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Synthesis of dexterity measure of mechanisms by evolution of dissipative system

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Abstract

The paper deals with the new approach of solving traditional kinematical synthesis of mechanisms. The kinematical synthesis is reformulated as nonlinear dynamical problem. All searched parameters of the mechanism are in this dynamical dissipative system introduced as time-varying during motion of mechanism's dimension iteration. The synthesis process is realized as the time evolution of such system. One of the most important objectives of the machine synthesis is the dexterity measure. The new approach is applied to optimization of this property.

Keywords: synthesis, optimization, dexterity, nonlinear dynamic

1. Introduction

Mechanical synthesis is necessary method that is used during design of mechanism. This method gives the optional kinematical parameters of a designed mechanism. Solution of such difficult task usually requires large amount of iterations. The current applied methods are either very specific for simple mechanisms or they are based on iterative solution of kinematical description of mechanism's motion in certain limited number of so called precision points or they are based on more general methods of optimization approaches, recently using evolutionary methods like genetics algorithm (e.g. [3]).

The general synthesis methods seem to be enough powerful and to find the solutions for all problems. They are based on performing mechanism's synthesis rely on an attempt to redefine the dimensions of the system in such a way that a deviation from the desired behaviour is minimized by the use of optimization methods. However, all current methods suffer from two related problems. The first problem is that the proposed dimensions of the mechanism being synthesized do not allow the mechanism assembly in all positions required for the desired motion. The second problem is that if a mechanism's synthesis iteration fails for certain parameter because of constraint and/or assembly violation the whole knowledge from this iteration is lost. Moreover the mechanism's synthesis requires different properties during different motion phases and this selective knowledge from different motion phases is not available from one parameter setting for the whole motion. The solution of the first problem has been proposed by the usage of time-varying dimensions during motion of mechanism's dimension iteration [2]. However, this scheme requires large amount of iterations.

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This insufficiency has been overcome by the approach based on nonlinear dynamical or nonlinear control [6]. This approach reformulates mechanism's synthesis as a nonlinear dynamical problem or nonlinear control problem. It proved that nonlinear dynamical or nonlinear control could be used for kinematical synthesis of a simple guiding mechanism. The approach based on nonlinear dynamical system had been further developed and proved on the latest version of redundant parallel kinematical machine sliding star [1].

This paper deals with synthesis of mechanism using dexterity measure optimization. The dexterity describes the quality of the motion and the force transfer between input and output [7].

The theory has been tested on a simple planar parallel kinematical machine. This means that the method tested previously on the mechanisms has been extended in order to optimize more complex kinematical properties.

2. Geometrical synthesis of the mechanism

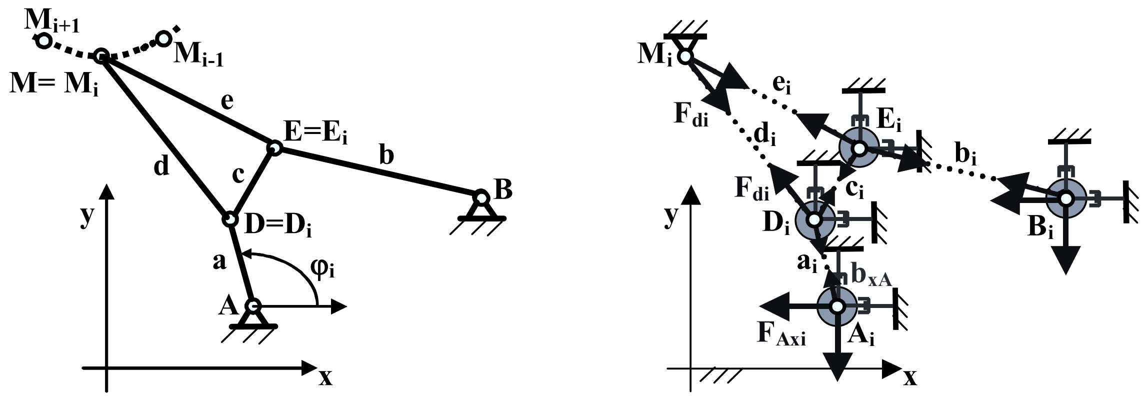
As it has been mentioned this method transforms problem of kinematical synthesis into a problem of solution of associated dissipative dynamical system. All searched (synthesized) parameters of the mechanism are in this dynamical dissipative system introduced as time-varying during motion of mechanism's dimensions iteration. The associated dynamical dissipative system consists of n subsystems for individual positions of end-effector point. End-effector point passes through demanded workspace and describes asked net. The sought mechanism is then decomposed as follows. Dimensions of the mechanism that are synthesized are replaced by linear springs with prescribed stiffness. Joints of the mechanism which are in touch with synthesized dimensions are replaced by prescribed masses. The nonzero force acts into relevant masses whenever the corresponding dimension differs among subsystems. Masses are damped by damper elements. These damper elements are connected to the inertial frame according to sky-hook idea [4] or [5] and ensure the stabilization of the whole system. The solution then consists of synthesis of dimensions of the particular bodies. The dimensions of the bodies assemble the overall dynamical system that enables separate parameters but requires their ultimate equality. The synthesis process is then realized as the time evolution of such system.

Based on the decomposition, final dynamical equations for subsystem i are as follows

$$
\begin{aligned}
m_P \ddot{x}_{Pi} &= \sum_{j=1}^{n} \left[k_{l1}(l_{1i} - l_{1j})\cos(\varphi_{l1i}) + k_{l2}(l_{2i} - l_{2j})\cos(\varphi_{l2i}) \right] - b_{xP}\dot{x}_{Pi}\,, \\
m_P \ddot{y}_{Pi} &= \sum_{j=1}^{n} \left[k_{l1}(l_{1i} - l_{1j})\sin(\varphi_{l1i}) + k_{l2}(l_{2i} - l_{2j})\sin(\varphi_{l2i}) \right] - b_{yP}\dot{y}_{Pi}\,.
\end{aligned}
\tag{1}
$$

Where m_P is mass of particular joint P. $\ddot{x}_{Pi}, \ddot{y}_{Pi}$ are accelerations of joint P in dynamical subsystem i in x axe and in y axe respectively. k_{l1} and k_{l2} are constant stiffness introduced in varying dimensions l_{1i} and l_{2i} of particular subsystem i and dimensions l_{1j} and l_{2j} of all simulated subsystems. φ_{l1i} and φ_{l2i} are the angles which are responsible for projection of force acting in the directions of dimensions l_{1i} and l_{2i} to the x axe or to the y axe. b_{xP} and b_{yP} are constant damper coefficients and finally $\dot{x}_{Pi}, \dot{y}_{Pi}$ are velocities of joint P in dynamical subsystem i.

The principle of the method is presented in fig. 1. The transformation of synthesis of an original mechanism into the associated dynamical dissipative system is illustrated in a simple guiding mechanism.



a) Original mechanism

b) Associated dynamical dissipative system

Fig. 1. Associated dynamical dissipative system of given example.

3. Geometrical synthesis using dexterity measure optimization

Also synthesis using dexterity measure optimization consists in transformation of dexterity computation of mechanical system into a problem of solution of associated dissipative dynamical system. The transformation is based on computation of dexterity for each demanded end-effector position and computation of force that ensures convergence to the demanded dexterity.

Synthesis of dexterity measure is in principal based on geometrical synthesis. Furthermore the geometrical synthesis is taken as a fundamental part of the synthesis of other parameters, e.g., dexterity measure. Also in this case all searched (synthesized) parameters of the mechanism are in the dynamical dissipative system introduced as time-varying during motion of mechanism's dimensions iteration and the synthesis process is realized as the time evolution of the system.

Let us introduce the dexterity and the condition number as one of important feature of the mechanism. Kinematical constraint is described as a function of dimensions **l**, the input coordinates in the joints **s** and the output coordinates, i.e. the position of the end-effector **v**

$$\mathbf{f}(\mathbf{l},\mathbf{s},\mathbf{v}) = \mathbf{0}\,. \tag{2}$$

The condition number C and the dexterity D are derived from the equation 2 as follows

$$\begin{gathered}\mathbf{J}_S\dot{\mathbf{s}} + \mathbf{J}_V\dot{\mathbf{v}} = \mathbf{0}\,,\\ \dot{\mathbf{s}} = -\mathbf{J}_S^{-1}\mathbf{J}_V\dot{\mathbf{v}},\end{gathered} \tag{3}$$

$$C = cond(\mathbf{J}_S^{-1}\mathbf{J}_V)\,, \tag{4}$$

$$D = \frac{1}{C}\,, \tag{5}$$

where $\mathbf{J}_S$, $\mathbf{J}_V$ are corresponding Jacobi matrices of the constraint 2. Instead of the dexterity parameter D, the condition number C of the Jacobi matrices of the system is treating as the synthesized parameter in following notation and example.

Based on principle of virtual power we can count for force contributions of the particular parts (dimensions). For the most clearness of the transformation two parameters (dimensions) l_1 and l_2 are taken as parameters influence the system.

$$F\dot{C} = F\frac{\partial C}{\partial l_1}\dot{l}_1 + F\frac{\partial C}{\partial l_2}\dot{l}_2 \tag{6}$$

and

$$F = k_{lC}(C - C_D)\,. \tag{7}$$

Where k_{lC} is again constant stiffness, C is computed (actual) dexterity and C_D is demanded dexterity. Acting forces for particular dimension are taken form substitution of equation (7) into equation (6). In the following the stiffness k_{lC} is divided into k_{l1C} and k_{l2C}

$$F_{l1} = k_{l1C}(C - C_D)\frac{\partial C}{\partial l_1}, F_{l2} = k_{l2C}(C - C_D)\frac{\partial C}{\partial l_2}\,. \tag{8}$$

The final dynamical equations of mass particles of the joint P with dexterity demand for subsystem i are as follows

$$\begin{aligned} m_P\ddot{x}_{Pi} &= \sum_{j=1}^{n}\left[\begin{array}{l}\left(k_{l1}(l_{1i} - l_{1j}) + k_{l1C}(C_i - C_{Di})\dfrac{\partial C_i}{\partial l_{1i}}\right)\cos(\varphi_{l1i}) + \\ + \left(k_{l2}(l_{2i} - l_{2j}) + k_{l2C}(C_i - C_{Di})\dfrac{\partial C_i}{\partial l_{2i}}\right)\cos(\varphi_{l2i})\end{array}\right] - b_{xP}\dot{x}_{Pi}\,, \\ m_P\ddot{y}_{Pi} &= \sum_{j=1}^{n}\left[\begin{array}{l}\left(k_{l1}(l_{1i} - l_{1j}) + k_{l1C}(C_i - C_{Di})\dfrac{\partial C_i}{\partial l_{1i}}\right)\sin(\varphi_{l1i}) + \\ + \left(k_{l2}(l_{2i} - l_{2j}) + k_{l2C}(C_i - C_{Di})\dfrac{\partial C_i}{\partial l_{2i}}\right)\sin(\varphi_{l2i})\end{array}\right] - b_{yP}\dot{y}_{Pi}\,. \end{aligned} \tag{9}$$

4. Example

The realization of the method is presented in fig. 2. The used example of kinematical system has two synthesizable dimensions (l_l, l_r). The point M of mechanism (end-effector of the simple planar parallel kinematical machine) should pass through given points M_i (i = 1, 2,…,n) inside of a given workspace.

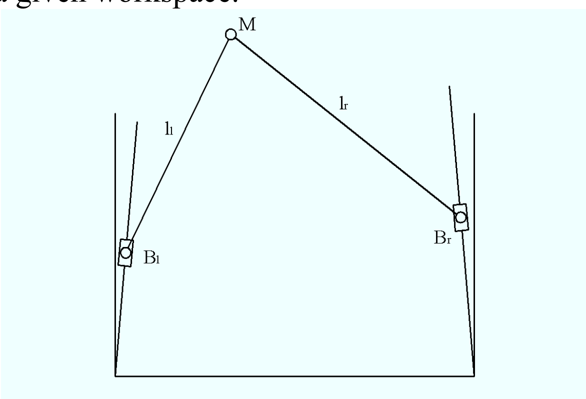


Fig. 2. Original mechanism.

The associated dynamical dissipative system then consists of n subsystems for individual positions of point M, fig. 3. The masses m_{Bl}, m_{Br} are introduced in points B_{li}, B_{ri}. The interactions among the subsystems are ensured by forces F_{lli}, F_{lri} of linear spring nature. The nonzero force acts into relevant masses whenever the corresponding dimension differs between subsystems i and j (i, j = 1, 2,….n). The stabilization of the whole system is ensured by damper elements.

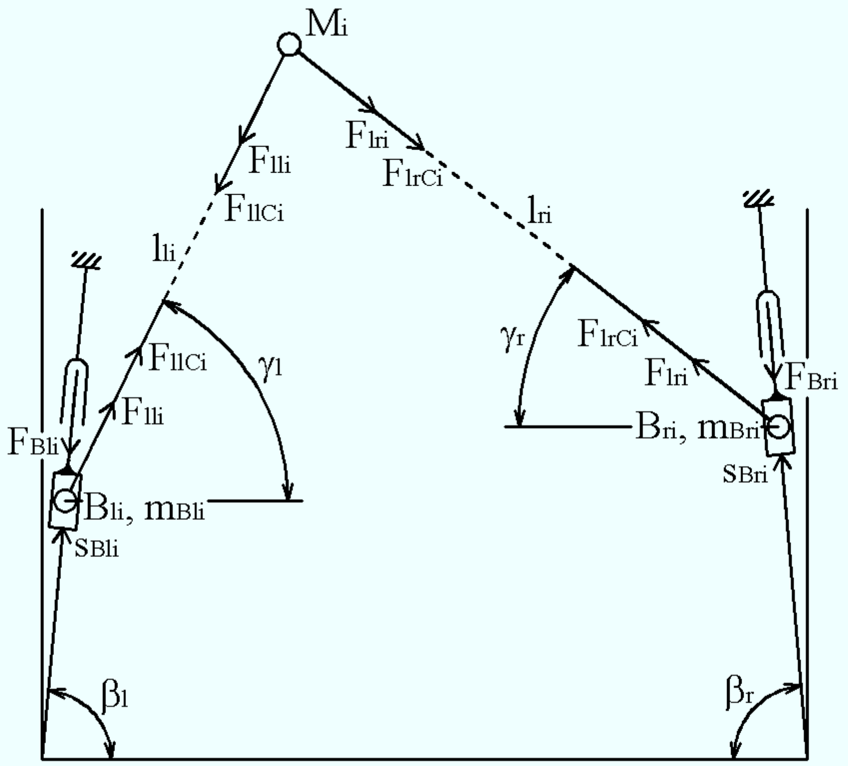


Fig. 3. Associated dynamical dissipative system.

Forces that comes from geometrical synthesis act in the dynamical system as follows

$$F_{lli} = \sum_{j=1}^{n} k_{ll}(l_{li} - l_{lj}); F_{Bli} = b_{sBl}\dot{s}_{Bli} ,$$
$$F_{lri} = \sum_{j=1}^{n} k_{lr}(l_{ri} - l_{rj}); F_{Bri} = b_{sBr}\dot{s}_{Bri} . \qquad (10)$$

Acting forces that comes from synthesis using condition number optimization for particular dimensions of the system are then

$$F_{llCi} = k_{llC}(C_i - C_{Di})\frac{\partial C_i}{\partial l_{li}} ,$$
$$F_{lrCi} = k_{lrC}(C_i - C_{Di})\frac{\partial C_i}{\partial l_{ri}} . \qquad (11)$$

The final dynamical equations for mass particles in the points B_l and B_r with demand of condition number are as follows

$$m_{Bl}\ddot{s}_{Bli} = \sum_{j=1}^{n}\left[k_{ll}(l_{li} - l_{lj}) + k_{llC}(C_i - C_{Di})\frac{\partial C_i}{\partial l_{li}}\right]\cos(\beta_l - \gamma_{li}) - b_{sBl}\dot{s}_{Bli}\,,$$
$$m_{Br}\ddot{s}_{Bri} = \sum_{j=1}^{n}\left[k_{lr}(l_{ri} - l_{rj}) + k_{lrC}(C_i - C_{Di})\frac{\partial C_i}{\partial l_{ri}}\right]\cos(\beta_r - \gamma_{ri}) - b_{sBr}\dot{s}_{Bri}\,. \quad (12)$$

The simulation of the associated dynamical system has been realized within Matlab-Simulink. The demanded condition number has been set equal 1 for all subsystems (for all end-effector's positions). This value means that the forces F_{llC} and F_{lrC} will act to the system until the mechanism reaches such dimensions for which the dexterity is maximal.

5. Results

The system coordinates (l_{li}, l_{ri}, i = 1, 2,…, 16) for all subsystems come to stay on equilibrium values, fig. 4. These equilibrium values can be interpreted as searched parameters of the mechanism. Also the condition number of each of the 16 end-effector's position has decreased and reached equilibrium values close to number 1.2, fig. 5. This means that mechanism's dexterity has increased to maximum value.

It's necessary to remark that for getting these results the stiffness coefficients had to be set to different values. For the equal stiffness parameters the system hasn't reached equilibrium values of the system coordinates. The forces F_{llC} and F_{lrC} had been so great and the system hasn't been able to converge to the equilibrium state with the same dimensions for each subsystem. For $k_{ll} > 20.k_{llC}$ the system reaches the equilibrium values in a short time.

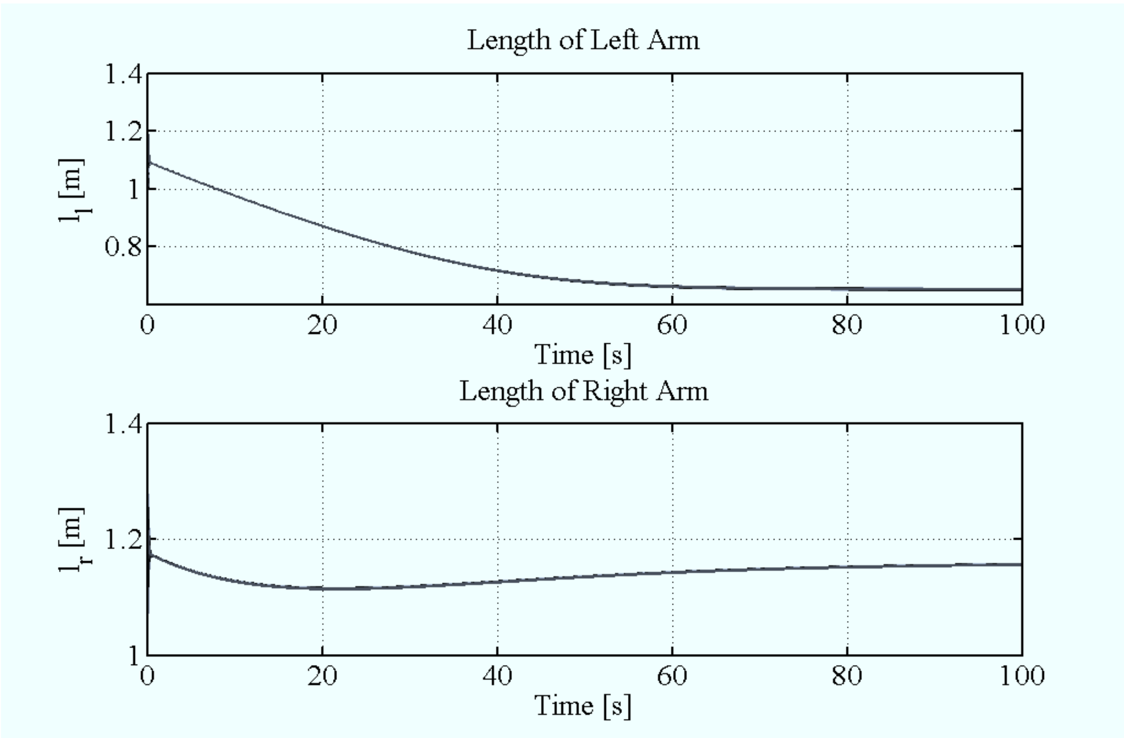


Fig. 4. Dynamical response of the dimensions l_l, l_r.

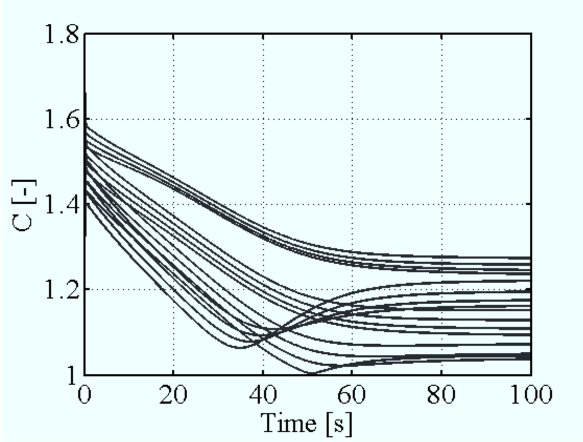
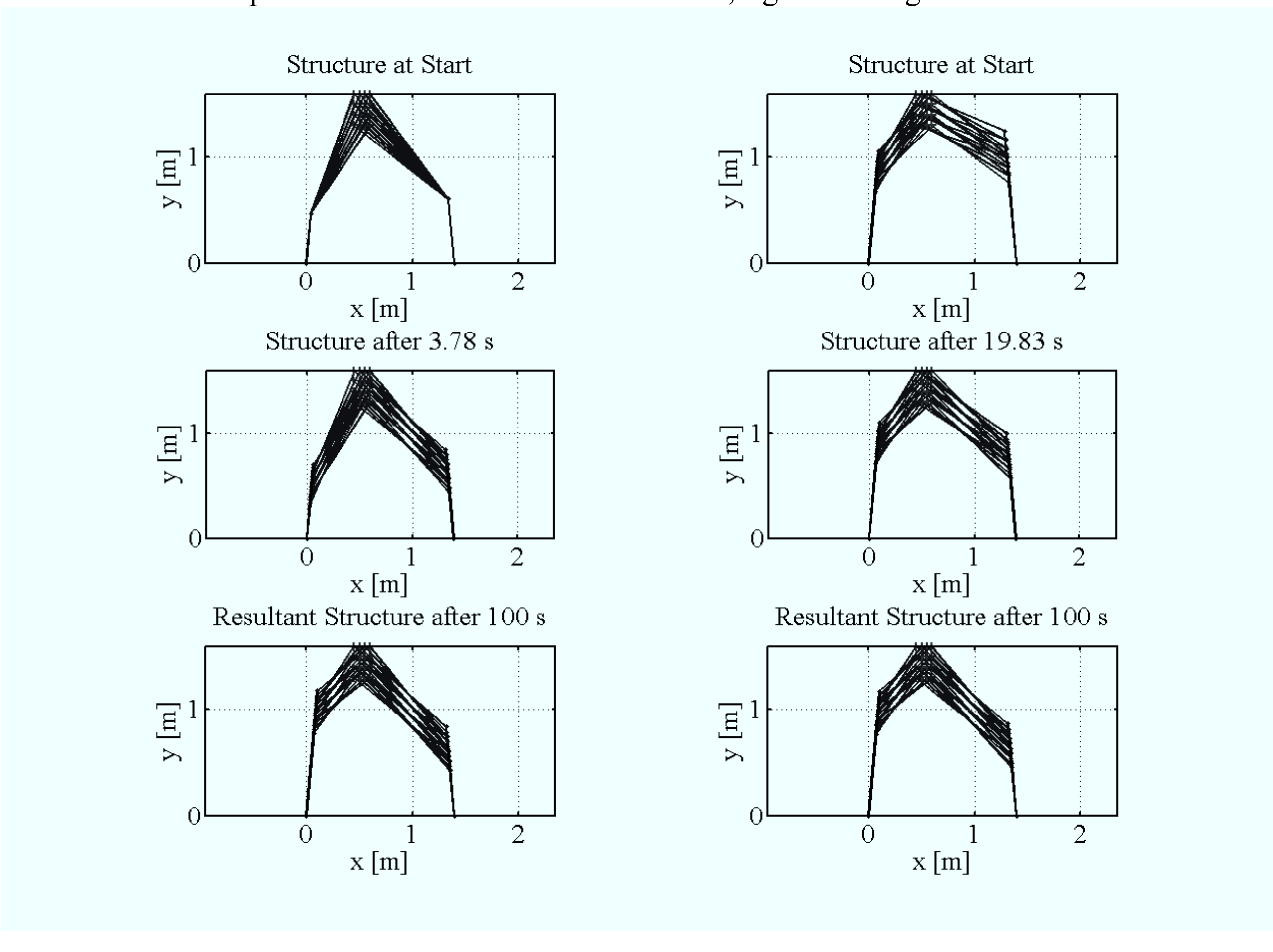


Fig. 5. Evolution of the condition numbers of all positions.

Two examples of optimization runs are illustrated in fig. 6. The pictures in the column fig. 6a correspond to time evolution of parameters and measure presented in fig. 4 and fig. 5. This simulation has started from different dimensions (lengths) of the arms, fig. 6a top for the different positions of points M_i. Simulation in fig. 6b has started from constant dimensions of the arms, fig. 6b top for the different positions of points M_i. It can be seen that the system has reached the same equilibrium values for both simulations, fig. 6a and fig. 6b bottom.



a) Start from different dimensions b) Start from constant dimensions

Fig. 6. Evolution of the mechanism’s structure.

6. Conclusion

The new approach of solving traditional kinematical synthesis of mechanism has been proposed and tested. The presented method introduces the associated dissipative dynamical system. The synthesis process is realized as the time evolution of such system. The approach could be used for basic synthesis tasks as well as for more complex problems of synthesis of parallel machines with dexterity demand. The method is very robust and currently solved benchmarks indicate its ability for global optimization in context of mechanism synthesis.

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