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Comparative analysis of influence selected geometrical parameters on stress concentration in the surrounding of inclusion

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Abstract

In this paper we are focused on influence of selected geometrical characteristics as are: inscribed circle diameter, circumscribed circle diameter, eccentricity, ovality and radius of curvature of inclusion on stress concentration around these defects modelling using by FEM. This task was solved as plane stress. From this point of view there are monitored and evaluated there factors: maximum value of stress along the loading, across the loading, shear stress and equivalent stresses. There will be also presented algorithms for automatic generation of models that make possible to practice statistical data processing.

Keywords: stress concentration, geometrical model, finite element methods, inclusion

1. Introduction

Calculations on the strength of structures are primarily based on the theory of elasticity. If the yield stress is exceeded plastic deformation occurs, more complex theory of plasticity has to be used [5]. The macroscopic elastic behaviour of an isotropic is characterized by three elastic constants, the elastic modulus or Young's modulus E, shear modulus G and Poisson's ratio μ [7]. The well-known relation between the constants is:

$$E = 2G(1+\mu), \tag{1}$$

In the structure, geometrical notches such as holes cannot be avoided. The notches are causing an inhomogeneous stress distribution, see fig. 1, with a stress concentration at the root of the notch [1] and [4]. The theoretical stress concentration factor K_t is defined as a ratio between peek stress at the root of the notch and the nominal stress witch would be present if a stress concentration does not occur, i.e.

$$K_t = \frac{\sigma_{peak}}{\sigma_{no\min al}}, \tag{2}$$

The severity of the stress concentration depends on the geometry of the notch configuration, generally referred to as the shape of the notch. The term *notch* is defined as a geometric discontinuity that may be introduced either by design, such as hole, or by the manufacturing process in the form of material and fabrication defect such as inclusion, weld defects, casting defects or machining marks [3]. For a component with a surface notch, the maximum elastic notch stress σ^e can be determined by the product of a nominal stress σ and the elastic stress concentration factor K_t, i.e.

$$\sigma^e = \sigma \cdot K_t, \tag{3}$$

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The maximum elastic notch stress can be calculated from an elastic finite elements analysis and is sometimes referred to as the pseudo-stress if the material at a notch is actually inelastic. Because notch stress and strains are controlled by net section material behaviour, the nominal stress for determination K_t is defined by an engineering stress formula based on basic elasticity theory and the net section properties that do not consider the presence of the notch [2] and [6].

It is known that the elastic stress concentration factor is a function of the notch geometry and the type of loading. For the case in which the component geometry and load conditions are relatively simple and the nominal stress can be easily defined. However, because of complexities of the geometry and loads in most real components are different, the value of σ^e can be directly obtained from the elastic finite element analysis.

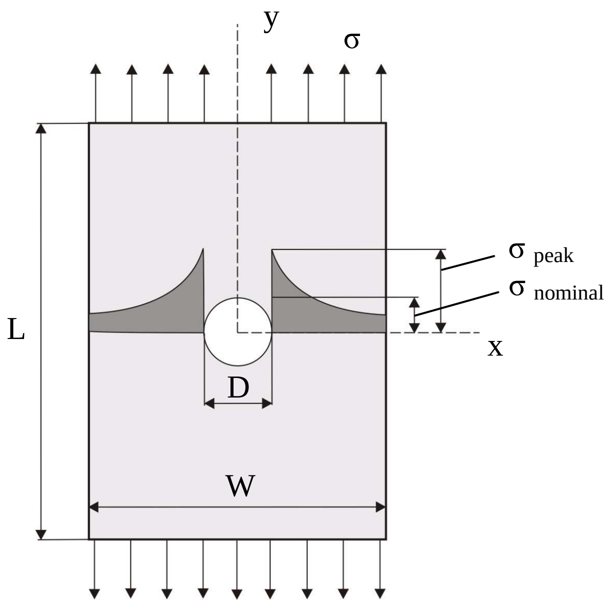

Fig. 1. Strip with central hole as a prototype of a notched part.

2. Effect of the inclusion geometry on stress concentration

In an infinite sheet for a circular hole is important only diameter *D*. However, for the simple tension specimen with a central hole, see fig. 1, there are already three dimensions: the specimen width *W*, the specimen length *L* and the hole diameter *D*. The specimen thickness is not yet considered here. In fig. 2 two specimens are shown, which are geometrical similar, but the size is different. A geometrical similarity implies that all ratios of the dimensions are the same, in the present case the same *D/W* and *L/W*.

Because stress concentration factor K_t is a dimensionless ratio, it can depend on dimensionless geometrical ratios only. Assume that all dimensions of specimen *2* in fig. 2 are two times larger then the dimension of specimen *1*. As a result of geometric similarity, all displacements are also two times larger, but the relative displacements will be the same. As a result, the strains are the same. Consequently, a geometrical similar stress distribution should occur in both specimens as depicted in fig. 2. The same peak stress will be found, and stress concentration factor K_t is the same. However, due to the difference in size, the stress gradient is not the same in the two specimens because the gradient is not dimensionless, but the gradient is inversely proportional to the root radius *r*. The consequence is that larger specimens have larger volumes and larger inclusions surface areas of highly stressed material, which is significant for the size effect [8].

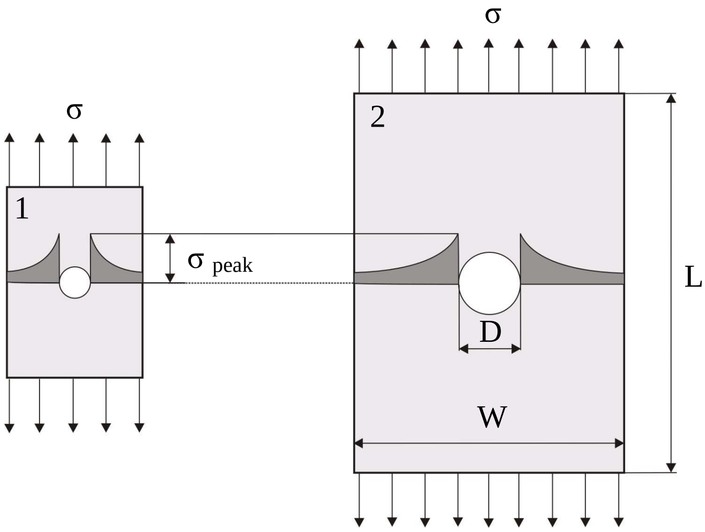

Fig. 2. Geometrically similar specimens.

3. Computational model description

Calculations of stress concentration are solved by several authors. These solutions are assigned for homogeneous, elastics, isotropic material and also for single inclusion with the defined profile. The theoretical value of stress concentration factor for sphere inclusion is $K_t = 3$. The real inclusions have a random shape and orientation, so stress concentration factor have a random value too [4] and [5].

If we want to determine a relationship between the shape of the inclusion and stress distribution in the surround of inclusion than it is necessary to prepare software generator of the random geometrical models of inclusion. By the reason of modelled inclusion shape it was defined next geometrical parameters:

- inscribed circle diameter,
- circumscribed circle diameter,
- eccentricity,
- ovality,
- radius of curvature at point of maximal stress concentration.

The stress analysis of inclusion model was made by software ADINA R&D. These solutions have next simplifications. The matrix of material is modelled as isotropic and elastic material model. The inclusion was subtracted from the model, it is a hole without mesh. This kind of inclusion models is acceptable on the ground of their properties: low tensile strength and often decohesion from the matrix (e.g. graphite in the nodular cast iron). The stress concentration factor was simply calculated from maximum value of stress in the direction of applied load traction $\sigma = 1$. This task was solved as plane stress with simple tension of loading. The illustration of a simple FEM model is given in fig. 3. The geometry of the component has been modelled by a large number of small interconnected elements.

With the view of statistical evaluation of results, it was needs to make a lot of FEM analysis. Therefore it was necessary to prepare simple algorithm for random model generation. The next step was the FEM analysis and evaluation of the required results. The best way is to create some simple evaluation programs. All these programs were created in user friendly software named OCTAVE.

The algorithm for generation of random inclusion model is based on following parameters:

- diameter of inclusion,
- the number of apex interval for control polygon of the inclusion,
- the interval of apex value.

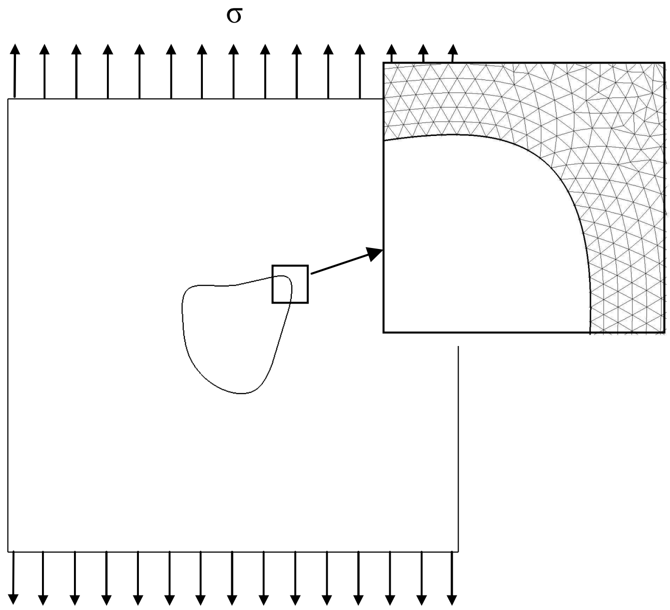

Fig. 3. FEM model of inclusion.

4. Results of FEM analysis

In first step, we must build the algorithm, which satisfied requirements for generation simplified geometrical models of inclusions of various shapes. When it was generated a sufficient number of models than it was realized stress analysis in program ADINA. As one of representative parameters for evaluation of stress concentration in the surrounding of inclusion was specified stress concentration factor K_t. The aim of FEM analysis was obtained results to describing the influence of geometrical characteristics on stress distribution in the surrounding of inclusion. These results are shown in fig. 4 - 7.

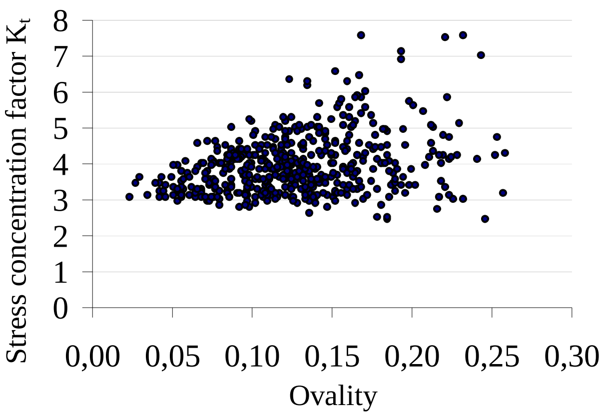

Fig. 4. Analysis of stress concentration in the surrounding of inclusion along the loading $\sigma_{yy.}$

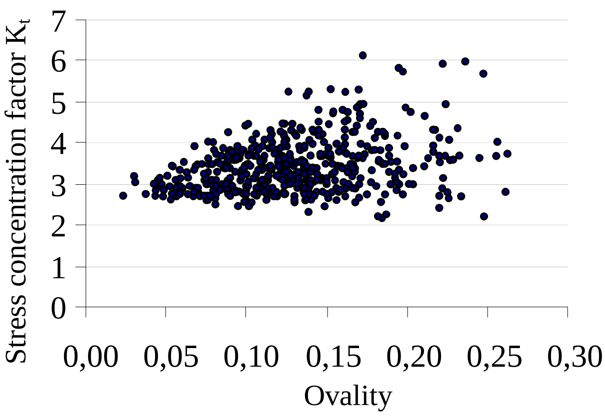

Fig. 5. Analysis of stress concentration in the surrounding of inclusion using by HMH theory.

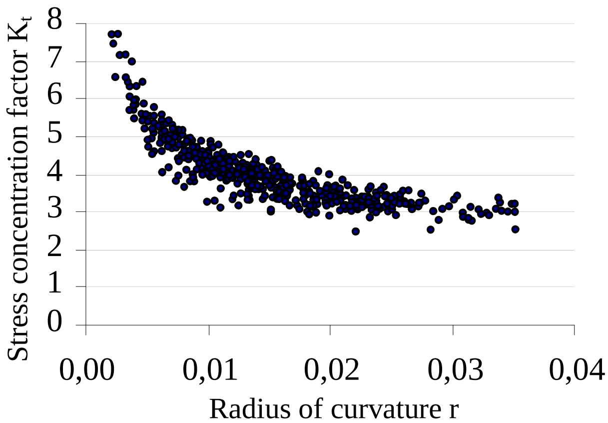

Fig. 6. Analysis of stress concentration in the surrounding of inclusion along the loading σ_{yy}.

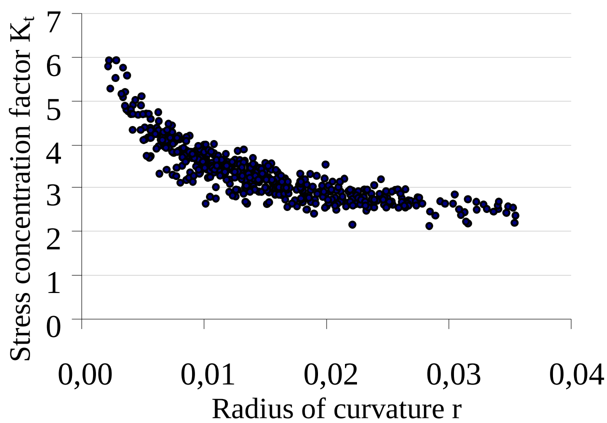

Fig. 7. Analysis of stress concentration in the surrounding of inclusion using by HMH theory.

On the basis of these results we can say that it was confirmed influence of some geometrical characteristics of inclusion shape on magnitude distribution of stress concentration in the surrounding of inclusion. The most expressive influence have only two from all monitored factors: ovality and radius of curvature of particle in region with highest stress concentration. As it results from FEM analysis, other geometrical characteristics (inscribed circle diameter, circumscribed circle diameter and eccentricity) have no direct (immediate) influence on magnitude of stress distribution in the surrounding of inclusion. These characteristics are important in relation to material characteristics. Mechanical properties of materials (Young's modulus E, shear modulus G and Poisson's ratio μ) specified the value of stress concentration in relation to magnitude and distribution of inclusions in matrix. From the analysis we can also generally say, that the minimum value of the stress at the edge of inclusion is three times the nominal stress in tension conditions.

Influence of ovality, in the sense of geometrical characteristic, on stress concentration factor K_t is shown in fig. 4 and fig. 5. On the ground of performed FEM analysis is possible to say that value of stress concentration factor K_t continually increase from value $K_t = 3$ (for spherical shape of inclusion) to value $K_t = 7$ (for elliptical shape of inclusion). This is in good agreement with theoretical background which related with stress distribution. It is important, however, to remember that elastic stress concentration factors for homogeneous isotropic materials depend only on geometry (independent of material) and mode of loading, and that they apply only when the inclusion is in the linear elastic deformation condition.

FEM analysis results have shown fig. 6. and fig. 7., that the most expressive influence from monitored geometrical characteristics have radius of curvature reviewed in region with highest stress, i.e. in region with the smallest radius of curvature of inclusion. It is possible to observe that if radius of curvature is increased in reviewed region of inclusion than stress concentration factor K_t decrease from value $K_t = 7$ (for sharp notch) to value $K_t = 3$ (for spherical shape of notch). This kind of continual decrease of stress concentration factor K_t is good agreement with obtained results that was presented above. This fact was documented in a lot of published works.

Similar values of stress concentration factor K_t calculated for equivalent stresses due to HMH theory (fig. 5 and fig. 7) we can be caused by presence of the other nonzero components of the stress tensor in the surrounding of inclusion. With respect to this fact, it is possible to assume the presence of multiaxial stress state in the surrounding of these components.

5. Conclusion

Stress concentration around the inclusion was considered in this article with stress concentration factor K_t as an important parameter for characterizing the severity of the stress distribution in the surrounding of inclusion. It's assumed idealized model of inclusions. If stress concentration factors or stress distributions are needed, calculations should be made with finite element techniques for which computational model have been developed. The solutions obtained are not exact because of continuum material is replaced by a simplified material model. However, very satisfactory results can be achieved. Application of loads to the model, boundary conditions and mesh distributions should be given careful attention. Obtained results can be summarized as follows:

- created algorithm satisfied requirements for generation of simplified geometrical models of inclusions of various shapes,
- by means of FEM was realized analysis of the state of stress,
- the most expressive influence on stress concentration from monitored geometrical characteristics was confirmed for ovality and radius of curvature. Other monitored geometrical parameters unconfirmed direct influence on stress concentration in the surrounding of inclusion,
- highest values of stress or stress concentration factor K_t was calculated in the surrounding of inclusion. In these regions was obtained highest change of geometrical shape in comparison with spherical shape,
- accurate K_t values can be calculated with FEM techniques. With the current computers, these calculations should be more accurate and cost-effective in comparison to measurements of stress distribution in the surrounding of inclusions,
- the stress concentration produced by a given inclusion is not a unique number, it is depend on the mode of loading and on the type of stress that is use to calculated K_t.

Acknowledgements

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