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Contribution to the dynamic behaviour of bladed disks

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Abstract

At this time is dominant to design rotors as assembly of few separated bladed disks. Another type of design ordering is that blade buckle together with a part of shaft creates bladed disk. It is necessary to look on dynamic behaviour of this rotors not only as whole rotors, but also as individual bladed disks. This contribution deals with an analysis of dynamic behaviour of disks. The main for this analysis is acquaintance frequency-modal phenomenons at quiescent state. For modes excitation analysis in rotation, especially for bladed disks, is necessary to build "full" model, it means whole disk, not only cyclic sector. Analysis in time domain presents numerical simulations of rotating bladed disks dynamic behaviour. In the mentioned case it ts possible to suppose steady operating state. The response to noted rotation speed change should be analysed too. In this paper is presented theoretic analysis of bladed disks and analysis of real bladed disk.

Keywords: bladed disk, modal analysis, transient dynamic analysis

1. Introduction

A steam turbines are mechanical device that extracts thermal energy from pressurized steam, and converts it into useful mechanical work [4]. Present-day steam turbines consist of few rotor disks and few rows of stator blades. By reason of higher effectivity are big steam turbines divided into high-pressure and low-pressure, eventually mid-pressure stage [4]. Each rotor stage presents bladed disk. Behind stator blades rising pressure wake, which is crosscut by moving blades relevant rotor stage. Inappropriately designed dimensions together with an inappropriately combination of number of rotor blades and stator blades should lead to strong vibrations. Danger situation is coming when disk excitation frequency is equal to natural frequency, but also when excitation frequency is equal to some of significant multiple of natural frequency. In this case the wave on disk stands that means the nodal diameters on disk do not travel and occurs expressive vibrations of the same points. This occurs that the disk rotating speed is equal critical speed. After the time of running it should be caused the crack nucleation. If they aren't duly locate and repair, they can lead to the disaster.

In the case which is presented in paper, the resonance vibration occurs and mode shape with four nodal diameters is excited. If excited wave is fixed (the nodal diameters aren't travelling) at the disk's coordinate system then the same points on disk will be oscillate. This phenomena evokes cyclic loading and can motives the crack nucleation.

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2. Disk vibrations

2.1. Mode shape

All real mechanical systems are continuous that means their number of degrees of freedom is infinite, thereby numbers of natural frequencies and mode shapes are also infinite. The theoretic basement describes dynamic behaviour at quiescent state is given for example in [1]. Basic mode shapes are characterized by nodal diameters and nodal circles. These points aren't oscillates. Fig. 1 shows some of mode shapes with nodal diameters, circles or theirs combinations. The disk is fixed on axe.

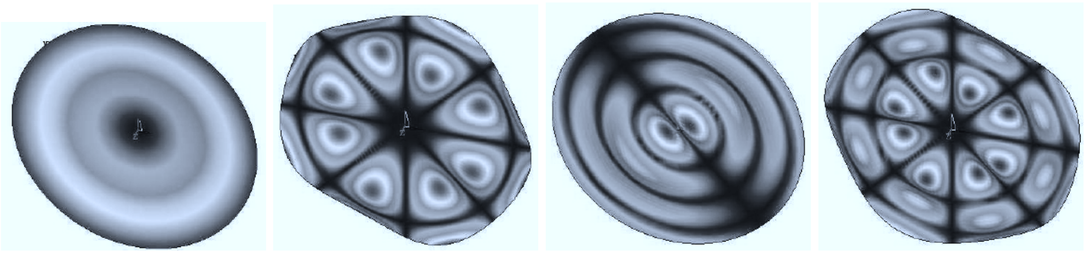

Fig. 1. Mode shape example.

2.2. Dynamic behaviour of rotating disc

In rotating disks may come stiffening. Therefore in the case of running analysis by the using FEM it's appropriate to add in the computation centrifugal forces effect. The result of analysis is relation between frequency and rotating speed. In experimental analysis of exciting rotating disk are formated two waves. The waves are moving in the opposite to each other. The response is standing wave. In rotating is the wave witch is moving in the same direction as is rotation direction is called forward (symbol $+$) and the second one, which is moving opposite is called backward (symbol $-$). From the stationary viewpoint the angular velocity of both waves for $k-$multiple of rotating can be expressed as:

$$\Omega_{m+} = \Omega_{m0} + k\omega, \quad \Omega_{m-} = \Omega_{m0} - k\omega \tag{1}$$

For the possibility to excite particular mode shape is most important so-called "standing wave in space", it occurs when angular velocity of backward wave is equal zero. In this case the nodal points are in quiescent state. Rotating speed, of which there is significant response is called critical speed. It comes when multiple of rotating speed is equate to the number of nodal diameters so:

$$\omega_{krit} = \frac{\Omega_{m0}}{m} \tag{2}$$

A plot of frequencies against rotating speed is known as a Campbell diagram. It's in fig. 2. Natural frequency is named as curve 'A'. It isn't always invariable. It can be changed by reason of, for example, centrifugal forces. There are two other curves in the diagram. Curve 'B' represents frequence of the backward wave and curve 'F' represents frequence of the forward wave [2].

If the number of nodal diameters is equate to $k-$multiple of excitation frequency, thus $m = k$, the disk oscillates significant every time, when natural frequency curve, at which is response mode with m nodal diameters, cross the line presented $k-$multiple of excitation frequency (fig. 2).

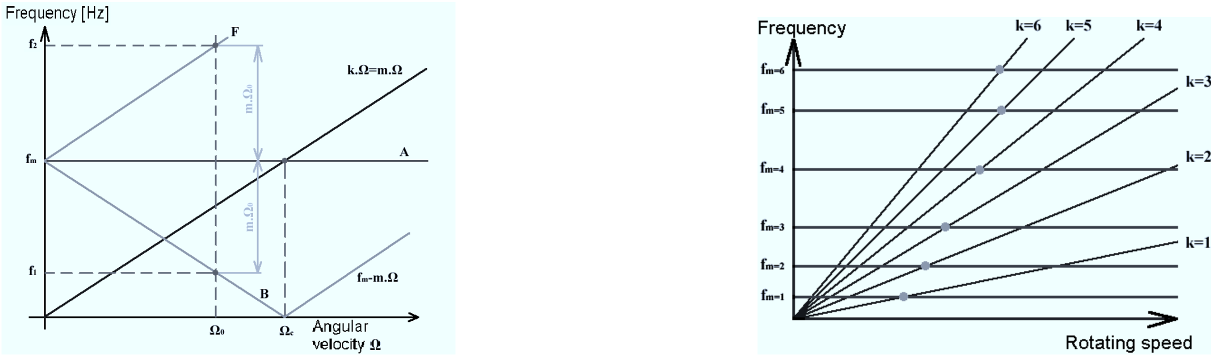

Fig. 2. Campbell diagram and significant resonant states.

2.3. Bladed disk vibrations

The critical speed of bladed disk depends on the natural frequency and number of nodal diameters, but also on the number of rotor blades, eventually on the number of cyclic sectors. There are significant oscillations not just if $k = m$ but also, as shown in fig. 3, for:

$$k = s - m \tag{3}$$

$$k = s + m \tag{4}$$

$$k = 2s - m \tag{5}$$

Where s is number of rotor blades or number of cyclic sectors created, for example, by connecting several blades by bandage. In many cases considering to stiffness effect and weight parameters, there may appear nodal circles passing through the blades, which basically means that blades oscillate by some of the higher shapes. Complete Campbell diagram drawn for mode shape with m nodal diameters is in fig. 3.

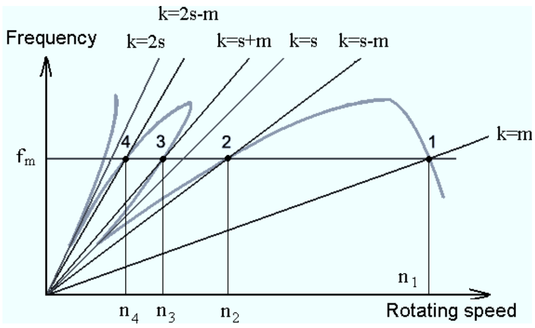

Fig. 3. Complete Campbell diagram.

3. Creating the model geometry

Model proportions has been given by the model of cyclic sector with three blades joint by bandage on the top (fig. 4). This model was provided by SKODA POWER. The modal analysis results of this model was provided too. It was necessary to create model of the whole disk, because it is complicated to do an assessment excitation of appropriate mode shape only on cyclic sector. With respect to hardware potentialities was necessary to simplify the model. Original model had created clevis attachment in detail and was created as an assembly including contact between individual parts. It leads to nonlinear problem. Contact elements were replaced with proportional coupling and shaft segment was removed. Comparison of the original model's natural frequencies with those of simplified model shows that eigenvalues of the both models in analysed range are not significantly different.

Fig. 4. Cyclic sector and whole disk.

4. Material model

Disk has been made of steel. It was presumed that the area of linear-elastic behaviour won't be exceeded, thus selected material was linear and isotropic. Young's modulus E was selected $E = 2.1E5MPa$ and Poisson's ratio $\mu = 0.3$. There are the most common steel constants. In problems where the mass matrix is in equations, the material density has to be defined. If measures are in millimetres, Young's modulus in megapascals, density must be in tons to cubic millimeters. So $\rho = 7.85E - 9t.mm^{-3}$

5. Generating the mesh

To analyse dynamic characteristics is the regular mesh useful. The mesh should be fine enough to resolve the highest mode shape of interest. It was necessary to generate such mesh which leeds to the computation time as short as possible, without the eigenvalues of original model and simplify model are too different. There arised a problem with a connecting of the blades and the disk during the meshing. At the mentioned place would be difficult to create the mapped mesh. Using of the contact elements with selected MPC (Multipoint constraint) algorithm seemed to be the best. The blade mesh and disk mesh can be generated separately, no matter if the boundary nodes are connected. Thanks to the MPC algorithm the problem stay linear. Connecting of the blades and the bandage was managed similary. Damping straps were modeled as coupled with bandage and with neighbour parts of damping strap also by MPC algorithm (fig. 5).

6. Modal analysis

Gripping the disk on the shaft was represented by boundary conditions. Shaft was considered to be a rigid body without oscillating. Inside disk surface was fixed in axial direction, also in radial and in tangential direction.

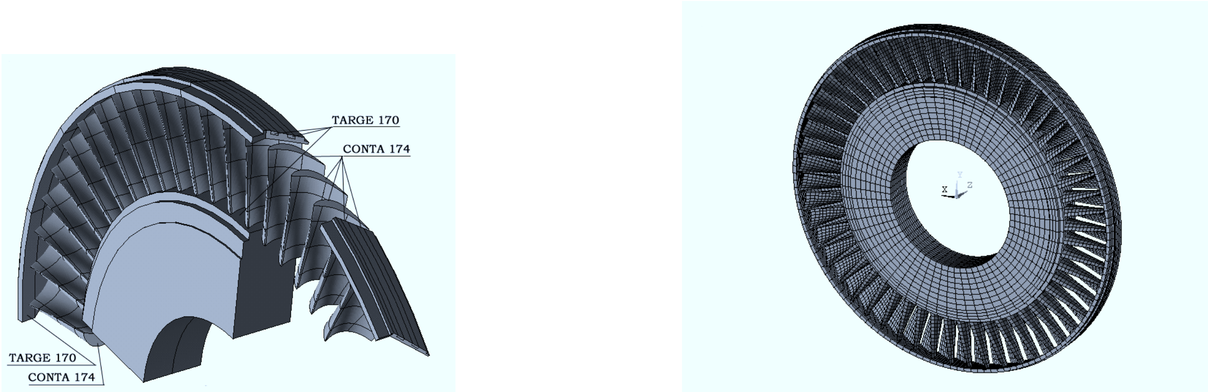

Fig. 5. Use contact elements and meshed model.

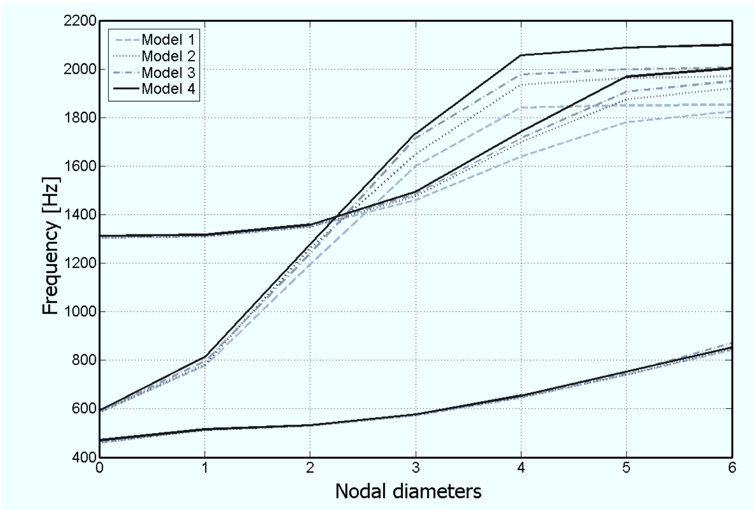

Fig. 6. Modal analysis results.

Frequency range of the modal analysis was selected from 0 Hz to 2500 Hz. Previous computing shows that stiffening due to centrifugal forces has no significant effect to natural frequencies. That's why the stiffening has been regarded immaterial. Number of modes to expand was chosen 100.

As the modal analysis method was selected Block Lanczos. The solution was repeated several times with different mesh density and different element types. Results comparison is in fig. 6. First curve (labeled as 'Model 1') presents the results of modal analysis at the original model (cyclic sector) and in this figure is just for comparing. Another three curves present the results of modal analysis at the simplified whole disk model. 'Model 2' had 188000 nodes, 'Model 3' had 81000 nodes. Element type used on the both of this models were quadratic structural SOLID 95 elements, 'Model 4' had 32000 nodes and mesh is generated by linear SOLID 45 elements. Natural frequencies difference are just several Hertz, but computation time is significantly shorter. For transient analysis must be used the same model and this analysis is much more time expensive than modal analysis, so it had been decided to work with last model.

7. Numerical simulation

7.1. Excitation

Bladed disk was excited by pressure wake behind the stator blades. In real turbine the stationary blading is in quiescence and disk is rotating. During rotation the rotor blades crosscut pressure wake, which causes excitation. Creating this model was complicated, easier was to assume the disk in quiescence and rotaring pressure array. The excitation might be defined as time-variable force affecting to each blade.

7.2. Speed mode selection

It was necessary to define speed modes to analyse the response. On the basis of the modal analysis results, it was decided to focus on excite mode shape with four nodal diameters, it means $m = 4$, it was shape, in which the disk probably oscillated in reality. As it can be seen in fig. 7 the mode shape with four nodal diameters can be excided by several frequencies, because various number of nodal circles can be excited. Frequency $\Omega_m = 1740Hz$ matches to the selected mode shape. Critical speed can be rated from the Campbell diagram as it's shown in fig. 3. Number of rotor blades is 54. Each three blades are joined on the top by bandage. It creates 18 cyclic sectors ($s = 18$). The values of critical speeds for different multiplies excitation frequency are in tab. 1.

Multiple	$k = 2s - m$	$k = s + m$	$k = s - m$	$k = m$
Multiple	32	22	14	4
Critical speed [min^{-1}]	3262.5	4745.5	7457	26100

Tab. 1. Critical speeds.

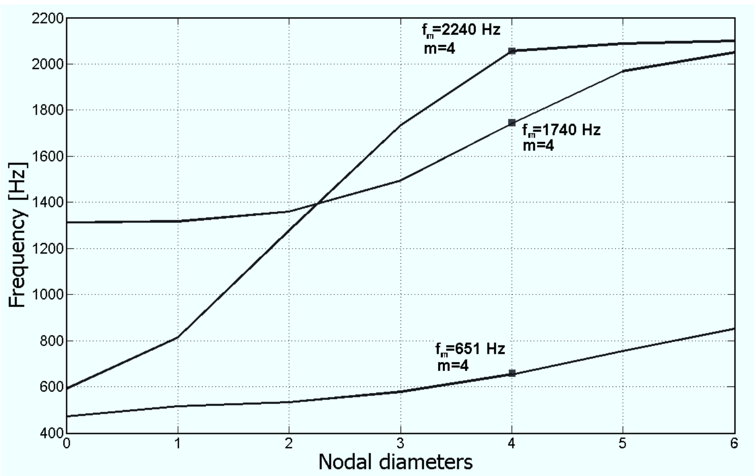

Fig. 7. Excitation of four nodal diameters mode shape.

7.3. Assesment magnitude of the force

Numerical simulations in CFD shown that the time behaviour was relatively well-approach by harmonic function. Amplitude of general force was decomposite to axial and tangential parts. Values was determined by SKODA POWER like this: Axial force amplitude is $F_{A0} = 130N$ and tangential force amplitude is $F_{T0} = 115N$. It is harmonic excitation, so forces can be determine through the time variable functions:

$$F_A(t) = F_{A0} \sin(\omega t + \alpha(l_r - 1)), \quad F_T(t) = F_{T0} \sin(\omega t + \alpha(l_r - 1)) \tag{6}$$

where ω [rad.s^{-1}] is the harmonic excitation angular frequency (values is in tab. 2), l_r is the rotor blade ordinal number where force is defined. On every next blade the force differs by phase difference α.

Multiple	32	22	14	4
Excitation frequency ω [Hz]	10934	15904	24992	87472

Tab. 2. Harmonic excitation frequency.

7.4. Proportional damping

Damping is defined by coefficients α and β. Proportional damping is referenced to mass matrix and stiffness matrix. This damping can be expressed by

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K} \tag{7}$$

where **C** is damping matrix. Term $\alpha\mathbf{M}$ presents the outside structural damping and term $\beta\mathbf{K}$ presents the inner material damping that is function of inner material properties (analogy to the stiffness of elastic element in an assembly) [3]. Coefficients were selected as follows: $\alpha = 2.5$ a $\beta = 1E - 5$. Coefficients may be defined from experiment.

7.5. Analysis method and time step

Selected method is in ANSYS known as ”Mode-Superposition Method”. This method works with a modal reduction. This selection seems to be very well, because later there will be requirement to reduce simulation range only to concrete frequency zone. It will be easy due to modal reduction.

The time step size Δt determines the accuracy of the solution. Recommended time step size should be shorten than tenth of the shortest period.

$$\Delta t < \Delta t_{max} = \frac{T_{min}}{10} \tag{8}$$

The smaller its value, the higher the accuracy, but a time step that is too small waste computer resources.

Simulation time was chosen according to the ten-times turns of the disk. Time step sizes for analyzed speed modes are in tab. 3.

Multiply	32	22	14	4
Time step size [s]	1.84E-1	1.26E-1	8.05E-1	2.30E-1

Tab. 3. Time step size.

7.6. Simulations results

Transient analysis results were data files containing displacements of all nodes in all three directions and in all time steps. It was decided to work up only with z direction displacements and only with selected nodes. Selected nodes lie at the outside circumference, on damping strap edge. Displacement values mentioned above were written in separated file that was processed in MATLAB software. At first it is needed to perform Fourier transform for each speed mode to find out frequencies in spectrums (fig.8). There appears the natural frequency and excitation frequency in all the spectrums. For multiply $k = 32$ they are identical, because number of stator blades is also 32. Except these two frequencies there is another one. Its value is about 650 Hz. Perhaps it is the blade natural frequency. This frequency is corresponding with point on lower curve in fig. 7. Excitation of this frequency was undesirable. To remove it, the modal reduction was used.

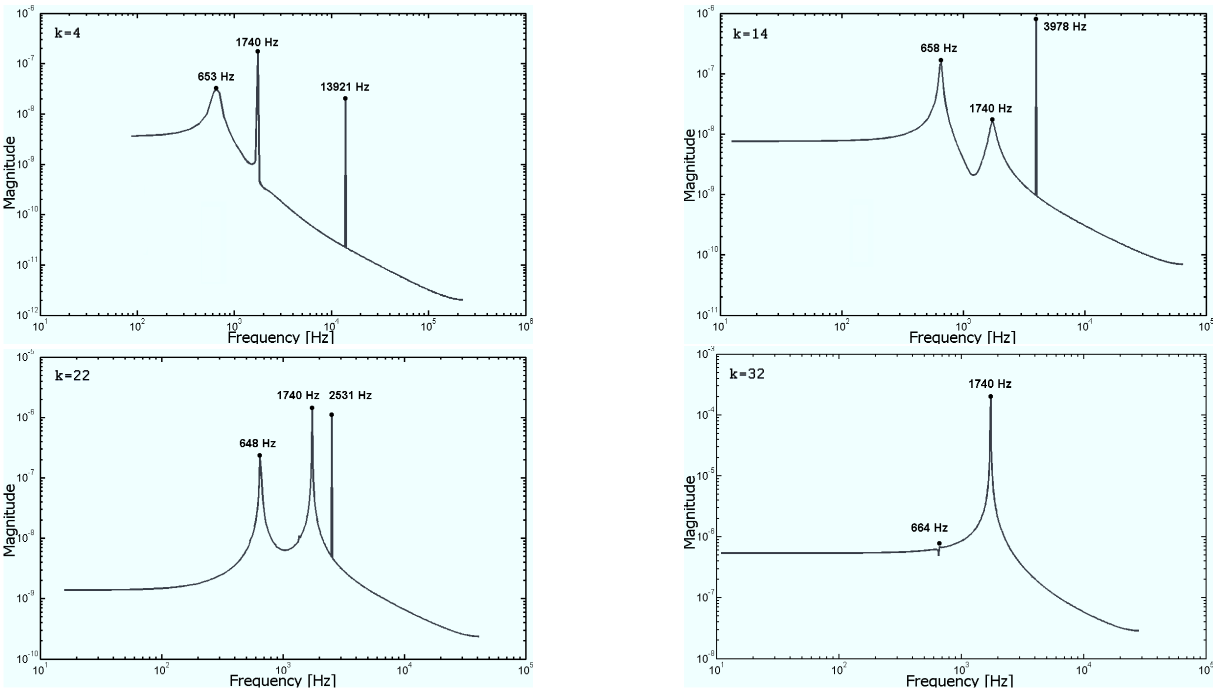

Fig. 8. Fourier's spectrums.

7.7. The modal reduction

The modal reduction makes possible to use only certain number of mode shapes. The mode superposition method sums factored mode shapes (obtained from the modal analysis) to calculate the dynamic response. Perform transient dynamic analysis, reduced to certain interval of frequencies, can be managed either by reducing the frequency range of modal analysis, or by direct choosing of modal shapes (which should be included to the solution) in the transient analysis setting. If the frequency range was reduced to the lower frequency 1000 Hz, it will lead to removing the above mentioned frequency 650 Hz. To prevent potential exciting of any other frequency was range reduced by the high frequency 2000 Hz. So selected range was 1000 Hz - 2000 Hz. Now the shape with four nodal diameters can be excited only by the frequency 1740 Hz (fig.9).

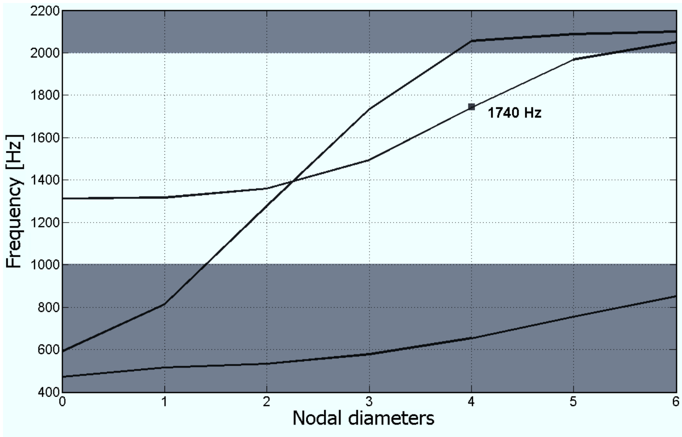

Fig. 9. Reduced frequency range.

7.8. Reduced problem results

Postprocessing was the same as in case of nonreduced problem. Spectrums created by Fourier transform are in fig. 10. Due to modal reduction there are only excitation frequency and natural frequency. Excitation frequency isn't damped that's why it looks like vertical line in spectrums.

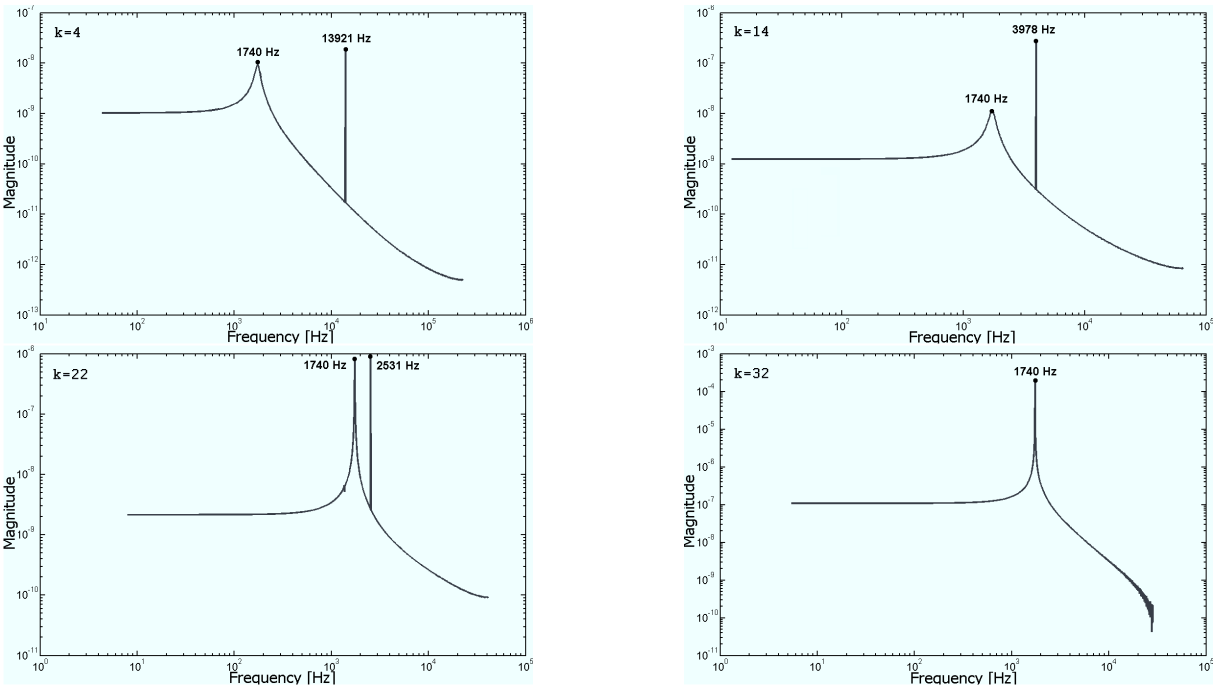

Fig. 10. Fourier spectrums.

Fig. 11 shows the oscillation of individual points on disk circumference in time. It can be seen, there are existing some points that oscillates more expressively than others. This points aren't travelling on disk.

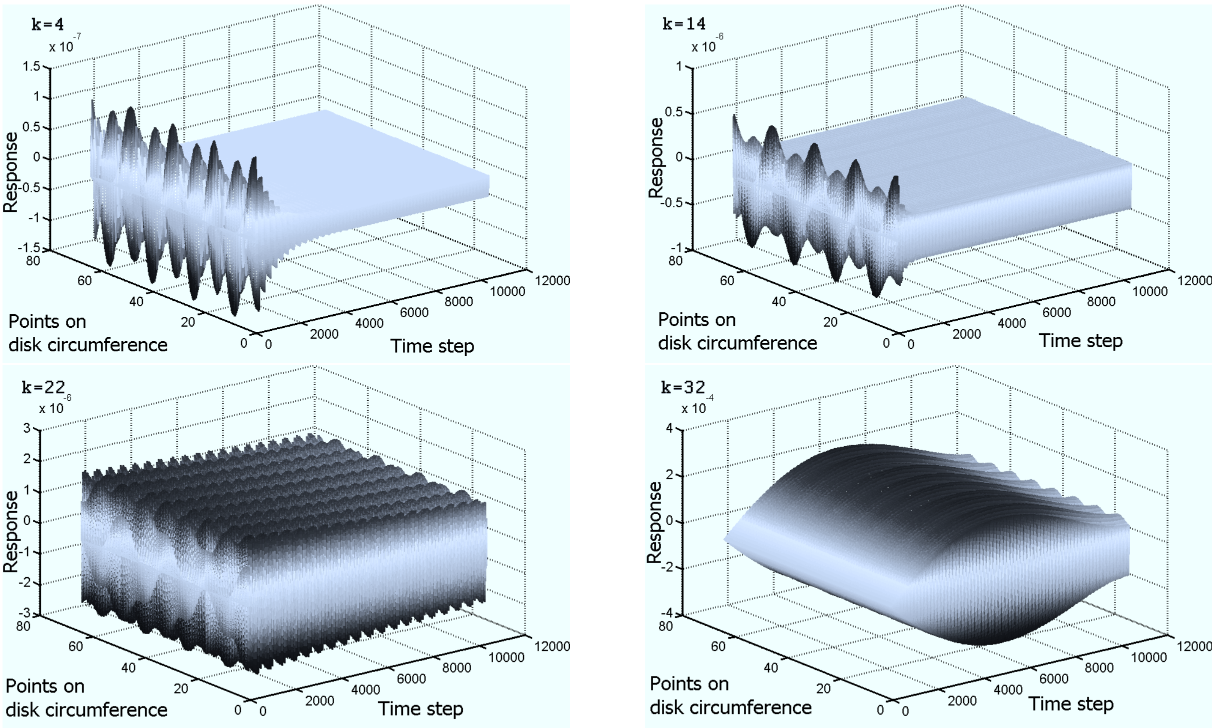

Fig. 11. Disk response.

8. Conclusion

Base for complete analysis of dynamic behaviour of disk and bladed disk are the frequency-modal analysis and the numerical simulation of the rotating bladed disks dynamic behaviour. Frequency-modal analysis doesn't make any imagination about excitation of individual mode shapes. Modal analysis has to go before the numeric simulations because of the eigenvalues determination. Angular velocities are specified pursuant to Campbell diagram. Numeric simulation shows, there are some points which is oscillates more expressively than surroundings points in analysed rotation speed. This points aren't traveling on disk. It's not suitable to run turbine in one of this modes. This model didn't respect damping effect of damping strap in bandage. There was also supposed the perfect rotational symmetry, so it was unable to observe the rotation symmetry breach influence (co-called "mistune effect").

Acknowledgements

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