# MATHEMATICAL MODELLING OF DOUBLY-FED ASYNCHRONOUS GENERATOR

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***ABSTRACT***

*This paper deals with mathematical modelling of doubly-fed asynchronous generator for wind power plant by the space phasors method. In this paper, the basic principles and characteristics of the system are briefly described, the mathematical description of this machine is shown, the computer model in the Matlab/Simulink program is presented and its function is briefly explained. At the end, possibilities of its next improvement are also mentioned.*

***KEYWORDS***

Mathematical modelling, doubly-fed asynchronous generator, wound rotor, space phasors, Simulink

## 1. INTRODUCTION

Electrical machines form an integral and traditional part of the electrical engineering and their usage became the indispensable foundation for the electrical energy generation. The importance of this field of study is also connected with the development of wind power plants that can be equipped by various types of generator, synchronous or asynchronous, with various types of rotor, short circuit or wound, and also with various mode of connection, for constant or variable rotational speed. The connection and operation of these renewable energy sources is also connected with a number of problems that originate partly owing to changeability of natural conditions and partly owing to phenomena that are characteristic for their individual components.

To be possible to study individual phenomena that are associated with the operation of wind power plants and with influences of these sources on the operation of distribution and transmission systems, it is necessary to do all sorts of computer simulations. Therefore, it is firstly necessary to make models of all parts of the simulated system, included wind, wind turbine, drive train, generator, power electronic elements, transformer, part of the grid and measuring elements. There are several methods for the modelling of electrical machines such as a symbolic-complex method, suitable for the analysis of steady states, or a phase coordinates and a space phasors method, suitable for the analysis of various transient effects.

## 2. BASIC PRINCIPLES AND CHARACTERISTICS OF DFIG SYSTEM

One of the most used systems for wind power plants is the doubly-fed asynchronous generator with variable rotational speed, which is able to keep practically constant torque because of the possibility of rotor speed changes. It is usually the asynchronous generator, which stator is directly connected to the grid and which wound armature is supplied over partial scale frequency converter, as you can see in Figure 1. In this way, electric current with appropriate amplitude, frequency, sequential order of the phases and phase difference in the face of the stator magnetic flux is fed into the rotor winding. This double feed is useful mainly owing to the fact that the converter has to control only about 20 to 30 percent of total electric output and power losses caused by this element are much lower in comparison to systems where the converter has to control total transmitted output. The smaller frequency converter makes this concept attractive also from economical point of view. The rotor speed can be changed within the range of the 30 percent of synchronous speed and can be adapted to changes of wind turbine speed. One of the main advantages of this technology is also possibility of the regulation of reactive energy, which is ensured by current in the rotor circuit.

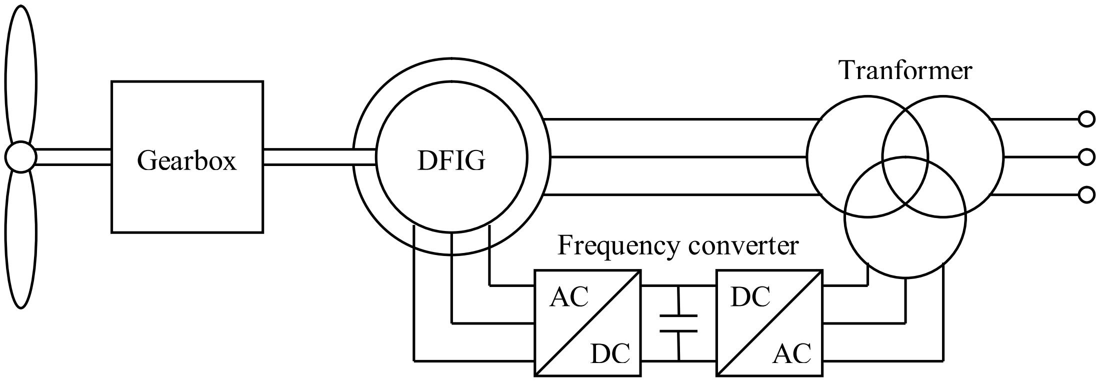


Figure 1 - Circuit diagram of doubly-fed asynchronous generator

## 3. MATHEMATICAL DESCRIPTION OF DFIG BY SPACE PHASORS

The most widely used process, in the field of the modelling of the alternating current machines, is the space phasors method, which is closely described in publication [1]. It is mainly owing to considerable simplification of differential equations of the asynchronous machine, especially in the transformation of variable parameters to constant. This possibility of transition to solution of differential equations with constant coefficients can be implemented in several ways such as Dq0 transformation or Park's transformation that are based on substitution of three-phase stator system of coordinates by other system of coordinates, two-phase and positionally bound for example with rotor.

For the basic model, it is possible to suppose some simplifications like symmetrical three-phase windings of stator and rotor, constant air gap, harmonic distribution of magnetic potential difference and induction in air gap, neglect of skin-effect in stator and rotor wires or neglect of losses in magnetic circuit. Under these conditions, the asynchronous machine is described by the non-linear system of differential equations, which becomes linear for constant angular speed of the rotor. The schematic configuration of stator and rotor windings is shown in Figure 2.

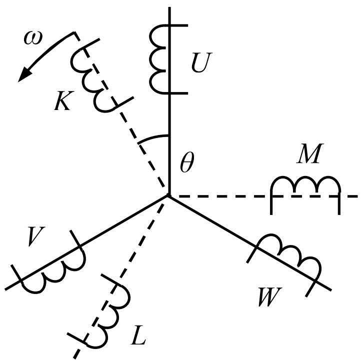


Figure 2 - Configuration of three-phase stator and rotor windings

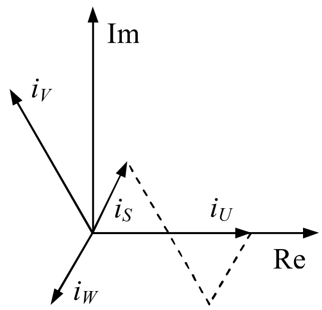


Figure 3 - Currents in three-phase stator winding in complex plane

The space phasors method results from analogy between time and space vectors. The instantaneous size of resultant space current wave in certain place on the machine circumference can be described with the projection of space phasor of this wave to the axis passing through this place. It is the same as the expression of the instantaneous size of current in certain phase by the projection of this time vector to, for example, real axis. In using space phasors, as well as in the case of time vectors, it is used the symbolic-complex method and phasors are displayed in the complex plane, which is perpendicular to the axis of rotor rotation. On the supposition that course of resultant current wave is harmonic, the space phasor of this wave is determined by geometric sum of space phasors of currents of individual phases as it is shown in Figure 3, where the situation of stator is illustrated.

For two-pole machine with double feeding, it means machine with six coils, three stator and three rotor windings, it is possible to write the next voltage equations, only for fundamental harmonic, of individual stator and rotor phases:

$$\overline{v_k} = \overline{i_k} R_k + \frac{d\overline{\Psi_k}}{dt}, \tag{01}$$

where $k$ is index of phase, $R$ is winding resistance and $\Psi$ is coupled magnetic flux. By reason of the assumption that stator and rotor windings are symmetrical, it is possible to write:

$$\begin{aligned} R_U &= R_V = R_W = R_S \\ R_K &= R_L = R_M = R_R \end{aligned}, \tag{02}$$

where $R_S$ is stator resistance and $R_R$ is rotor resistance. By reason that the air gap of the asynchronous machine is constant, it is possible to write the next:

$$\begin{aligned} L_{UU} &= L_{VV} = L_{WW} = L_{Ss} \\ L_{KK} &= L_{LL} = L_{MM} = L_{Rs} \end{aligned}, \tag{03}$$

where $L_{Ss}$ is stator self inductance and $L_{Rs}$ is rotor self inductance. For mutual inductances of stator and rotor windings that also aren't depend on position, can be written:

$$\begin{aligned} L_{UV} &= L_{UW} = L_{VW} = M_S = L_{Sm} \cdot \cos 120° = L_{Sm} \cdot \cos 240° = -0{,}5 L_{Rm} \\ L_{KL} &= L_{KM} = L_{LM} = M_R = L_{Rm} \cdot \cos 120° = L_{Sm} \cdot \cos 240° = -0{,}5 L_{Sm} \end{aligned}, \tag{04}$$

where $L_{Sm}$, or $L_{Rm}$, is mutual inductance between two stator phases, or between two rotor phases, in the same axis. Mutual inductances between stator and rotor depend on the rotor position:

$$\begin{aligned} L_{UK} &= L_{KU} = L_{VL} = L_{LV} = L_{WM} = L_{MW} = M_1 = L_h \cdot \cos\vartheta \\ L_{UL} &= L_{LU} = L_{VM} = L_{MV} = L_{WK} = L_{KW} = M_2 = L_h \cdot \cos\left(\vartheta + \frac{2}{3}\pi\right), \\ L_{KV} &= L_{VK} = L_{LW} = L_{WL} = L_{MU} = L_{UM} = M_3 = L_h \cdot \cos\left(\vartheta - \frac{2}{3}\pi\right) \end{aligned} \tag{05}$$

where $L_h$ is the maximal value of mutual inductance between stator and rotor windings. For values of coupled magnetic fluxes can be expressed by means of inductances as:

$$\begin{bmatrix} \Psi_U \\ \Psi_V \\ \Psi_W \\ \Psi_K \\ \Psi_L \\ \Psi_M \end{bmatrix} = \begin{bmatrix} L_S & M_S & M_S & M_1 & M_2 & M_3 \\ M_S & L_S & M_S & M_3 & M_1 & M_2 \\ M_S & M_S & L_S & M_2 & M_3 & M_1 \\ M_1 & M_3 & M_2 & L_R & M_R & M_R \\ M_2 & M_1 & M_3 & M_R & L_R & M_R \\ M_3 & M_2 & M_1 & M_R & M_R & L_R \end{bmatrix} \cdot \begin{bmatrix} i_U \\ i_V \\ i_W \\ i_K \\ i_L \\ i_M \end{bmatrix}, \tag{06}$$

For representation of space phasors positions in the complex plane, it is suitable to use unit vectors, which means vectors with unit length and phase differences 0, $2\pi/3$ and $4\pi/3$:

$$\bar{1} = e^{j0} = 1, \; \bar{a} = e^{j\frac{2}{3}\pi} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \text{ and } \bar{a}^{2} = e^{j\frac{4}{3}\pi} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}, \tag{07}$$

The first vector is equal to one and by this the axis of the first phase is identical with the real axis, such as in Figure 3. It is the same as with time phasors interpretation in the complex plane. The resultant space phasor is determined like the sum of individual space phasors that appertain to individual phases. Generally, it is possible to write for space phasors of current, voltage, magnetic flux or other arbitrary three-phase quantity $f$ the next mathematical equation:

$$\overline{f} = \frac{2}{3}\left(f_A \cdot \overline{1} + f_B \cdot \overline{a} + f_C \cdot \overline{a}^2\right) \tag{08}$$

Because of magnetic coupling, which is between stator and rotor windings and changes depending on the alignment of stator and rotor, it is advantageous to express all equations in uniform, stator or rotor coordinates. Coefficients in equations are constant after that and voltage equations for stator and rotor circuits, in stator coordinates, have the next form:

$$\begin{aligned} \overline{v_S^S} &= \overline{i_S^S} R_S + \frac{d\overline{\Psi_S^S}}{dt} \\ \overline{v_R^S} &= \overline{i_R^S} R_R + \frac{d\overline{\Psi_R^S}}{dt} - jp_p\omega_{gen}\overline{\Psi_R^S} \end{aligned}, \tag{09}$$

where $p_\mathrm{p}$ is a number of pole-pairs and $\omega_\mathrm{gen}$ is mechanical generator speed. To be possible to solve these equations in the field of real numbers, it must be made the decomposition of space phasors to components, to projections to real and imaginary axis of the chosen system of coordinates:

$$\begin{aligned} v_\alpha &= \frac{2}{3}\left[\overline{v_A}\cos\vartheta + \overline{v_B}\cos\left(\vartheta + \frac{2}{3}\pi\right) + \overline{v_C}\cos\left(\vartheta - \frac{2}{3}\pi\right)\right] \\ v_\beta &= \frac{2}{3}\left[\overline{v_A}\sin\vartheta + \overline{v_B}\sin\left(\vartheta + \frac{2}{3}\pi\right) + \overline{v_C}\sin\left(\vartheta - \frac{2}{3}\pi\right)\right] \end{aligned} \tag{10}$$

Owing to this, the solution of voltage equations converts from three phases to two phases, whereas transforms components form symmetrical two-phase winding, which induce the same operating wave as the original. For the reverse transformation of resultant currents can be written:

$$\begin{aligned} \overline{i_A} &= i_\alpha \cos\vartheta + i_\beta \sin\vartheta \\ \overline{i_B} &= i_\alpha \sin\left(\vartheta + \frac{2}{3}\pi\right) + i_\beta \sin\left(\vartheta + \frac{2}{3}\pi\right) \\ \overline{i_C} &= i_\alpha \sin\left(\vartheta - \frac{2}{3}\pi\right) + i_\beta \sin\left(\vartheta - \frac{2}{3}\pi\right) \end{aligned} \tag{11}$$

The next quantity, which is important for the model, is the inner electromagnetic torque, which value is can be calculated from the equation of rotor short-circuit in stator coordinates. From inner energy balance of the machine result that the torque of the generator can be expressed as:

$$T_{gen} = \frac{3}{2} p_p L_m \operatorname{Re}\left\{ j \cdot \overline{i_R^S} \cdot \overline{i_S^S}^* \right\} = \frac{3}{2} p_p L_m \left( i_{S\beta} \cdot i_{R\alpha} - i_{S\alpha} \cdot i_{R\beta} \right) \tag{12}$$

Values of coupled magnetic fluxes can be expressed by self and mutual inductances and currents. On the assumption that the sum of stator currents, or the sum of rotor currents, is zero, it is possible to write, for the basic rotor position, the next equations:

$$\begin{aligned} \overline{\Psi_S} &= (L_{Ss} + L_{Sm}) \cdot \overline{i_S} + \frac{3}{2} L_h \overline{i_R} = L_S \overline{i_S} + L_m \overline{i_R} \\ \overline{\Psi_R} &= (L_{Rs} + L_{Rm}) \cdot \overline{i_R} + \frac{3}{2} L_h \overline{i_S} = L_R \overline{i_R} + L_m \overline{i_S} \end{aligned}, \tag{13}$$

where $L_\mathrm{S}$ and $L_\mathrm{R}$ are inductances of stator and rotor that include self inductance of assigned phase and mutual inductances of other phases and $L_\mathrm{m}$ is magnetizing inductance, which represents the maximal magnetic coupling of the three-phase electric machine between stator and rotor phase.

## 4. COMPUTER MODEL OF DFIG IN MATLAB/SIMULINK PROGRAM

For the modelling of all sorts of systems in all fields of engineering, the Simulink program, which is one of tools of mathematical program Matlab and which enables to use blocks of basic mathematical functions to creation of more complicated systems, is successfully used. In this software is made the next simplified model of doubly-fed asynchronous generator, which is shown in Figure 4 and which is based on previous thinking and derived mathematical equations This model enables to watch courses and changes of stator and rotor currents and also course and changes of inner electromagnetic torque depending on courses and changes of power supply voltages of stator and rotor and also on course and changes of mechanical speed of generator.

Basic parameters of this model, whose values can be changed in the mask of block, are resistances and inductances of stator and rotor, magnetizing inductance between stator and rotor windings and also the number of pole-pairs of the machine. In the block, there is firstly made the transformation of power supply voltages from three-phase to two-phase system. After that, currents are calculated with the aid of mentioned mathematical equations and integral blocks. And finally, resultant currents are transformed from two-phase system back to original three-phase system.

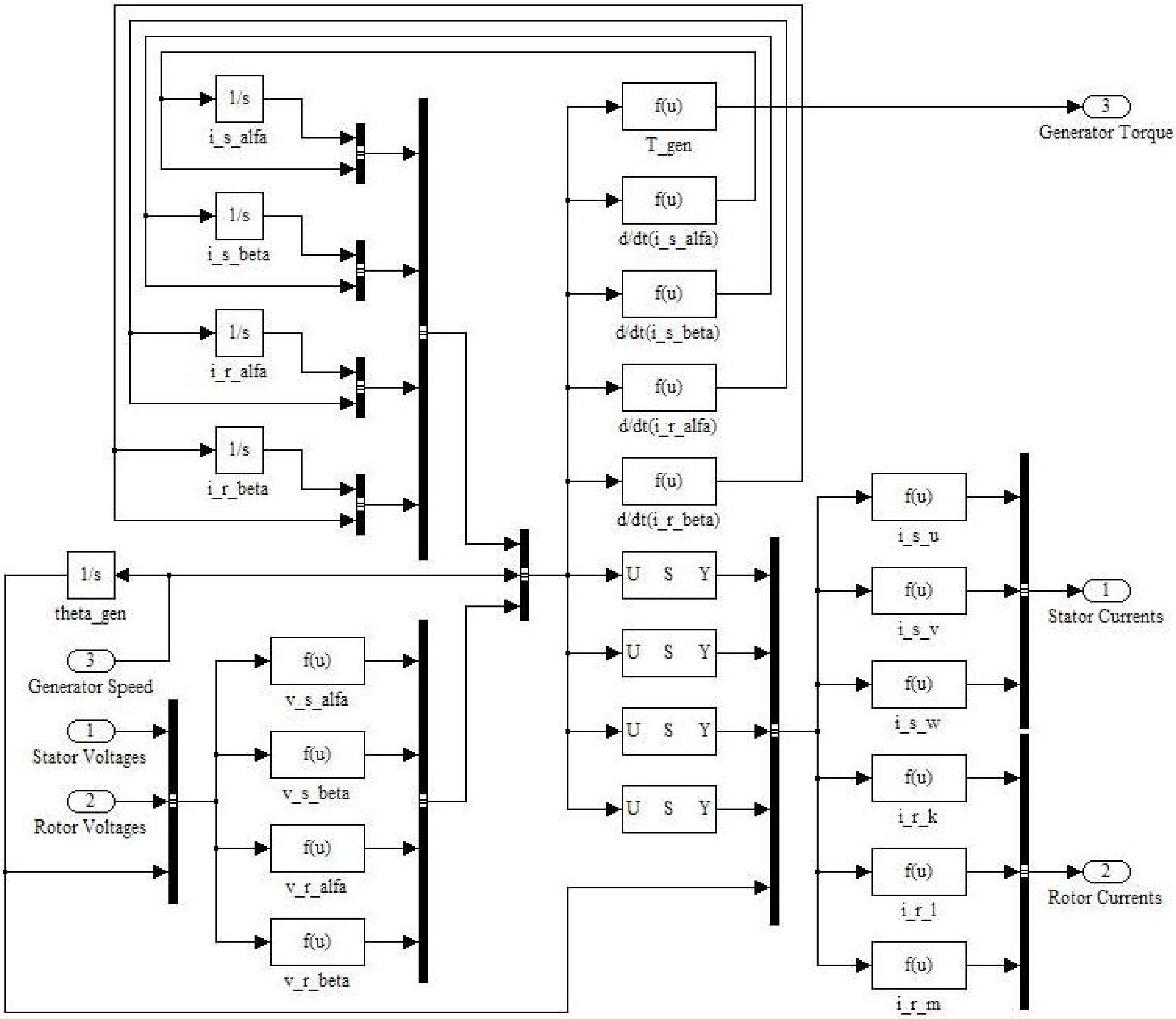


Figure 4 - Model of doubly-fed asynchronous generator

Input quantities are the above mentioned courses of phase voltages of the stator, phase voltages of the rotor and mechanical speed of the rotor. Outputs are courses of stator and rotor phase currents and electromagnetic torque on the rotor. Example of courses of individual rotor currents is illustrated in Figure 5, which shows these courses at harmonic stator and rotor feedings and constant angular speed of the rotor, close to synchronous speed. The transient effect at the beginning of the diagram is caused by the step change of the angular speed from zero to nonzero value and conjoined connection of the stator and rotor windings ot power supply at the beginning of the simulation. After this transient there are shown courses of these currents in steady state.

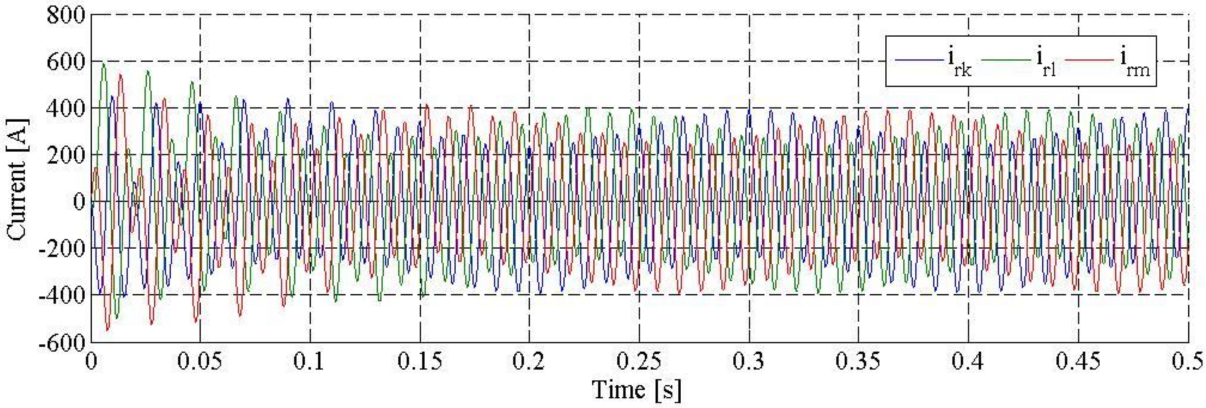


Figure 5 - Courses of individual rotor currents

## 5. CONCLUSIONS

This model is only one of the base parts of much more complicated model of whole wind power plant system, which enables to do real simulations of the operation of these units and watch phenomena that are connected with these sources of renewable energy. It is designed to be able to be interconnected with models of other system parts, such as drive train, which ensures the mechanical coupling between wind turbine and generator and transmits mechanical torques, or pitch regulation, whose input quantity is the size of the electric output supplied to the grid. However, this simplified model can be improved by the implementation of unsymmetrical windings, skin-effect in wires of stator and rotor, losses in magnetic circuit and inharmonious power supply.

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