STABILITY ALGORITHMS FOR NEWTON-RAPHSON METHOD IN LOAD FLOW ANALYSIS

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ABSTRACT

This paper deals with possible algorithms, which may ensure numerical stability of Newton-Raphson method in load flow analysis. Although the Newton-Raphson method is frequently used, it may have difficulties to obtain convergence. Oscillations, divergence or even convergence to unfeasible solutions may appear using traditional procedures. Therefore, various techniques (such as update truncation, factor relaxation, etc.) can be broadly employed to increase the reliability of the results obtained. The aim of this paper is to find the best available stability algorithm providing solutions with minimum number of iterations and lowest CPU time requirements.

KEYWORDS

Load Flow Analysis, Newton-Raphson, Update Truncation method, Factor Relaxation method, One-Shot State Variable Update.

1. INTRODUCTION

For decades, traditional numerical methods have been used for load flow analysis of electric power systems. From the historical point of view, first numerical algorithm employed was the Gauss-Seidel method successfully solving the system of nonlinear algebraic equations with complex coefficients due to fair convergence properties. Although the CPU time per iteration was relatively small, strong dependency of network size on total number of iterations excluded the Gauss-Seidel method from simulations of larger power systems.

Currently, the Newton-Raphson method is frequently used especially for large-scale strongly nonlinear problems for its quadratic rate of convergence. Although the CPU time per iteration is higher than for the Gauss-Seidel method, solutions can be mostly obtained in 2 to 7 iterations with intended accuracy and without the reference to the problem size. Unfortunately, this behaviour can be expected only when initial values are chosen near the physical solution. Otherwise, because of nonlinear nature of a problem, divergence or convergence to non-physical solutions can often appear. Beside high sensitivity on starting values, update instability also belongs to its most severe weaknesses.

In literature (e.g. in [4], [5] and [6]), the Newton-Raphson method is well described and fully understood. However, not enough space is given to handle the numerical instability. Several different approaches can be applied to improve its numerical behaviour. Possible algorithms to find suboptimal value of a relaxation factor for state variable update are introduced in [1] and [2] by Heckmann *et al.* and by Koh, Ryu and Fujiwara, respectively. Tate in [9] calls attention to fractal behaviour of the load flow analysis using the Newton-Raphson method and gives detailed advices, such as r/x ratio modifications, state update truncations and one-shot fast-decoupled method, to avoid possible divergence or convergence to non-physical load flow solutions. Several different procedures using second Taylor series expansion, Jacobian adjustments and Levenberg-Marquardt method are introduced in [3]. Schmidt in [8] offers several changes in input data files of tested power systems, which may lead to better numerical stability and convergence.

This paper is organized as follows. Section 2 describes negative numerical properties of the Newton-Raphson algorithm in more detailed way. In Section 3, each possible stability approach is fully introduced. Testing of stability algorithms is provided in Section 4 comparing the flexibility of individual procedures, reliability of obtained results, total number of iterations, total CPU time, level of robustness and complexity to the original Newton-Raphson code. In Section 5, some concluding remarks close the paper with the best-evaluated stability algorithm obtained.

2. NUMERICAL INSTABILITY OF THE NEWTON-RAPHSON METHOD

Convergence problems can appear when solving the networks, which are kept "on the edge" of their operating conditions. These networks, referred to as ill-conditioned power systems, are very sensitive to small changes of state variables (V,θ). Non-convergent behaviour is strongly connected to the reactive power problem of examined power systems. Excessively high loadings, moving final voltage values far away from the initial guess, can cause numerical divergence. Often used PQ loads, i.e. constant voltage-independent loads, have only approximate values, which are higher than real demands in the system. When reaching critical loading values, the network is close to the voltage collapse and cannot be solved due to singularity of the Jacobian matrix.

Sparse network topology (e.g. radial systems) and small number of interconnections between individual areas of the system can also cause rather poor convergence due to insufficient reactive power distribution in the network. Too narrowed operation limits, such as var constraints for PV buses and ranges of tap-changing transformers, along with incorrectly chosen slack bus, negative line reactances (i.e. series capacitors) and long lines are the most common reasons of the numerical instability. Especially, lightly loaded long lines may consume significant volume of transmitted reactive power.

To avoid these problems, it is recommended to primarily switch PQ-loads to Z-loads, replace PV-generators by PQ-generators (with relaxed var limits), disconnect long lines, add additional shunt compensators, release tap ratio limits of LTC transformers, change the slack bus position and divide the network into smaller areas which can be solved separately.

In the industry, several modifications of the network topology are performed during the day. Due to above mentioned factors, the Newton-Raphson method may not be able to perform the load flow analysis of such networks previously successfully solved. Therefore, robust stability algorithms are essential for simulations of majority of load flow cases by the Newton-Raphson method.

3. STABILITY ALGORITHMS FOR NEWTON-RAPHSON METHOD

3.1. Start Point Estimation Methods

High sensitivity of the Newton-Raphson method on starting values has been already introduced in Section 2. In load flow analysis, usually chosen voltage start (flat) point is 1.0 for all PQ buses with zero angles ensuring the numerical stability for majority of well-conditioned load flow cases.

Probably the most powerful approaches for better estimations of initial values are One-Shot Fast-Decoupled and One-Shot Gauss-Seidel methods. These procedures, often located in pre-processing part of the Newton-Raphson method, improve the start points to better values which may be closer to the final solution. Unfortunately due to their linear simplifications, networks with high R/X ratios can significantly influence the convergence. Possible approach lies in the use of special technique presented in [9] to slightly amend high R/X rates to avoid divergence. Currently, the One-Iteration Fast-Decoupled algorithm is also used in commercial software package PowerWorld Simulator [7].

3.2. State Update Methods

Serious difficulties may also appear when updating state variables V and θ in the Newton-Raphson code. Therefore, the following approaches can be broadly used to avoid non-convergent scenarios:

I. <u>Power mismatch minimization using relaxation factor α</u>

This method, similarly as in [1] and [2], offers better values of α for updating voltage magnitudes and angles (Eqn. 1). In some cases, usually used unity value of α may either increase the number of iterations for the convergence or even cause divergence behaviour.

$$\begin{aligned} \theta_i^{(p+1)} &= \theta_i^{(p)} + \alpha\,\Delta\theta_i \\ V_i^{(p+1)} &= V_i^{(p)} + \alpha\,\Delta V_i \end{aligned} \tag{1}$$

To find optimal factor value, mean mismatches (Eqn. 2) from actual (α = 1) and previous (α = 0) iteration have to be compared.

$$M_\alpha = \sqrt{\frac{1}{n-1}\sum_{\substack{i=1 \\ i\neq ref}}^{n}\left(\Delta P_i^2 + \Delta Q_i^2\right)} \tag{2}$$

Quadratic relation between mismatches M and relaxation factors α is employed to find optimal value of relaxation factor. When M_1 is smaller than a half of M_0, relaxation factor value is unity. Otherwise, new α is calculated and used for the updating process.

Possible convergence improvements can be achieved for majority of networks. In ill-conditioned load flow cases, value of α can be pushed close to zero producing only an approximate solution, which may be far from the physical one. Nevertheless, these solutions can be also applied as new start values for following simulations of power systems.

II. Jacobian modifications

In order to reduce nonlinear dependence of the Jacobian, the rows related to reactive power mismatches can be divided by particular bus voltage magnitudes. This approach, recommended by [5] and [9], is broadly applied in professional load flow programs.

Another approach, presented in [3], includes also the second order Taylor series expansion of objective function *F(x)* for numerically more reliable correction vectors $\Delta\theta$ and ΔV. Unfortunately, updating of Hessian matrix is relatively time demanding, especially with no sparsity techniques employed.

III. State update truncation approaches

Possible technique to avoid convergence problems is to set upper limits for correction vector values. It is obvious, that if the corrections are too great, convergence to non-physical solution or even divergence can occur. Probably, when using continuous approach limiting correction vectors according to their actual values, as suggested in [9], better numerical performance can be observed. Suggested procedure applies state update truncation function (defined by Eqn. 3) to obtain new (truncated) correction vectors. Relation between computed Δx and truncated Δx_T values is as follows.

$$\Delta x_T = \begin{cases} \Delta x & if\,|\Delta x| < DXT \\ 2\operatorname{sgn}(\Delta x)DXT - \dfrac{DXT^2}{\Delta x} & if\,|\Delta x| \geq DXT \end{cases} \tag{3}$$

Due to my personal experience, values 0.2 and 0.3 have been selected for correction vectors ΔV and Δθ in the testing part of this paper, respectively.

4. NUMERICAL RESULTS

Following stability algorithms have been investigated and compared to the original Newton-Raphson code (marked as No. 1). No. 2 contains the truncation algorithm with modified DXT limit values. One-Shot Fast-Decoupled procedure with V updates only is included in No. 3. One-Shot Fast-Decoupled and One-Shot Gauss-Seidel techniques (V updates only) combined with the truncation method are implemented in No. 4 and No. 5, respectively. Power mismatch minimization approach with α relaxation factors is realized in No. 6.

For evaluation of presented stability algorithms, following networks with convergence difficulties have been prepared (see Tab. 1). For transparency reasons, they are labelled with capital letters A – F.

Table 1 – Overview of analyzed test power systems

Networks	Description
A	Reduced Mato Grosso System (11-bus)
B	Simplified Czech Transmission System (56-bus)
C	Simplified 14-Gen SE Australian System (59-bus)
D	South England Power System (61-bus)
E	Power System of Iowa (145-bus)
F	Simplified Scottish System (629-bus)

Main emphasis of the evaluation process is placed on degree of numerical stability obtained, reduction of total number of iterations, CPU time minimization and computational burden of individual stability techniques. For each load flow case and numerical approach applied, total number of iterations and total CPU time are presented in Tab. 2 and Tab. 3, respectively.

Table 2 – Total number of iterations for individual load flow cases

		Examined stability algorithms				
Networks	No. 1.	No. 2	No. 3	No. 4	No. 5	No. 6
A	5*	6	5	7	8	7*
B	11	6	5	5	5	6
C	50**	30*	18*	9	6	8
D	50**	50**	5	5	5	5
E	50**	8	50**	8	8	10
F	8*	7	4	4	7	10

* convergence to non-physical solutions ** divergence

Table 3 – Total CPU time per iteration [in ms]

		Examined stability algorithms		
Networks	No. 1.	No. 4	No. 5	No. 6
A	14.6	13.4	11.7	19.4
B	71.5	89.5	86.4	95.4
C	73.2	84.5	90.4	102.9
D	75.1	101.1	98.9	118.7
E	425.2	480.5	472.6	592.2
F	9791.6	11653.6	10114.6	12753.1

Note: No sparsity techniques applied.

It can be seen from results above, that the most reliable stability algorithms are procedures No. 4 and 5. Subsequent testings of various 42 real test power systems have been accomplished preferring approach No. 4 for smallest number of iterations needed. Algorithm No. 6 has been evaluated as relatively reliable. Unfortunately, in some cases it can converge to non-physical solutions and increase total number of iterations.

The lowest CPU requirements have been observed in approach No. 5 (+11.8%) when compared to approaches No. 4 and No. 6 with increase of 16.5% and 39.0%, respectively.

5. CONCLUSIONS

In this paper, introductory analysis of different stability algorithms for Newton-Raphson method has been performed. Finally, the One-Shot Fast-Decoupled and One-Shot Gauss-Seidel approaches with V updates only in combination with the truncation algorithm have been found the most reliable in terms of convergence to physical solutions, total number of iterations, CPU time and modification level of the original Newton-Raphson code.

However, the R/X modifications and Jacobian adjustments (e.g. Hessian inclusion) have not been considered. Therefore, future research of this problem is intended to find more reliable techniques, which would show better convergence properties also for ill-conditioned load flow cases.

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