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Parametric studies in elastic stability of nonconservative beams

T. Nánási [a,*]

[a]*Department of Applied Mechanics, Faculty of Materials Science and Technology in Trnava, Slovak University of Technology in Bratislava, Paulínska 16, 917 24 Trnava, Slovak Republic*

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Abstract

In this paper the nonconservative Beck`s column loaded with follower compressive force is generalized by introduction of the possibility to be subjected to subtangential follower force with excentric point of action. For corresponding boundary eigenvalue problem the frequency equation is derived. Results of parametric studies are presented for the lowest eigencurves, which describe the relation between the compressive load magnitude and the eigenfrequencies and indicate whether stability loss by divergence or by flutter occurs.

Keywords: elastic stability, generalized Beck-Reut`s column, divergence, flutter

1. Introduction

The Beck`s column is a classical example of relatively simple nonconservative system. The cantilever loaded by a compressive force **P** of constant magnitude gives rise to different types of behaviour in dependence of the way the load interacts with the beam structure. The most common case is given by the “dead” load, see fig. 1a, when the direction of the compressive force maintains its orientation whatever the beam deflections are. Gravitational forces are typical example, the corresponding boundary problem is selfadjoint and as far as the elastic stability is concerned, the Euler`s classical approach gives satisfactory results. In other words, the statical criterion of stability can be used. Beck`s column is a cantilever loaded by a compressive force oriented in direction tangential to the deflection curve, fig. 1b. This type of load is labeled as follower force. One of few true possibilities to realize such follower force is the application of a reactive force at the free end of the cantilever. Beck`s column is a nonconservative system. To asses its stability the dynamic criterion of stability is required in which inertia effects are included. The loss of stability in the case of conservative dead load corresponds to divergence, while under the follower force the mechanism of stability loss may correspond to flutter – i.e., vibration with increasing amplitude. Attractive feature of the Beck`s column is the fact, that theoretical critical compressive force is more than eight times higher than the critical force at the conservative case.

Reut`s column arises when the compressive force maintains both the direction and the line of action, fig. 1c. Obviously a platform at the beam tip is required to keep its interaction with the deflecting beam. Here due to excentricity ε additional moment acts at the beam end. This case is also nonconservative, moreover, in mathematical terms the Reut`s column is adjoint of the Beck`s column and as such has the same critical loads as the Beck`s column. Generalization of the above mentioned three beams we obtain if we admit subtangential compressive force together with partial excentricity as illustrated in fig. 1d. The aim of this

[*]Corresponding author. Tel.: +421 335511601, e-mail: tibor.nanasi@stuba.sk.

paper is the study of the influence of the angular declination θ and the excentricity ε on the lowest eigencurves and to draw conclusions on possible transition from divergence mechanism to flutter mechanism of stability loss.

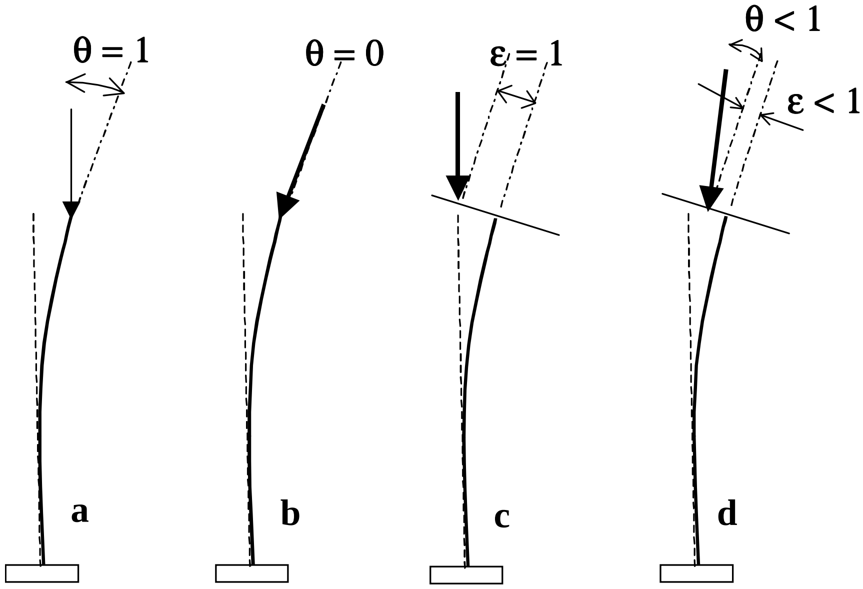

Fig. 1. Different types of behaviour of the compressive force: a – dead load, b – Beck`s column, c – Reut`s column, d – generalized Beck-Reut`s column.

2. Governing equations

We consider a straight slender elastic prismatic clamped beam loaded by a compressive force **P** of constant magnitude *P*. The directional features of the behaviour of the load vector with respect to the deflecting beam are specified via boundary condition. The governing equation for constant cross-section and for harmonic vibration in a single plain is of form

$$y^{(4)}(x) + \alpha y^{(2)}(x) = \lambda^2 y(x)\,, \tag{1}$$

where $y(x)$ is the nondimensional flexural deflection, $\alpha = Pl^2/EJ$ is constant non-dimensional magnitude of the compressive force and $\lambda^2 = \omega^2 m/EJ$ is the nondimensional frequency with ω standing for circular frequency. Boundary conditions corresponding to the generalized case according to fig. 1d are of form

$$y(0) = 0, \quad y^{(1)}(0) = 0, \quad y^{(3)}(1) + \theta\alpha y^{(1)}(1) = 0, \quad y^{(2)}(1) + \varepsilon\alpha y(1) = 0, \tag{2}$$

in which the parameter θ describes the directional behaviour of the loading vector and the parameter ε is the measure of the excentricity of the point of action of the loading vector with respect to the beam axis. The combination $\theta = 1,\ \varepsilon = 0$ corresponds to the classical dead load on fig. 1a, the Beck`s column we have for $\theta = 0,\ \varepsilon = 0$ and the Reut`s column we obtain for values $\theta = 0,\ \varepsilon = 1$. The variation of θ in interval $0 < \theta < 1$ describes the transition from follower force $(\theta = 0)$ to partial (subtangential) follower force $(0 < \theta < 1)$ to conservative dead load $(\theta = 1)$. Similar variation of the parameter ε indicates transition from centric load $(\varepsilon = 0)$ to partially centric load $0 < \varepsilon < 1$ and fully excentric load $(\varepsilon = 1)$.

The frequency equation corresponding to boundary eigenvalue problem (1), (2) is obtained after rather lenghty calculations in the form

$$\begin{aligned}&\lambda\{\alpha^2+2\lambda^2+\alpha\lambda Sin(k_1)Sinh(k_2)+2\lambda^2 Cos(k_1)Cosh(k_2)+\\&+\alpha\theta(\alpha Cos(k_1)Cosh(k_2)-2\lambda Sin(k_1)Sinh(k_2)-\alpha)\ \}+\\&\varepsilon\alpha[\alpha\lambda Cos(k_1)Cosh(k_2)+(\alpha^2+2\lambda^2)Sin(k_1)Sinh(k_2)-\alpha\lambda]+\\&\varepsilon\theta\alpha^2[2\lambda-2\lambda Cos(k_1)Cosh(k_2)-\alpha Sin(k_1)Sinh(k_2)]=0,\end{aligned} \tag{3}$$

where

$$k_1=\sqrt{\sqrt{\left(\frac{\alpha}{2}\right)^2+\lambda^2}+\frac{\alpha}{2}},\quad k_1=\sqrt{\sqrt{\left(\frac{\alpha}{2}\right)^2+\lambda^2}-\frac{\alpha}{2}}\,. \tag{4}$$

3. Numerical results

To understand the vibration and stability of the generalized Beck-Reut`s column, numerical solution of the frequency equation (3) was performed for selected sets of parameters θ and ε. Here we present the results for various combinations of parameters θ and ε as series of curves $\alpha = f(\lambda)$ only for the first two eigenfrequencies.

There are basically only to types of eigencurves:

In the first type there are two completely separated nonintersecting curves for the first and second eigenfrequency. The loss of stability is due to the mechanism of divergence and here the critical load is traced out by the point of intersection of the lower eigencurve with the vertical axis, where $\lambda = 0$. This is the highest load applicable to avoid loss of stability. Above this load the second eigencurve has only theoretical meaning unless special measures are taken to avoid the divergence.

The second type is characterised by the coalescence of the two lowest eigencurves. At the flutter point the tangent to the eigencurves is horizontal, this is the geometrical interpretation of the flutter point. Above the flutter point there are no more real eigenfrequencies, the lowest two eigenfrequencies are complex and conjugated and the loss of stability appears as vibration with increasing amplitude. The flutter point can be computed numerically from simultaneous equations

$$F(\alpha,\lambda,\theta,\varepsilon)=0,\quad G(\alpha,\lambda,\theta,\varepsilon)=\frac{\partial F(\alpha,\lambda,\theta,\varepsilon)}{\partial\lambda}=0, \tag{5}$$

where θ,ε are fixed and $F(\alpha,\lambda,\theta,\varepsilon)$ is the left hand side of the frequency equation (3). The explicit form of the function $G(\alpha,\lambda,\theta,\varepsilon)$ is not given here due to its extend, explained mainly by the formulas (4) giving the arguments of the goniometric and hyperpolic functions.

Fig. 2 presents the transition of the pairs of the lowest eigencurves from the case of dead load *(θ = 1)* to the Beck`s column *(θ = 0)* for the partially follower force applied without excentricity *(ε = 0)*. With decreasing declination θ the critical divergence load increases until the point θ = *0.5*. Here the critical divergence load suddenly jumps from the value approximately four four to the value of critical flutter load which is slightly above six. With further decrease of the parameter θ the critical flutter load is increasing until the value of *8.12* correspondig to Beck`s column is achieved.

Similar tendencies are valid for the case of follower compressive force *(θ = 0)*, which is applied excentrically, see fig. 3. Here the transition between the divergence and flutter occurs at the point ε = *0.175*. Obviously, with increasing excentricity the tendency to the loss of stability by divergence is strengthned.

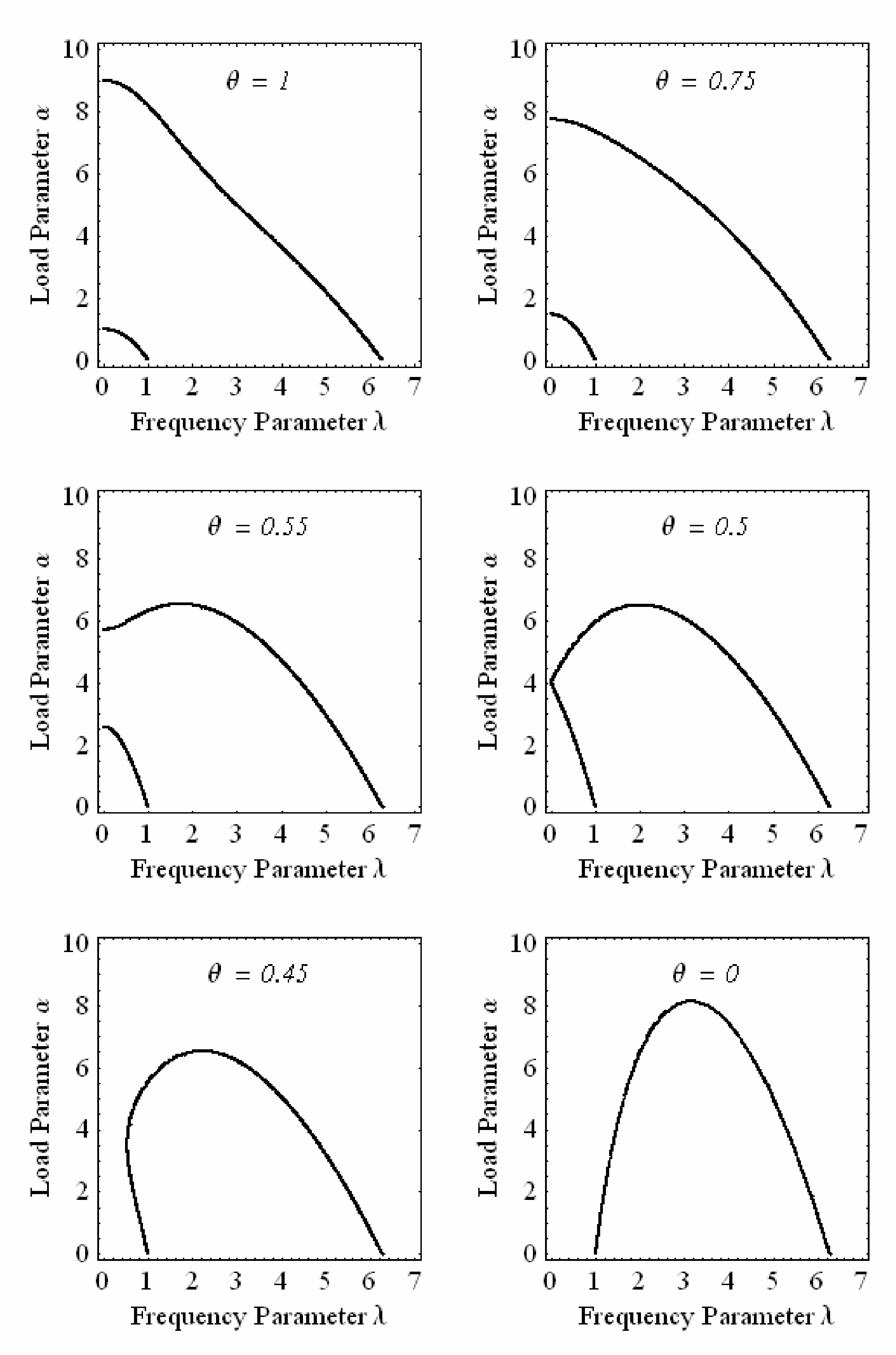

Fig. 2. Lowest eigencurves for partially follower force with no excentricity.

General insight to the qualitative nature of the mechanism of the loss of stability can be obtained from fig. 4, where the lines of transition from flutter to divergence are indicated. It is interesting, that the results are symmetric with respect to the line $\varepsilon = 1\text{-}\theta$. One of demonstrations of this symmetry is the fact, that results for the Reut`s column coincide with those for the Beck`s column. The equation $\varepsilon = 1\text{-}\theta$ is the condition for selfadjointnes of the boundary eigenvalue problem (1), (2), as derived in [1].

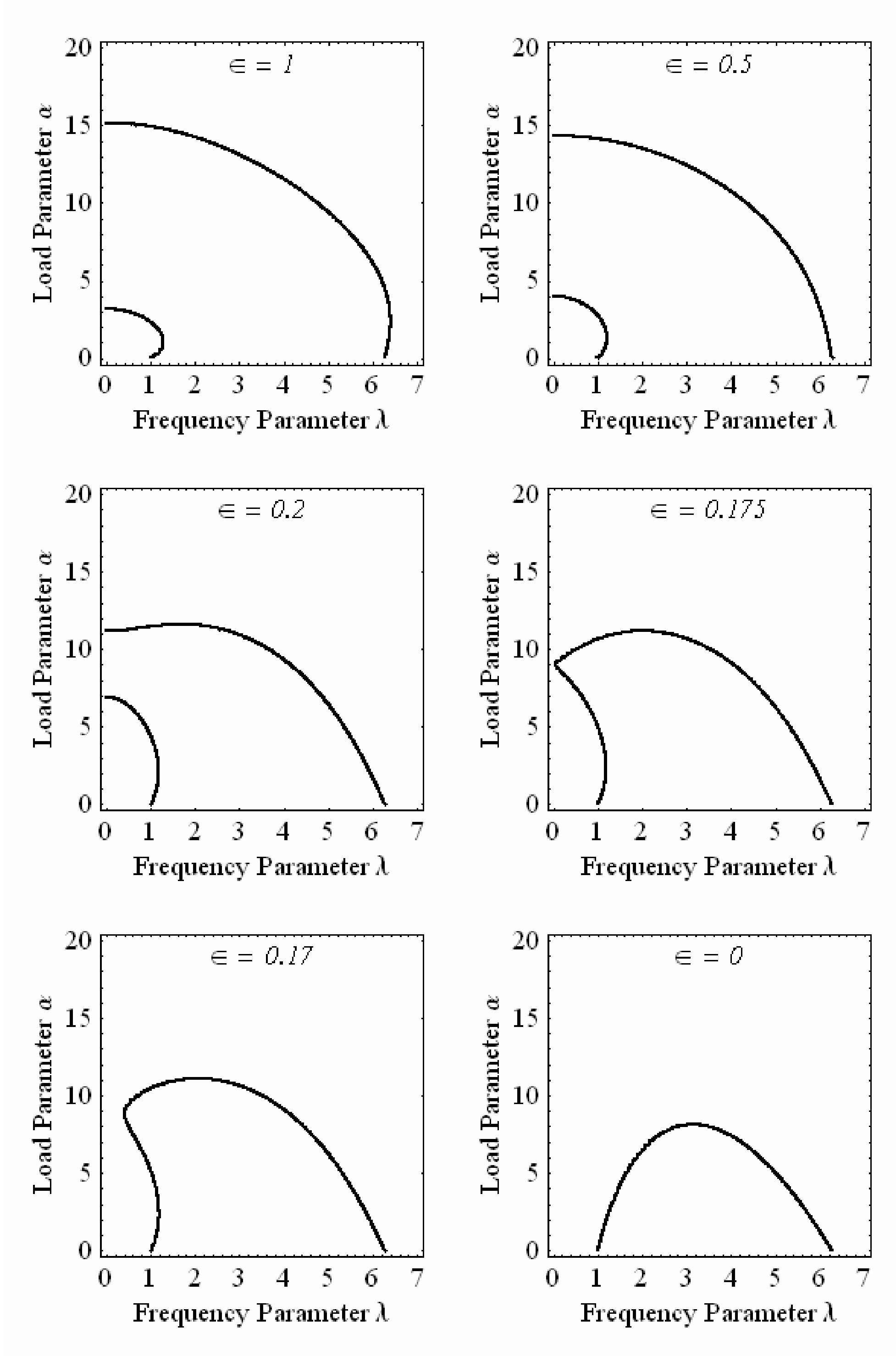

Fig. 3. Lowest eigencurves for follower force with excentricity.

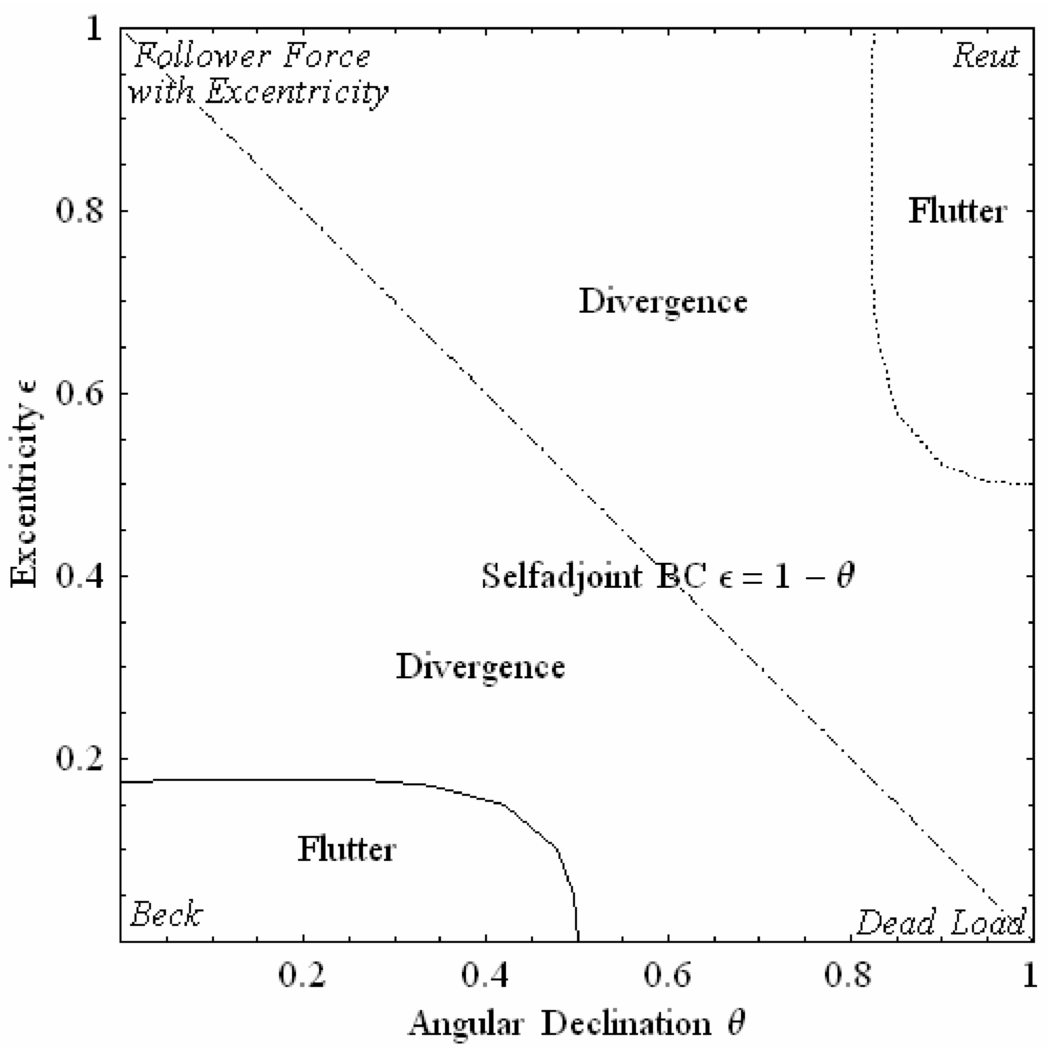

Fig. 4. Stability map for generalized Beck-Reut`s beam.

4. Conclusion

Frequency equation and flutter condition has been derived for generalized Beck-Reut`s column. Presented parametric studies allow to understand how the angular declination of the partially follower force and the excentricity influence the vibration and the stability of compressed beam.

Acknowledgements

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References

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