Antimagic labeling of Cubic circulant graphs

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1 Introduction

In 1990 Hartsfield and Ringel [3] conjectured that every connected graph is antimagic except K_2. Antimagic labeling is an injective labeling of the edges of G with the labels $1, \ldots, V(G)$. We define f on the vertex set of G by setting $f(v)$ to be the sum of the labels on edges containing v. If f is an injective function then we say that both the edge labeling and G are antimagic.

Harsfield and Ringel also proved that paths, cycles, wheels and complete graphs are antimagic. The most significant progress on this problem was made by Alon et al. [1]. They proved among others following theorems. If G has $|V(G)| \geq 4$ vertices and $\Delta(G) \geq |V(G)|-2$ then G is antimagic. They also proved that all complete partite graphs (other then K_2) are antimagic. Cranston [2] proved that every regular bipartite graph (with degree at least 2) is antimagic.

The last theorem gives us a motivation for our result. We prove that cubic circulant graphs $C_{2P}(1, P)$ are antimagic. We will use the fact that cubic circulant graphs $C_{2P}(1, P)$ are isomorphic to a Möbius ladder.

2 Cubic circulant graphs

For a sequence of positive integers $1 \leq d_1 < d_2 < \ldots d_\ell \leq \lfloor \frac{n}{2} \rfloor$, the circulant graph $G = C_n(d_1, d_2, \ldots, d_\ell)$ has a vertex set $V = \{0, 1, \ldots, n-1\}$, with two vertices x, y being adjacent iff $x \equiv (y \pm d_i) \, mod \, n$ for some i, $1 \leq i \leq \ell$.
We focus on Cubic Circulant Graphs $C_{2P}(1, P)$.

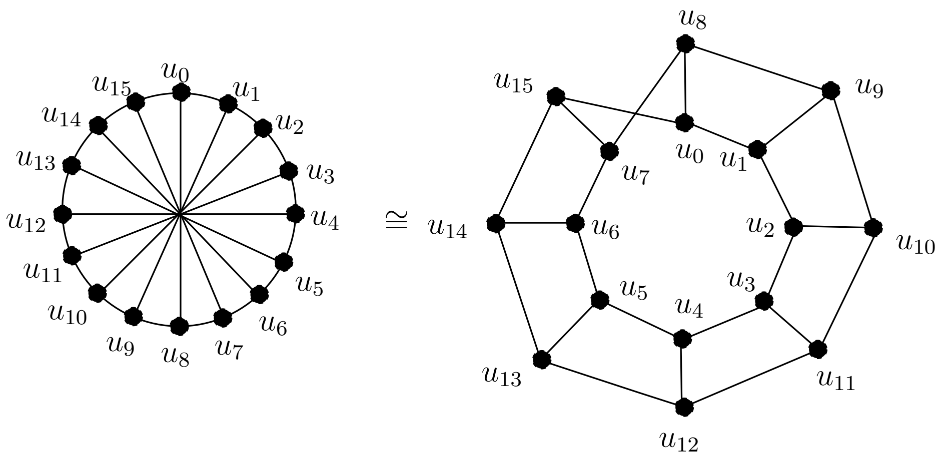

Figure 1: Cubic circulant graphs $C_{16}(1, 8)$ is ismorphic to the Möbius ladder

Our work is motivated by a known result for Regular bipartite graphs.

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Lemma 1 *[2] Every 3–regular bipartite graph is antimagic.*

We prove the following result.

Theorem 1 *Cubic circulant graphs $C_{2P}(1, P)$ are antimagic.*

We make use of the fact that in the Möbius ladder we can choose a perfect matching using strip edges. The resulting 2–factor has then one cycle.

We devide the proof of the theorem into two cases (bipartite and nonbipartite). For a bipartite case we will use the next result which is a special case of a result due to Cranston [2].

Lemma 2 *[2] Bipartite cubic circulant graphs $C_{2P}(1, P)$ are antimagic.*

For the nonbipartite case we modify the proof of Lemma 1 due to Cranston. For a nonbipartite graph we will split up this case into two subcases.

Lemma 3 *Nonbipartite cubic circulant graphs $C_{2P}(1, P)$ with P mod $4 \equiv 0$ are antimagic.*

And

Lemma 4 *Nonbipartite cubic circulant graphs $C_{2P}(1, P)$ with P mod $4 \neq 0$ are antimagic.*

Combining the above three lemmas we have proved Theorem 1.

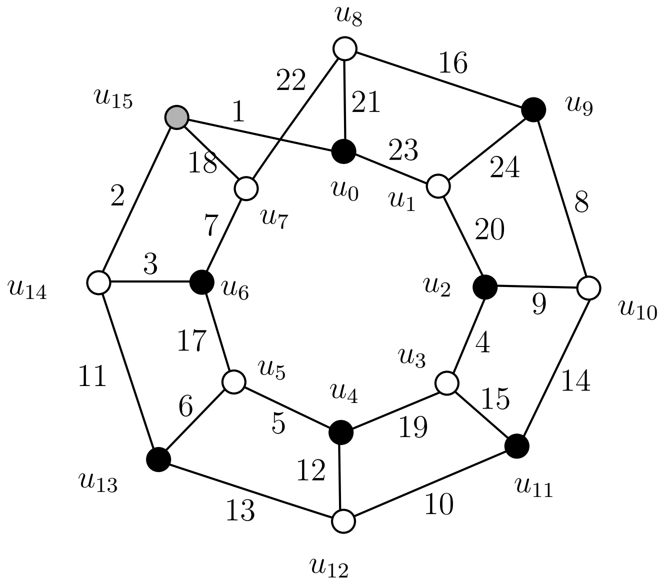

Figure 2: Antimagic labeling of Cubic circulant graph $C_{16}(1, 8)$.

References

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[2] Daniel W. Cranston. Regular bipartite graphs are antimagic. *J. Graph Theory*, 60(3):173–182, March 2009.

[3] Nora Hartsfield and Gerhard Ringel. *Pearls in graph theory - a comprehensive introduction (Reviewed Edition)*. Academic Press, 1994.