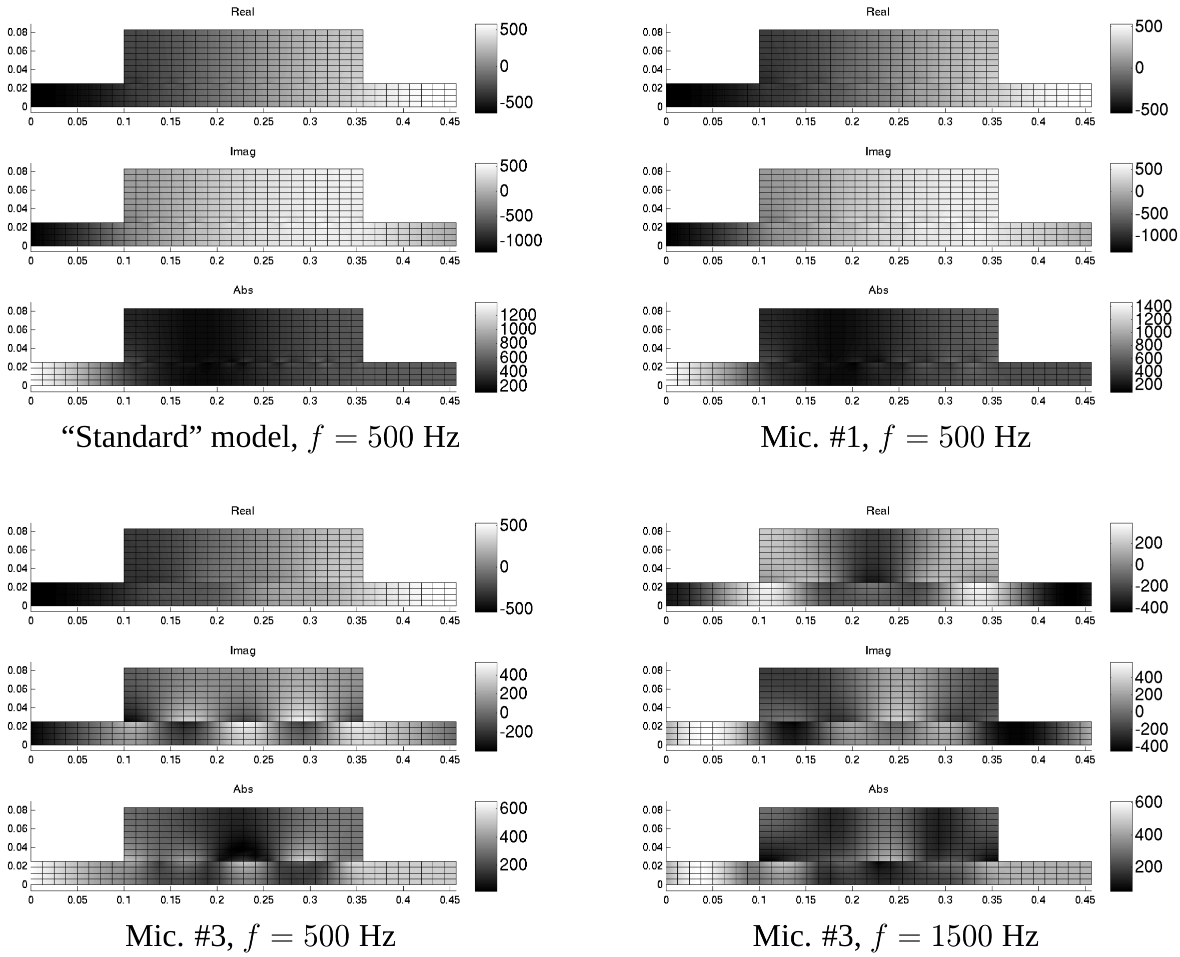

Fig. 4. Influence of perforation type (Mic. #1 and Mic. #3) and frequency ($f = 500$ Hz, $f = 1500$ Hz) on the acoustic pressure in the waveguide. Comparison of the “standard” model with the homogenized one which involves the corresponding microstructure.

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