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The behaviour of a tube bundle near the stability limit

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Abstract

Tube heat exchangers are inseparable components of a great number of energetic machinery, where one fluid flows through tubes and the other fluid flows around the tubes. Heat transfer occurs between these two fluids. Apart from the problem of heat transfer, the problem of fluid-structure interaction is very important too. Mainly the fluid flowing around the tubes may be very dangerous, because it causes vibrations of these tubes. Intensity of vibration depends on the velocity of the flow. Under the certain, so-called critical velocity, vibration amplitudes can have random pattern. The objective of this article is to determine the probability of up crossing of some fixed level. It is necessary to avoid such regimes of operations, in which the damage of a heat exchanger as a consequence of flow-induced vibration could be caused.

Keywords: fluidoelastic interaction, critical velocity, random vibration, stability limit

1. Introduction

The cases, in which it is necessary to transfer heat from one fluid to another fluid, are very important and very frequent in power engineering. The heat exchangers are used for these purposes. The exchangers consisting of many parallel tubes are mostly used. One fluid flows inside the tube and the other fluid flows cross-tube. The heat is transferred from the fluid with a higher temperature to the fluid with lower temperature. The fluid flowing around tubes can be the source of aerodynamic excitation. Besides the problems dealing with heat transfer it is necessary to solve very important questions dealing with the dynamic behaviour of the whole equipments. Under the certain conditions the vibration of tubes can be so intensive, that a serious damage can occur. This happens under the certain, so called critical, velocity of flow. We say that the loss of stability began.

More and more requirements on power production increase the call for designers, in order to machines operated near the stability limit. It is necessary to devote great attention to the stability limit determination for safeguarding the reliable and safety operation of the machinery. The problem of aerodynamic excitation is discussed in this paper. Physical nature of phenomena connected with the origins of non-stable states must be studied very carefully, too. When the turbulent excitation of cross-flowed tubes takes place, it is necessary to know the probability of up crossing of some fix level of amplitudes. In this paper it is supposed that vibration of tubes can be described by the equation of an oscillator with a light non-linear damping and a white noise excitation.

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2. Mathematical formulation

The tubes are modelled as slender bars. The axes of tubes are identical with axis z. Tubes can vibrate in plane (x,z) and (y,z). The displacement in plane (x,z) is marked $u(z,t)$ and the displacement in plane (y,z) is marked $v(z,t)$. We choose a cell containing N tubes. If we suppose, that all tubes in the cell are identical, we can express equations of motion for tube number j in the form

$$EJ\frac{\partial^4 u_j(z,t)}{\partial z^4} + \mu EJ\frac{\partial^5 u_j(z,t)}{\partial z^4 \partial t} + M\frac{\partial^2 u_j(z,t)}{\partial t^2} = F_{uj}(z,t) \ , \tag{1}$$

$$EJ\frac{\partial^4 v_j(z,t)}{\partial z^4} + \mu EJ\frac{\partial^5 v_j(z,t)}{\partial z^4 \partial t} + M\frac{\partial^2 v_j(z,t)}{\partial t^2} = F_{vj}(z,t) \ , \tag{2}$$

where E is the modulus of elasticity of tube material, $J = \pi(D_1^4 - D_2^4)/64$ is the moment of inertia of a tube, μ is the coefficient of material damping, M is the mass of tubes per one meter length (including the mass of fluid inside of a tube), $F_{uj}(z,t)$ is the force acting on a tube in plane (x,z) and $F_{vj}(z,t)$ is force acting on the tube in plane (y,z).

Very detailed analysis is made in [4]. It is supposed, that the resulting aerodynamic force can be divided in two components. The former component is independent on movement of a tube, and the latter component depends on the harmonic movement. The existence of memory effects between the tube displacements and fluid forces follows from this analyse, [5]. These memory effects are generalisation of time delay, which was included in [11].

Forces $F_{uj}(z,t)$ and $F_{vj}(z,t)$ can be expressed in the form of sum of aerodynamic forces $f_{uj}^a(z,t)$, $f_{vj}^a(z,t)$ and forces $f_{uj}^m(u(L_p))$, $f_{vj}^m(v(L_p))$ expressing interactions of tubes with dividing walls. Parameter L_p denotes the length of segment from the left end of tube. We can write

$$F_{uj}(z,t) = f_{uj}^a(z,t) + \sum_{p=1}^{P} f_{uj}^m(z,t)\delta(z - L_p), \tag{3}$$

$$F_{vj}(z,t) = f_{vj}^a(z,t) + \sum_{p=1}^{P} f_{vj}^m(z,t)\delta(z - L_p), \tag{4}$$

where $\delta(z - L_p)$ is Dirac δ - function.

3. Suppositions of the solution

The so-called displacement mechanism of the aerodynamic excitation is mostly used for the mathematical formulation of tube bundles vibration. This mechanism is based on the assumption that the pressure field around considered tube and simultaneously pressure field around neighbouring tubes are changed by the tube deviation from its equilibrium position. The aerodynamic forces cause the tubes to begin vibrate. The aerodynamic couplings between tubes exist. These couplings are expressed by means of coefficients of added mass, aerodynamics damping and aerodynamics stiffness, [7], [8], [9]. In this approach the knowledge of these coefficients is required to solve the stability problem. Besides these deterministic forces it is necessary consider the stochastic forces $f_{uj}^{turb}(z,t)$, $f_{vj}^{turb}(z,t)$, which are caused by turbulent fluctuations. We can write

$$f_{uj}^{a}(z,t) = -\frac{\pi D_1^2 \rho}{4}\sum_{k=1}^{N}\left(\alpha_{jk}^{xx}\frac{\partial^2 u_k}{\partial t^2} + \alpha_{jk}^{xy}\frac{\partial^2 v_k}{\partial t^2}\right) - \frac{1}{2}\pi D_1 \rho w_x \sum_{k=1}^{N}\left(\beta_{jk}^{xx}\frac{\partial u_k}{\partial t} + \beta_{jk}^{xy}\frac{\partial v_k}{\partial t}\right) + \\ -\frac{1}{2}\rho w_x^2 \sum_{k=1}^{N}\left(\gamma_{jk}^{xx}u_k + \gamma_{jk}^{xy}v_k\right) + f_{uj}^{turb}(z,t), \tag{5}$$

$$f_{vj}^{a}(z,t) = -\frac{\pi D_1^2 \rho}{4}\sum_{k=1}^{N}\left(\alpha_{jk}^{yx}\frac{\partial^2 u_k}{\partial t^2} + \alpha_{jk}^{yy}\frac{\partial^2 v_k}{\partial t^2}\right) - \frac{1}{2}\pi D_1 \rho w_x \sum_{k=1}^{N}\left(\beta_{jk}^{yx}\frac{\partial u_k}{\partial t} + \beta_{jk}^{yy}\frac{\partial v_k}{\partial t}\right) + \\ -\frac{1}{2}\rho w_x^2 \sum_{k=1}^{N}\left(\gamma_{jk}^{yx}u_k + \gamma_{jk}^{yy}v_k\right) + f_{vj}^{turb}(z,t). \tag{6}$$

Coefficients α_{jk}^{rs}, β_{jk}^{rs}, γ_{jk}^{rs} in these equations are the coefficients of added mass, added aerodynamic damping and aerodynamic stiffness, mentioned above. The subscripts and superscripts are $j,k = 1,2,...,N$, $= x,y$, $s = x,y$, N is the number of tubes, which are in aerodynamic coupling. For example α_{jk}^{rs} is added mass of tube number *j*, moving in direction *r*, influenced by tube number *k*, moving in direction *s*. It is necessary to determine these coefficients experimentally. Real heat exchangers consist of great number of tubes, but we suppose, that aerodynamic couplings are restricted on a few tubes only. We choose a cell containing a small number of tubes, usually N = 9, or N = 7 tubes, in the dependency on geometric shape and flow direction. In spite of that, the degree of freedom number of such system is very high.

The so-called *global model* was formulated in [3] for this purpose. The tube bundle consisting of the great number of cross-flowed elastic tubes (system with many degrees of freedom) is replaced by the system consisting of bundle of rigid tubes with one elastic tube, in this approach. Theory of this global model is based on the assumption that tubes are identical, have identical boundary conditions and the dominant frequency ω_0 (peak) exists in the response frequency spectrum. For circular natural frequency ω_k and viscous damping factor ζ_k it is possible to write

$$\omega_k = \omega_0(1+\varepsilon_k), \quad \zeta_k\omega_k = \zeta_0\omega_0(1+\eta_k), \tag{7}$$

where k is the tube number and $\varepsilon_k << 1, \eta_k << 1$. Introducing this model means that requirements on experimental works are much lower.

It was found out on the base of theoretical considerations [1], [2], [12], that for cross-flowed elastic bundle the two different mechanism for loss of stability exists. If the value of parameter $M\delta/\rho D_1^2$ is low, the loss of stability is mainly determined by the negative value of the damping aerodynamic coefficient. The influence of the aerodynamic couplings is suppressed in that case. If the value of parameter $M\delta/\rho D_1^2$ is high, the loss of vibration stability is determined by aerodynamic stiffness. Aerodynamic couplings are important in that case. Boundary value is $M\delta/\rho D_1^2 = 300$. That means that for $M\delta/\rho D_1^2 < 300$ it is possible to judge the vibration stability of elastic tube bundle on the base of vibration stability of one elastic tube vibrating inside of the rigid tube bundle. This approach (so called "semi rigid theory") is very popular, mainly for vibration of loosely supported tubes, because it simplifies stability limit calculations.

Let us suppose one elastic tube vibrating inside of tube bundle consisting from of rigid tubes. Aerodynamic exciting forces can be according [12] expressed in the form

$$f_u^a(z,t) = -m_p \frac{\partial^2 u}{\partial t^2} + \frac{1}{2a^2}\rho w_r^2 D_1\left(\overline{C}_L \sin\alpha + \overline{C}_D \cos\alpha\right) + f_u^{turb}(z,t), \tag{8}$$

$$f_v^a(z,t) = -m_p \frac{\partial^2 v}{\partial t^2} + \frac{1}{2a^2}\rho w_r^2 D_1\left(\overline{C}_L \cos\alpha - \overline{C}_D \sin\alpha\right) + f_u^{turb}(z,t). \tag{9}$$

Subscript j, which determines tube number in the bundle, is omitted here. In equations mentioned above is $\alpha \cong \dot{v}/w_r$, $w_r \cong U - \dot{u}$, $U = aw_x$, $a = T/(T-D)$, where T is the distance between tube centres. Parameters $\overline{C}_D, \overline{C}_L$ are drug and lift coefficients, respectively. These quantities depend on displacements $u(z,t-\Delta t), v(z,t-\Delta t)$, where Δt has a meaning of time delay between aerodynamic force and the tube displacement. In a view of the fact that coefficients C_D, C_L are usually established in the dependence of flow velocity w_x, it is possible to express aerodynamic forces acting on tube in the form

$$f_u^a(z,t) = \frac{1}{2}\rho w_x^2 L D_1\left[C_D\left(1 - \frac{2}{aw_x}\dot{u}(z,t)\right) + C_L \frac{\dot{v}(z,t)}{aw_x}\right] + f_u^{turb}(z,t), \tag{10}$$

$$f_v^a(z,t) = \frac{1}{2}\rho w_x^2 L D_1\left[C_L\left(1 - \frac{2}{aw_x}\dot{u}(z,t)\right) - C_D \frac{\dot{v}(z,t)}{aw_x}\right] + f_u^{turb}(z,t), \tag{11}$$

where $C_D = \overline{C}_D a^2$, $C_L = \overline{C}_L a^2$. These expressions are substituted to equations (1) and (2). We can see that resulting system of equations is non-linear.

4. Probability of boundary limit over-crossing

We know from the preceding explanation that it is possible to transpose the solving of the stability problem of the tube bundle vibration (consisting from elastic tubes) on the stability problem of one elastic tube vibrating inside the tube bundle (consisting of rigid tubes). This problem can be described by equation

$$\ddot{q}(t) + \varepsilon B(q,\dot{q}) + K(q) = f(t), \tag{12}$$

where $q(t)$ is the tube displacement and ε is a small parameter. The function $B(q,\dot{q})$ expresses the non-linear damping and the function $K(q)$ expresses the non-linear stiffness. We shall suppose that $B(q,\dot{q})$ is an odd function with respect to $\dot{q}(t)$. Function $f(t)$

expresses random excitation (excitation by turbulent fluctuations). We further suppose that this function has a form of white noise.

Under these assumptions the quantities $q(t)$ and $\dot{q}(t)$ are components of a two-dimensional Markov process. We designate the trajectory in the phase plane in a point t by symbols $x = q(t),\ y = \dot{q}(t)$. Let $x = x_0,\ y = y_0$ is starting point of the trajectory at time $t = 0$. The probability that the trajectory in phase plane, which starts at $x_0 = q(0),\ y_0 = \dot{q}(0)$, arrives within the differential element $dxdy$ at the later time is $p(x, y|x_0, y_0; t)dxdy$.

The appropriate Fokker-Planck equation is

$$\frac{\partial p}{\partial t} + \frac{\partial p}{\partial x} y - \frac{\partial}{\partial y}\{[\varepsilon B(x, y) + K(x)]p\} - \frac{1}{2} I \frac{\partial^2 p}{\partial y^2} = 0, \tag{13}$$

where I is the intensity of white noise which is given by expression

$$\mathsf{E}\{f(t).f(t+\tau)\} = I\delta(\tau), \tag{14}$$

E is so called expected value.

Kinetic energy related to tube of length one meter and mass one kilogram is $E_k = \frac{1}{2}\dot{q}^2$.

Potential energy is $E_p = \int_0^q K(\xi)d\xi$. Total energy is therefore given by the sum

$$E = E_k + E_p. \tag{15}$$

Equations describing probability density for parameters $q(t)$ and $E_k(t)$ can be obtained by transformation of variable x, y in equation (13) on variable x, η, [13]. It is useful to introduce following designation for this purpose: $S = p^+ + p^-, D = p^+ - p^-$, where $p^+ = p(x, y|\eta_0; t)$, $p^- = p(x, -y|\eta_0; t)$. Then we can write

$$\frac{\partial S}{\partial t} + y\frac{\partial D}{\partial x} - \frac{\partial}{\partial y}(\varepsilon BS) - K\frac{\partial D}{\partial y} - \frac{1}{2} I \frac{\partial^2 S}{\partial y^2} = 0, \tag{16}$$

$$\frac{\partial p}{\partial t} + \frac{\partial D}{\partial x} - \frac{\partial}{\partial \eta}\left\{\left[\varepsilon\sqrt{2(\eta - E_k)}B\left(x, \sqrt{2(\eta - E_k)}\right) - \frac{1}{2}I\right]p\right\} - \frac{1}{2} I \frac{\partial^2}{\partial \eta^2}[(\eta - E_k)p] = 0. \tag{17}$$

If we suppose that the response amplitude is of the unity order, variable I must be of ε order. From that it follows that the third and the forth terms on the left-hand side of equation (17) are of ε order. In order to the second term on the left-hand side of this equation was also of ε order, it must be $D = g(\eta) + 0(\varepsilon)$, where $g(\eta)$ is a function of η. That means that $p(x, y|\eta_0; t)$ can be expressed in the form of η function, with an error of ε order. We can write

$$p(x, \eta|\eta_0; t) = \frac{\overline{p}(\eta|\eta_0; t)}{2A_1(\eta)\sqrt{(\eta - E_k)}}, \tag{18}$$

where
$$\bar{p}(\eta|\eta_0;t) = \int_{\Delta x} p(x,\eta|\eta_0;t)dx\,, \tag{19}$$

$$A_1(\eta) = \frac{1}{2}\int_{\Delta x}\frac{dx}{\sqrt{\eta - E_k(x)}}\,. \tag{20}$$

It is evident that the integration range must be such that $\eta > E_k$. Under these considerations it is possible to establish that for a small value of parameter ε the one-dimensional Markov process with probability density $\bar{p}(\eta|\eta_0;t)$ can approximate the random vibration of tube. Corresponding Fokker-Planck equation has the form

$$\frac{\partial\bar{p}}{\partial t} - \frac{\partial}{\partial\eta}\left\{\left[\varepsilon A_2(\eta) - \frac{1}{2}I\right]\bar{p}\right\} - \frac{1}{2}I\frac{\partial^2}{\partial\eta^2}\left[A_3(\eta)\bar{p}\right], \tag{21}$$

$$A_2(\eta) = \frac{1}{A_1(\eta)\sqrt{2}}\int_{\Delta x} B\left[x, \sqrt{2(\eta - E_k(x))}\right]dx\,, \tag{22}$$

$$A_3(\eta) = \frac{1}{A_1(\eta)}\int_{\Delta x}\sqrt{\eta - E_k}\,dx\,. \tag{23}$$

Let us consider so called stationary solution $\bar{p}_\infty(\eta)$ at first, which is defined by expression

$$\bar{p}_\infty(\eta) = \lim_{t\to\infty}\bar{p}(\eta|\eta_0;t)\,. \tag{24}$$

This solution can be found from equation (21) if we put $\frac{\partial\bar{p}}{\partial t} = 0$. We get

$$\bar{p}_\infty(\eta) = kA_1(\eta)\exp\left[-(2\varepsilon/I)A_4\right], \tag{25}$$

where
$$A_4(\eta) = \int_0^\eta \frac{A_2(\xi)}{A_3(\xi)}d\xi \tag{26}$$

and k is a normalization constant.

Let us consider the non-linear damping in the form of equation (27). The same problem was solved in [13]. We can meet with the same form of damping in the case of vibration of the loosely supported tube (if the dry friction is supposed in the gap).

$$B(q,\dot{q}) = \dot{q}\left(1 + \varepsilon|\dot{q}|^n\right)\,. \tag{27}$$

Equation of motion describing tube vibration is then of the form

$$\ddot{q} + 2\zeta_0\omega_0\dot{q}\left(1 + \varepsilon|\dot{q}|^n\right) + \omega_0^2 = f(t)\,, \tag{28}$$

where ζ_0 is damping ratio and ω_0 is natural frequency in a case of the linear vibration problem, when is $\varepsilon = 0$ a $B = 2\zeta_0\omega_0$. It follows from expressions (22), (23) and (26) given above:

$$A_3(\eta) = \eta\,, \tag{29}$$

$$A_2(\eta) = \eta + \varepsilon \frac{2}{\pi}(2\eta)^{(n+2)/2} C_{n+2}, \tag{30}$$

$$A_4(\eta) = \eta + \alpha_n \varepsilon \eta^{(n+2)/2}, \tag{31}$$

$$C_n = \int_0^{\pi/2} \cos^n \psi d\psi, \tag{32}$$

$$\alpha_n = \frac{2^{(n+6)/2}}{(n+2)\pi} C_{n+2}. \tag{33}$$

We introduce the non-dimensional quantities

$$\tilde{q}(t) = \frac{q(t)}{\sigma_0}, \tag{34}$$

$$\dot{\tilde{q}}(t) = \frac{\dot{q}(t)}{\sigma_0 \omega_0}, \tag{35}$$

$$\sigma_0 = \left(\frac{I}{4\zeta_0 \omega_0^3} \right)^{1/2}. \tag{36}$$

The probability $p(x,y)$ has in that case the form

$$p(x,y) = c p_0(x,y) \exp\left\{ -\alpha_n \varepsilon^* \left[(x^2 + y^2)/2 \right]^{(n+2)/2} \right\}, \tag{37}$$

$\varepsilon^* = \varepsilon \omega_0^n \sigma_0^n$ is non-dimensional parameter characterizing non-linearity and $p_0(x,y)$ is the solution of linear problem for the case $\varepsilon^* = 0, c = 1$. We can write this solution in the form

$$p_0(x,y) = \frac{1}{2\pi} \exp\left\{ -\left[(x^2 + y^2)/2 \right] \right\}. \tag{38}$$

The normalization constant c in equation (37) is given by expression

$$c = \left[\int_0^\infty \exp\left\{ -A_4^*(\eta) \right\} d\eta \right]^{-1}. \tag{39}$$

We substitute $\varepsilon = \varepsilon^*$ in to the expression (30) and calculate the function $A_4^*(\eta)$. Expected value of the momentum of order m (if m is even) of non-dimensional quantity $\tilde{q}(t) = q(t)/\sigma_0$ can be calculated from the relation

$$\mathsf{E}\left\{ \left[\frac{q(t)}{\sigma_0} \right]^m \right\} = \frac{2^{(m+2)/2} C_m}{\pi} \frac{\int_0^\infty \eta^{m/2} \exp\left\{ -A_4(\eta) \right\} d\eta}{\int_0^\infty \exp\left\{ -A_4(\eta) \right\} d\eta}. \tag{40}$$

This quantity equals zero if m is odd. We express the exponential functions in (38) and (39) in the form of power series in ε^*. After integration we obtain

$$\mathsf{E}\left\{\left[\frac{q(t)}{\sigma_0}\right]^m\right\} = \frac{2^{(m+2)/2} C_m}{\pi}\left(1-\varepsilon^* A_{mn}\right)\Gamma\left(\frac{m+2}{2}\right) + O\left(\varepsilon^{*2}\right), \tag{41}$$

constants A_{mn} are

$$A_{mn} = \left\{\left[\frac{\Gamma\left(\frac{m+n+4}{2}\right)}{\Gamma\left(\frac{m+2}{2}\right)}\right] - \Gamma\left(\frac{n+4}{2}\right)\right\}. \tag{42}$$

For $m = 2$ we get from equation (41) the expression for the mean square response $\sigma^2 = \mathsf{E}\{q^2(t)\}$. If n is odd, we have

$$\frac{\sigma^2}{\sigma_0^2} = 1 - \varepsilon^*\left[(n+1)(n-1)\cdots(1)\right] + O(\varepsilon^{*2}). \tag{43}$$

If n is even, we have

$$\frac{\sigma^2}{\sigma_0^2} = 1 - \varepsilon^* \frac{\sqrt{2}}{\pi}\left[(n+1)(n-1)\cdots(2)\right] + O(\varepsilon^{*2}). \tag{44}$$

It is interesting to compare this result with the result of solution of the same problem solved by method of equivalent linearization, [14].

Instead of equation (28) we use equation in the form

$$\ddot{q}(t) + \varepsilon_{eq}\dot{q} + \omega^2 q = f(t). \tag{45}$$

The equivalent damping in this equation must be chosen to minimize mean square of the difference between the response of equation (28) and the response of equation (45). The optimal value of ε_{eq} is then given by expression

$$\varepsilon_{eq} = \varepsilon\left[1 + \frac{\mathsf{E}\left\{|\dot{q}|^{n+2}\right\}}{\mathsf{E}\left\{\dot{q}^2\right\}}\right]. \tag{46}$$

Let us consider the response $q(t)$ of non-linear tube vibration, which is described by the non-linear differential equation (28). Expected frequency of the up-crossing of certain fixed level a is given by expression

$$\Omega = 2\pi\int_0^\infty y p(\tilde{q}_a, y)\,dy, \tag{47}$$

where $\tilde{q}_a = a/\sigma_0$. For expressing the function p we use equation (37).

It is possible to get an analytical expression for Ω in special case $n = 2$

$$\frac{\Omega}{\omega_0} = \frac{1 - erf\left(\psi + q_a^2/4\psi\right)}{1 - erf\psi} \exp\left(q_a^2/2\right). \tag{48}$$

The frequency ω_0 is natural frequency of the linear problem solution, that means for $\varepsilon^* = 0$. Knowledge of the frequency Ω is very important for assessment of heat exchangers reliability. Let us remark that for $q_a = 0$ is $\Omega = \omega_0$.

5. Conclusion

In the first part of the article the problem of the cross-flowed tube bundle vibration is described. The phenomenon of the vibration stability limit is mentioned. The loss of stability limit means, that for certain flow velocity, so called critical velocity, the amplitudes of vibration suddenly increase. It can be very dangerous, because the vibration can be so intensive, that the heat exchanger may be destroyed.

The fundamental equations describing the tube vibration are introduced. It is supposed that the so-called displacement mechanism takes place. This mathematical model of vibration is based on the knowledge of the coefficients of added mass, added aerodynamic damping and added aerodynamic stiffness. These coefficients express aerodynamic couplings between the tubes. The coefficients are determined experimentally. The experimental equipment for the research of aerodynamic couplings between tubes is built in the laboratory of the University of West Bohemia.

The justification of semi rigid theory based on the solving of vibration of one elastic tube inside the tube bundle, which consists of rigid tubes, is introduced further. The main part of the article is devoted to the theory of non-linear tube vibration for the case of random turbulent excitation. It is supposed that the excitation has a form of white noise. Under these assumptions the displacements $q(t)$ and vibration velocity $\dot{q}(t)$ are the components of two-dimensional Markov process. The probability of that process is described by the Fokker-Planck equation. For a special form of the non-linear damping the analytical solution is introduced. The main objective is to determine the probability of the over-crossing of some fixed vibration amplitude. The expected frequency of up-crossings is calculated. The knowledge of the frequency is very important for the assessment of the heat exchangers safety work.

It is known that near the stability limit some interesting phenomena take place. Further works will be therefore concentrated on that field. It is mainly the intermittent behaviour [6], [10], [15] and the triggering mechanism of non-stability. It is important to study also other types of non-linear systems, for example non-linear models of loosing supported tubes or non-linear vibration of the turbine and the compressor blades.

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