

MULTI-CRITERIA GROUP DECISION-MAKING USING AN EXTENDED EDAS METHOD WITH INTERVAL TYPE-2 FUZZY SETS

*Mehdi Keshavarz Ghorabae, Maghsoud Amiri,
Edmundas Kazimieras Zavadskas, Zenonas Turskis*

Introduction

Multi-criteria decision-making (MCDM), which is a sub-discipline of operations research (OR), includes many methods and problems, which are related to many fields such as economy, engineering, military and management (Zavadskas et al., 2014; Lazauskas et al., 2015; Tošenovský, 2015). There are different classifications of MCDM problems and methods. We can classify them as multi-criteria evaluation problems and multi-criteria design problems (Chakraborty et al., 2015; Triantaphyllou, 2013). The first class is also known as the multi-attribute decision-making and consists of a finite number of alternatives known in the beginning of the decision-making process, and each alternative is represented by its performance in multiple criteria (Mardani et al., 2016; Faraji Sabokbar et al., 2016). The second class is also known as the multi-objective decision-making, and an alternative (solution) can be found by solving a mathematical model (Yu et al., 2016; Mahdiraji et al., 2016). This study focuses on the first class of MCDM problems.

Because of the characteristics of the decision-making problems, the uncertainty of information is usually inevitable in the MCDM problems. Fuzzy set theory is an efficient tool for modeling the human knowledge and evaluations which are used in the decision-making process (Stanujkic et al., 2015). To handle the uncertainty of multi-criteria decision-making problems, the weights of criteria and the performance of alternatives are usually characterized by type-1 fuzzy sets that proposed by Zadeh (1965). Many researchers have studied the MCDM methods and problems in the type-1 fuzzy environment (Ecer, 2015; Karabasevic et al., 2016; Aliakbari Nouri et al., 2015; Li et al., 2015; Razavi Hajiagha et al., 2015). Mardani et al. (2015) reviewed the MCDM methods and their applications in fuzzy environment.

Type-1 fuzzy sets are efficient in modeling the multi-criteria decision-making problems and have many applications for extending MCDM methods in an uncertain environment. However, we may confront with situations that more degrees of flexibility are needed to deal with the decision-making process. Type-2 fuzzy sets (T2FSs), which was introduced by Zadeh (1975), are more flexible than type-1 fuzzy sets in the modeling of uncertainty. Interval type-2 fuzzy sets (IT2FSs) are a special type of T2FSs. Some basic definitions of IT2FSs were proposed by Mendel et al. (2006). IT2FSs have increasingly been considered by researchers in applications and extensions of multi-criteria decision-making methods. Chen and Lee (2010) developed a new ranking method for interval type-2 fuzzy sets and used it in a new fuzzy MCDM method. Chen et al. (2012) proposed a new ranking method and a new multi-criteria decision-making method with interval type-2 fuzzy sets. Wang et al. (2012) introduced a multi-criteria group decision-making (MCGDM) method in type-2 fuzzy environment, which can be used with incomplete information about criteria weights. Hu et al. (2013) developed a new ranking method based on the possibility degree for IT2FSs and applied it in multi-criteria decision-making process. Keshavarz Ghorabae et al. (2014) presented a new fuzzy ranking method and extended COPRAS (Complex Proportional Assessment) method in the context of IT2FSs to evaluate suppliers in a supply chain. Celik et al. (2014) proposed an interval type-2 fuzzy MCDM method to identify and evaluate critical success factors for humanitarian relief logistics management. Balin and Baraqli (2015) developed a fuzzy MCDM methodology based on the IT2FSs for evaluating renewable energy alternatives in Turkey. Chen (2015) proposed a new likelihood-based interval type-2 fuzzy

MCDM method using the concepts of likelihood-based performance indices, likelihood-based comprehensive evaluation values, and signed distance-based evaluation values. Sang and Liu (2016) presented ranking method for IT2FSs and extended an IT2FSs-based TODIM method for green supplier selection in automobile manufacturers. Keshavarz Ghorabae et al. (2016b) extended the WASPAS method with IT2FSs and applied it to evaluation of green suppliers. Celik et al. (2015) performed a comprehensive review of MCDM methods with interval type-2 fuzzy sets.

Keshavarz Ghorabae et al. (2015b) introduced the evaluation based on distance from average solution (EDAS) method. This method was also extended for decision-making in fuzzy environment and was applied to supplier selection problem (Keshavarz Ghorabae et al., 2016c). However, the previous versions of this method are not appropriate to deal with MCGDM problems with IT2FSs. In this study, we propose a new extended EDAS with interval type-2 fuzzy sets (EDAS-IT2FSs). A numerical example is employed to illustrate the process and show the effectiveness of the proposed method. A comparison and a sensitivity analysis are also performed to represent the validity and stability of the ranking result. The results of these analyses show that the proposed extended EDAS method is stable in different weights of criteria and well-consistent with some existing methods.

The rest of this paper is organized as follows. In Section 1, some basic concepts and arithmetic operation of T2FS are summarized. In Section 2, EDAS-IT2FSs is presented to deal with MCGDM with interval type-2 fuzzy sets. In Section 3, we use a numerical example to illustrate the procedure of using the EDAS-IT2FSs method. In Section 4, a sensitivity analysis is performed to show the validity and stability of the results of the proposed method. Finally, conclusions are discussed.

1. Concepts and Arithmetic Operations

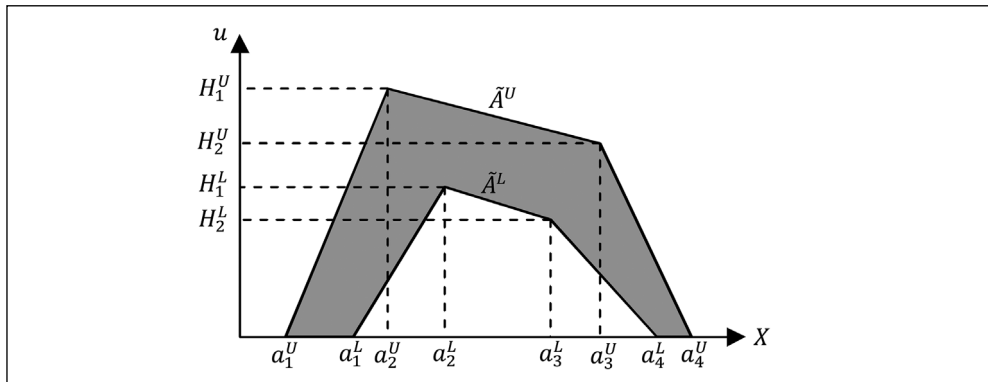
Type-2 fuzzy sets are one of the main extensions of the type-1 fuzzy sets. T2FSs are represented by primary and secondary membership values. These types of fuzzy sets could be very useful in many fields of sciences, especially decision-making theory. In this section, the basic concepts and arithmetic operations of this type of fuzzy sets are defined.

Definition 1. A T2FS \tilde{A} is described by a type-2 membership function, expressed as follows (Mendel et al., 2006):

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x, u) / (x, u) \tag{1}$$

where X denotes the domain of \tilde{A} , $\mu_{\tilde{A}}$ refers to the membership function (secondary membership function) of \tilde{A} , $J_x \subseteq [0, 1]$ denotes the primary membership function and \int denotes the union over all admissible x and u . For a T2FS \tilde{A} , if all $\mu_{\tilde{A}}(x, u) = 1$, then \tilde{A} is called interval type-2 fuzzy set.

Fig. 1: An example of a trapezoidal IT2FS



Source: Chen and Lee (2010)

Definition 2. An IT2FS is called trapezoidal IT2FS if and only if the UMF (Upper Membership Function) and the LMF (Lower Membership Function) are both trapezoidal fuzzy sets. Let \tilde{A} be a trapezoidal IT2FS. \tilde{A} can be expressed as follows (Chen and Lee, 2010):

$$\tilde{A} = (\tilde{A}^U | T \in \{U, L\}) = (a_i^T; H_{1A}^T; H_{2A}^T | T \in \{U, L\}, i = 1, 2, 3, 4) \quad (2)$$

where \tilde{A}^U and \tilde{A}^L denote the UMF and LMF of \tilde{A} , respectively, and $H_j^U \in [0, 1]$ and $H_j^L \in [0, 1]$ ($j = 1, 2$) denote the membership values of the corresponding elements a_{j+1}^U and a_{j+1}^L , respectively. An example of a trapezoidal IT2FS is shown in Fig. 1.

Definition 3. Suppose that \tilde{A} and \tilde{B} are two trapezoidal IT2FSs and d is a crisp number where,

$$\tilde{A} = (\tilde{A}^T | T \in \{U, L\}) = (a_i^T; H_{1A}^T, H_{2A}^T | T \in \{U, L\}, i = 1, 2, 3, 4)$$

$$\tilde{B} = (\tilde{B}^T | T \in \{U, L\}) = (b_i^T; H_{1B}^T, H_{2B}^T | T \in \{U, L\}, i = 1, 2, 3, 4)$$

Then the arithmetic operations of IT2FSs are defined as follows (Keshavarz Ghorabae et al., 2015a; Keshavarz Ghorabae et al., 2016a):

- Addition:

$$\tilde{A} \oplus \tilde{B} = (a_i^T + b_i^T; \min(H_{1A}^T, H_{1B}^T), \min(H_{2A}^T, H_{2B}^T) | T \in \{U, L\}, i = 1, 2, 3, 4) \quad (3)$$

$$\tilde{A} + d = (a_i^T + d; H_{1A}^T, H_{2A}^T | T \in \{U, L\}, i = 1, 2, 3, 4) \quad (4)$$

- Subtraction:

$$\tilde{A} \ominus \tilde{B} = (a_i^T - b_{5-i}^T; \min(H_{1A}^T, H_{1B}^T), \min(H_{2A}^T, H_{2B}^T) | T \in \{U, L\}, i = 1, 2, 3, 4) \quad (5)$$

- Multiplication:

$$\tilde{A} \otimes \tilde{B} = (X_i^T; \min(H_{1A}^T, H_{1B}^T), \min(H_{2A}^T, H_{2B}^T) | T \in \{U, L\}, i = 1, 2, 3, 4) \quad (6)$$

$$X_i^T = \begin{cases} \min(a_i^T b_i^T, a_i^T b_{5-i}^T, a_{5-i}^T b_i^T, a_{5-i}^T b_{5-i}^T) & \text{if } i = 1, 2 \\ \max(a_i^T b_i^T, a_i^T b_{5-i}^T, a_{5-i}^T b_i^T, a_{5-i}^T b_{5-i}^T) & \text{if } i = 3, 4 \end{cases} \quad (7)$$

$$d \cdot \tilde{A} = \begin{cases} (d \cdot a_i^T; H_{1A}^T, H_{2A}^T | T \in \{U, L\}, i = 1, 2, 3, 4) & \text{if } d \geq 0 \\ (d \cdot a_{5-i}^T; H_{1A}^T, H_{2A}^T | T \in \{U, L\}, i = 1, 2, 3, 4) & \text{if } d \leq 0 \end{cases} \quad (8)$$

Definition 4. The crisp score of a trapezoidal IT2FS is defined as follows (Keshavarz Ghorabae et al., 2015a):

$$\sigma(\tilde{A}) = \frac{1}{2} \left(\sum_{T \in \{U, L\}} \frac{a_1^T + (1 + H_{1A}^T)a_2^T + (1 + H_{2A}^T)a_3^T + a_4^T}{4 + H_{1A}^T + H_{2A}^T} \right) \quad (9)$$

Definition 5. A function is defined in the following to find the maximum between a trapezoidal interval type-2 fuzzy set fuzzy number and zero.

$$Z(\tilde{A}) = \begin{cases} \tilde{A} & \text{if } \sigma(\tilde{A}) > 0 \\ \tilde{0} & \text{if } \sigma(\tilde{A}) \leq 0 \end{cases} \quad (10)$$

where $\tilde{0} = ((0, 0, 0, 0; 1, 1), (0, 0, 0, 0; 1, 1))$.

2. EDAS-IT2FSs Method

The EDAS method was developed by Keshavarz Ghorabae et al. (2015b). By comparing this method with some existing MCDM method, it was demonstrated that the EDAS method is efficient to handle decision-making problems with multiple criteria. In this section, an extended version of the EDAS method is proposed to deal with MCGDM problems in the interval type-2 fuzzy environment. The concepts and arithmetic operations of the IT2FSs, which has been presented in Section 1, are utilized for extending the EDAS method. The current study is focused on a situation that all evaluations of decision-makers are subjective. However, the proposed method can be used in a situation with both subjective and objective evaluations. Suppose that we have a set of n alternatives ($\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_n\}$), a set of m criteria ($\mathcal{C} = \{\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_m\}$) and k decision-makers ($\mathcal{D} = \{\mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_k\}$). The steps of EDAS-IT2FSs method are presented as follows:

Step 1. Construct the average decision matrix (X), shown as follows:

$$X = [\tilde{x}_{ij}]_{n \times m} \quad (11)$$

where,

$$\tilde{x}_{ij} = \frac{1}{k} \bigoplus_{p=1}^k \tilde{x}_{ij}^p \quad (12)$$

and \tilde{x}_{ij}^p denotes the performance value of alternative \mathcal{A}_i ($1 \leq i \leq n$) with respect to criterion \mathcal{C}_j ($1 \leq j \leq m$) assigned by the p th decision-maker ($1 \leq p \leq k$).

Step 2. Construct the matrix of criteria weights, shown as follows:

$$W = [\tilde{w}_j]_{1 \times m} \tag{13}$$

where,

$$\tilde{w}_j = \frac{1}{k} \bigoplus_{p=1}^k \tilde{w}_j^p \tag{14}$$

and \tilde{w}_j^p denotes the weight of criterion C_j ($1 \leq j \leq m$) assigned by the p th decision-maker ($1 \leq p \leq k$).

Step 3. Determine the matrix of average solutions, shown as follows:

$$AV = [\tilde{M}_j]_{1 \times m} \tag{15}$$

where,

$$\tilde{M}_j = \frac{1}{n} \bigoplus_{i=1}^n \tilde{x}_{ij} \tag{16}$$

The elements of this matrix (\tilde{M}_j) represents the average solutions with respect to each criterion. Therefore, the dimension of the matrix is equal to the dimension of criteria weights matrix.

Step 4. Suppose that B is the set of beneficial criteria and N is the set of non-beneficial criteria. In this step the matrices of positive distance from average (PDA) and negative distance from average (NDA) are calculated according to the type of criteria (beneficial and non-beneficial), shown as follows:

$$PDA = [\tilde{p}_{ij}]_{n \times m} \tag{17}$$

$$NDA = [\tilde{n}_{ij}]_{n \times m} \tag{18}$$

$$\tilde{p}_{ij} = \begin{cases} \frac{Z(\tilde{x}_{ij} \ominus \tilde{M}_j)}{\ominus(\tilde{M}_j)} & \text{if } j \in B \\ \frac{Z(\tilde{M}_j \ominus \tilde{x}_{ij})}{\ominus(\tilde{M}_j)} & \text{if } j \in N \end{cases} \tag{19}$$

$$\tilde{n}_{ij} = \begin{cases} \frac{Z(\tilde{M}_j \ominus \tilde{x}_{ij})}{\ominus(\tilde{M}_j)} & \text{if } j \in B \\ \frac{Z(\tilde{x}_{ij} \ominus \tilde{M}_j)}{\ominus(\tilde{M}_j)} & \text{if } j \in N \end{cases} \tag{20}$$

where \tilde{p}_{ij} and \tilde{n}_{ij} denote the positive and negative distance of performance value of i th

alternative from the average solution in terms of j th criterion, respectively.

Step 5. Calculate the weighted sum of positive and negative distances for all alternatives, shown as follows:

$$\tilde{s}\tilde{p}_i = \bigoplus_{j=1}^m (\tilde{w}_j \otimes \tilde{p}_{ij}) \tag{21}$$

$$\tilde{s}\tilde{n}_i = \bigoplus_{j=1}^m (\tilde{w}_j \otimes \tilde{n}_{ij}) \tag{22}$$

Step 6. The normalized values of $\tilde{s}\tilde{p}_i$ and $\tilde{s}\tilde{n}_i$ for all alternatives are calculated as follows:

$$\tilde{n}\tilde{p}_i = \frac{\tilde{s}\tilde{p}_i}{\max_i(\tilde{s}\tilde{p}_i)} \tag{23}$$

$$\tilde{n}\tilde{n}_i = 1 - \frac{\tilde{s}\tilde{n}_i}{\max_i(\tilde{s}\tilde{n}_i)} \tag{24}$$

Step 7. Calculate the appraisal score (\tilde{h}_i) for all alternatives, shown as follows:

$$\tilde{h}_i = \frac{1}{2} (\tilde{n}\tilde{p}_i \oplus \tilde{n}\tilde{n}_i) \tag{25}$$

Step 8. Rank the alternatives according to the decreasing ranking values of appraisal scores (\mathcal{RV}). In other words, the alternative with the highest appraisal score is the best choice among the candidate alternatives. It should be noted that the method proposed by Keshavarz Ghorabae et al. (2014) is used in this step for computing the ranking value of trapezoidal IT2FSs.

3. Numerical Example

In this section, the application of the EDAS-IT2FSs method in an MCGDM problem is represented. For this aim, we use an example of multi-criteria evaluation of subcontractors in the construction industry. Suppose that a company, which is the main contractor involved in a construction project, intends to subcontract some parts of the project to a qualified subcontractor. The board of directors of the company performed an initial screening, and eight alternatives (\mathcal{A}_1 to \mathcal{A}_8) remained for further appraisal. A group of three decision-makers (\mathcal{D}_1 , \mathcal{D}_2 and \mathcal{D}_3) from the experts of the

Tab. 1: Linguistic terms and their corresponding interval type-2 fuzzy sets

Linguistic terms	Interval type-2 fuzzy sets
Very low (VL)	((0,0,0,0.1;1,1),(0,0,0,0.05;0.9,0.9))
Low (L)	((0,0,0.1,0.15,0.3;1,1),(0.05,0.1,0.15,0.2;0.9,0.9))
Medium low(ML)	((0.1,0.3,0.35,0.5;1,1),(0.2,0.3,0.35,0.4;0.9,0.9))
Medium (M)	((0.3,0.5,0.55,0.7;1,1),(0.4,0.5,0.55,0.6;0.9,0.9))
Medium high (MH)	((0.5,0.7,0.75,0.9;1,1),(0.6,0.7,0.75,0.8;0.9,0.9))
High (H)	((0.7,0.85,0.9,1;1,1),(0.8,0.85,0.9,0.95;0.9,0.9))
Very high (VH)	((0.9,1,1,1;1,1),(0.95,1,1,1;0.9,0.9))

Source: Keshavarz Ghorabae et al. (2014)

Tab. 2: Performance values of alternatives

DMs	Alternatives	Criteria						
		C_1	C_2	C_3	C_4	C_5	C_6	C_7
D_1	A_1	M	ML	M	M	MH	ML	VH
	A_2	L	MH	MH	VH	VH	H	M
	A_3	VL	VH	MH	MH	H	H	M
	A_4	H	VH	VH	L	H	MH	VL
	A_5	VH	M	VH	L	MH	M	ML
	A_6	VH	H	MH	L	VH	H	VL
	A_7	M	MH	VH	H	ML	MH	MH
	A_8	MH	L	ML	MH	VL	M	VH
D_2	A_1	M	M	ML	M	MH	L	H
	A_2	ML	H	MH	H	H	MH	ML
	A_3	L	H	M	MH	MH	MH	M
	A_4	H	VH	H	L	H	M	L
	A_5	H	MH	VH	VL	M	M	L
	A_6	VH	H	H	ML	VH	VH	VL
	A_7	MH	MH	H	MH	M	M	M
	A_8	H	ML	L	MH	ML	ML	MH
D_3	A_1	MH	ML	ML	ML	H	M	H
	A_2	ML	MH	M	MH	VH	H	MH
	A_3	ML	MH	M	M	MH	MH	M
	A_4	MH	H	H	L	H	H	ML
	A_5	H	MH	H	VL	H	MH	M
	A_6	VH	H	H	VL	VH	VH	VL
	A_7	M	M	MH	M	M	H	MH
	A_8	M	VL	L	M	L	MH	VH

Source: own

company was formed for making a decision on these alternatives. These experts chose seven criteria (C_1 to C_7) to appraise alternatives. The list of the selected criteria is represented as follows (Vahdani et al., 2013; Plebankiewicz, 2012; Abbasianjahromi et al., 2013):

- Management capability (C_1);
- Technical capability (C_2);
- Experience level (C_3);
- Tender price (C_4);
- Safety and quality (C_5);
- Financial strength and stability (C_6);
- Completion time (C_7).

The tender price (C_4) and completion time (C_7) are non-beneficial criteria, and the other criteria are beneficial. To assess the importance of the criteria and appraise the performance values of alternatives with respect to each criterion, the decision-makers use the linguistic terms shown in Tab. 1 (Keshavarz Ghorabae

et al., 2014). The corresponding interval type-2 fuzzy sets of these linguistic terms are also defined in Tab. 1. The performance values of the alternatives on each criterion given by the decision-makers are presented in Tab. 2. The importance weights of the criteria determined by these decision-makers are represented in Tab. 3. The process of using the extended EDAS method with IT2FSs is illustrated as follows:

Step 1. The average decision matrix X can be calculated based on the Tab. 1, Tab. 2 and Equations (11) and (12). Tab. 4 shows the IT2FSs related to the elements of X matrix

Step 2. The matrix of the criteria weights is calculated based on Tab. 3 and Equations (13) and (14).

$$W = [\tilde{w}_1 \ \tilde{w}_2 \ \tilde{w}_3 \ \tilde{w}_4 \ \tilde{w}_5 \ \tilde{w}_6 \ \tilde{w}_7]$$

The results are shown in Tab. 5.

Tab. 3: Weights of the criteria evaluated by the decision makers

Criteria	Decision-makers		
	D_1	D_2	D_3
C_1	H	VH	VH
C_2	MH	H	MH
C_3	M	M	ML
C_4	VH	VH	H
C_5	H	H	MH
C_6	M	MH	M
C_7	MH	MH	M

Source: own

Tab. 4: The elements of the average decision-matrix (X) – Part 1

	\tilde{x}_{ij}^U						\tilde{x}_{ij}^L					
	x_1^U	x_2^U	x_3^U	x_4^U	H_1^U	H_2^U	x_1^L	x_2^L	x_3^L	x_4^L	H_1^L	H_2^L
\tilde{x}_{11}	0.37	0.57	0.62	0.77	1	1	0.47	0.57	0.62	0.67	0.9	0.9
\tilde{x}_{21}	0.07	0.23	0.28	0.43	1	1	0.15	0.23	0.28	0.33	0.9	0.9
\tilde{x}_{31}	0.03	0.13	0.17	0.30	1	1	0.08	0.13	0.17	0.22	0.9	0.9
\tilde{x}_{41}	0.63	0.80	0.85	0.97	1	1	0.73	0.80	0.85	0.90	0.9	0.9
\tilde{x}_{51}	0.77	0.90	0.93	1	1	1	0.85	0.90	0.93	0.97	0.9	0.9
\tilde{x}_{61}	0.90	1	1	1	1	1	0.95	1	1	1	0.9	0.9
\tilde{x}_{71}	0.37	0.57	0.62	0.77	1	1	0.47	0.57	0.62	0.67	0.9	0.9

Tab. 4: The elements of the average decision-matrix (X) – Part 2

	\tilde{x}_{ij}^U						\tilde{x}_{ij}^L					
	x_1^U	x_2^U	x_3^U	x_4^U	H_1^U	H_2^U	x_1^L	x_2^L	x_3^L	x_4^L	H_1^L	H_2^L
\tilde{x}_{81}^{\approx}	0.50	0.68	0.73	0.87	1	1	0.60	0.68	0.73	0.78	0.9	0.9
\tilde{x}_{12}^{\approx}	0.17	0.37	0.42	0.57	1	1	0.27	0.37	0.42	0.47	0.9	0.9
\tilde{x}_{22}^{\approx}	0.57	0.75	0.80	0.93	1	1	0.67	0.75	0.80	0.85	0.9	0.9
\tilde{x}_{32}^{\approx}	0.70	0.85	0.88	0.97	1	1	0.78	0.85	0.88	0.92	0.9	0.9
\tilde{x}_{42}^{\approx}	0.83	0.95	0.97	1	1	1	0.90	0.95	0.97	0.98	0.9	0.9
\tilde{x}_{52}^{\approx}	0.43	0.63	0.68	0.83	1	1	0.53	0.63	0.68	0.73	0.9	0.9
\tilde{x}_{62}^{\approx}	0.70	0.85	0.90	1	1	1	0.80	0.85	0.90	0.95	0.9	0.9
\tilde{x}_{72}^{\approx}	0.43	0.63	0.68	0.83	1	1	0.53	0.63	0.68	0.73	0.9	0.9
\tilde{x}_{82}^{\approx}	0.03	0.13	0.17	0.30	1	1	0.08	0.13	0.17	0.22	0.9	0.9
\tilde{x}_{13}^{\approx}	0.17	0.37	0.42	0.57	1	1	0.27	0.37	0.42	0.47	0.9	0.9
\tilde{x}_{23}^{\approx}	0.43	0.63	0.68	0.83	1	1	0.53	0.63	0.68	0.73	0.9	0.9
\tilde{x}_{33}^{\approx}	0.37	0.57	0.62	0.77	1	1	0.47	0.57	0.62	0.67	0.9	0.9
\tilde{x}_{43}^{\approx}	0.77	0.90	0.93	1	1	1	0.85	0.90	0.93	0.97	0.9	0.9
\tilde{x}_{53}^{\approx}	0.83	0.95	0.97	1	1	1	0.90	0.95	0.97	0.98	0.9	0.9
\tilde{x}_{63}^{\approx}	0.63	0.80	0.85	0.97	1	1	0.73	0.80	0.85	0.90	0.9	0.9
\tilde{x}_{73}^{\approx}	0.70	0.85	0.88	0.97	1	1	0.78	0.85	0.88	0.92	0.9	0.9
\tilde{x}_{83}^{\approx}	0.03	0.17	0.22	0.37	1	1	0.10	0.17	0.22	0.27	0.9	0.9
\tilde{x}_{14}^{\approx}	0.23	0.43	0.48	0.63	1	1	0.33	0.43	0.48	0.53	0.9	0.9
\tilde{x}_{24}^{\approx}	0.70	0.85	0.88	0.97	1	1	0.78	0.85	0.88	0.92	0.9	0.9
\tilde{x}_{34}^{\approx}	0.43	0.63	0.68	0.83	1	1	0.53	0.63	0.68	0.73	0.9	0.9
\tilde{x}_{44}^{\approx}	0.00	0.10	0.15	0.30	1	1	0.05	0.10	0.15	0.20	0.9	0.9
\tilde{x}_{54}^{\approx}	0.00	0.03	0.05	0.17	1	1	0.02	0.03	0.05	0.10	0.9	0.9
\tilde{x}_{64}^{\approx}	0.03	0.13	0.17	0.30	1	1	0.08	0.13	0.17	0.22	0.9	0.9
\tilde{x}_{74}^{\approx}	0.50	0.68	0.73	0.87	1	1	0.60	0.68	0.73	0.78	0.9	0.9
\tilde{x}_{84}^{\approx}	0.43	0.63	0.68	0.83	1	1	0.53	0.63	0.68	0.73	0.9	0.9
\tilde{x}_{15}^{\approx}	0.57	0.75	0.80	0.93	1	1	0.67	0.75	0.80	0.85	0.9	0.9
\tilde{x}_{25}^{\approx}	0.83	0.95	0.97	1	1	1	0.90	0.95	0.97	0.98	0.9	0.9
\tilde{x}_{35}^{\approx}	0.57	0.75	0.80	0.93	1	1	0.67	0.75	0.80	0.85	0.9	0.9
\tilde{x}_{45}^{\approx}	0.70	0.85	0.90	1	1	1	0.80	0.85	0.90	0.95	0.9	0.9
\tilde{x}_{55}^{\approx}	0.50	0.68	0.73	0.87	1	1	0.60	0.68	0.73	0.78	0.9	0.9
\tilde{x}_{65}^{\approx}	0.90	1	1	1	1	1	0.95	1	1	1	0.9	0.9
\tilde{x}_{75}^{\approx}	0.23	0.43	0.48	0.63	1	1	0.33	0.43	0.48	0.53	0.9	0.9
\tilde{x}_{85}^{\approx}	0.03	0.13	0.17	0.30	1	1	0.08	0.13	0.17	0.22	0.9	0.9
\tilde{x}_{16}^{\approx}	0.13	0.30	0.35	0.50	1	1	0.22	0.30	0.35	0.40	0.9	0.9
\tilde{x}_{26}^{\approx}	0.63	0.80	0.85	0.97	1	1	0.73	0.80	0.85	0.90	0.9	0.9

Tab. 4: The elements of the average decision-matrix (X) – Part 3

	\tilde{x}_{ij}^U						\tilde{x}_{ij}^L					
	x_1^U	x_2^U	x_3^U	x_4^U	H_1^U	H_2^U	x_1^L	x_2^L	x_3^L	x_4^L	H_1^L	H_2^L
\tilde{x}_{36}^u	0.57	0.75	0.80	0.93	1	1	0.67	0.75	0.80	0.85	0.9	0.9
\tilde{x}_{46}^u	0.50	0.68	0.73	0.87	1	1	0.60	0.68	0.73	0.78	0.9	0.9
\tilde{x}_{56}^u	0.37	0.57	0.62	0.77	1	1	0.47	0.57	0.62	0.67	0.9	0.9
\tilde{x}_{66}^u	0.83	0.95	0.97	1	1	1	0.90	0.95	0.97	0.98	0.9	0.9
\tilde{x}_{76}^u	0.50	0.68	0.73	0.87	1	1	0.60	0.68	0.73	0.78	0.9	0.9
\tilde{x}_{86}^u	0.30	0.50	0.55	0.70	1	1	0.40	0.50	0.55	0.60	0.9	0.9
\tilde{x}_{17}^u	0.77	0.90	0.93	1	1	1	0.85	0.90	0.93	0.97	0.9	0.9
\tilde{x}_{27}^u	0.30	0.50	0.55	0.70	1	1	0.40	0.50	0.55	0.60	0.9	0.9
\tilde{x}_{37}^u	0.30	0.50	0.55	0.70	1	1	0.40	0.50	0.55	0.60	0.9	0.9
\tilde{x}_{47}^u	0.03	0.13	0.17	0.30	1	1	0.08	0.13	0.17	0.22	0.9	0.9
\tilde{x}_{57}^u	0.13	0.30	0.35	0.50	1	1	0.22	0.30	0.35	0.40	0.9	0.9
\tilde{x}_{67}^u	0.00	0.00	0.00	0.10	1	1	0.00	0.00	0.00	0.05	0.9	0.9
\tilde{x}_{77}^u	0.43	0.63	0.68	0.83	1	1	0.53	0.63	0.68	0.73	0.9	0.9
\tilde{x}_{87}^u	0.77	0.90	0.92	0.97	1	1	0.83	0.90	0.92	0.93	0.9	0.9

Source: own

Tab. 5: The elements of the matrix of criteria weights

	\tilde{w}_j^U						\tilde{w}_j^L					
	w_1^U	w_2^U	w_3^U	w_4^U	H_1^U	H_2^U	w_1^L	w_2^L	w_3^L	w_4^L	H_1^L	H_2^L
\tilde{w}_1	0.83	0.95	0.97	1	1	1	0.90	0.95	0.97	0.98	0.9	0.9
\tilde{w}_2	0.57	0.75	0.80	0.93	1	1	0.67	0.75	0.80	0.85	0.9	0.9
\tilde{w}_3	0.23	0.43	0.48	0.63	1	1	0.33	0.43	0.48	0.53	0.9	0.9
\tilde{w}_4	0.83	0.95	0.97	1	1	1	0.90	0.95	0.97	0.98	0.9	0.9
\tilde{w}_5	0.63	0.80	0.85	0.97	1	1	0.73	0.80	0.85	0.90	0.9	0.9
\tilde{w}_6	0.37	0.57	0.62	0.77	1	1	0.47	0.57	0.62	0.67	0.9	0.9
\tilde{w}_7	0.43	0.63	0.68	0.83	1	1	0.53	0.63	0.68	0.73	0.9	0.9

Source: own

Step 3. The matrix of average solution can be calculated by using the results of Step 1 and Equations (15) and (16).

$$AV = [\tilde{M}_1 \ \tilde{M}_2 \ \tilde{M}_3 \ \tilde{M}_4 \ \tilde{M}_5 \ \tilde{M}_6 \ \tilde{M}_7]$$

The elements of the matrix of average solutions are represented in Tab. 6.

Step 4. Based on Tab. 4 and Tab. 6 and Equations (17) to (20), the positive and negative distances (the PDA and NDA matrices) are calculated.

The positive distances are shown in Tab. 7 and the negative distances are represented in Tab. 8.

Tab. 6: The elements of the matrix of average solutions

	$\tilde{\mathcal{M}}_j^U$						$\tilde{\mathcal{M}}_j^L$					
	\mathcal{M}_1^U	\mathcal{M}_2^U	\mathcal{M}_3^U	\mathcal{M}_4^U	H_1^U	H_2^U	\mathcal{M}_1^L	\mathcal{M}_2^L	\mathcal{M}_3^L	\mathcal{M}_4^L	H_1^L	H_2^L
$\tilde{\mathcal{M}}_1$	0.45	0.61	0.65	0.76	1	1	0.54	0.61	0.65	0.69	0.9	0.9
$\tilde{\mathcal{M}}_2$	0.48	0.65	0.69	0.80	1	1	0.57	0.65	0.69	0.73	0.9	0.9
$\tilde{\mathcal{M}}_3$	0.49	0.65	0.70	0.81	1	1	0.58	0.65	0.70	0.74	0.9	0.9
$\tilde{\mathcal{M}}_4$	0.29	0.44	0.48	0.61	1	1	0.37	0.44	0.48	0.53	0.9	0.9
$\tilde{\mathcal{M}}_5$	0.54	0.69	0.73	0.83	1	1	0.63	0.69	0.73	0.77	0.9	0.9
$\tilde{\mathcal{M}}_6$	0.48	0.65	0.70	0.83	1	1	0.57	0.65	0.70	0.75	0.9	0.9
$\tilde{\mathcal{M}}_7$	0.34	0.48	0.52	0.64	1	1	0.41	0.48	0.52	0.56	0.9	0.9

Source: own

Tab. 7: The positive distances from the average solution (PDA matrix) – Part 1

	$\tilde{\mathcal{P}}_{ij}^U$						$\tilde{\mathcal{P}}_{ij}^L$					
	\mathcal{P}_1^U	\mathcal{P}_2^U	\mathcal{P}_3^U	\mathcal{P}_4^U	H_1^U	H_2^U	\mathcal{P}_1^L	\mathcal{P}_2^L	\mathcal{P}_3^L	\mathcal{P}_4^L	H_1^L	H_2^L
$\tilde{\mathcal{P}}_{11}$	0	0	0	0	1	1	0	0	0	0	1	1
$\tilde{\mathcal{P}}_{21}$	0	0	0	0	1	1	0	0	0	0	1	1
$\tilde{\mathcal{P}}_{31}$	0	0	0	0	1	1	0	0	0	0	1	1
$\tilde{\mathcal{P}}_{41}$	-0.21	0.24	0.38	0.82	1	1	0.07	0.24	0.38	0.58	0.9	0.9
$\tilde{\mathcal{P}}_{51}$	0.01	0.40	0.52	0.87	1	1	0.25	0.40	0.52	0.69	0.9	0.9
$\tilde{\mathcal{P}}_{61}$	0.22	0.56	0.62	0.87	1	1	0.41	0.56	0.62	0.74	0.9	0.9
$\tilde{\mathcal{P}}_{71}$	0	0	0	0	1	1	0	0	0	0	1	1
$\tilde{\mathcal{P}}_{81}$	-0.42	0.05	0.20	0.66	1	1	-0.15	0.05	0.20	0.39	0.9	0.9
$\tilde{\mathcal{P}}_{12}$	0	0	0	0	1	1	0	0	0	0	1	1
$\tilde{\mathcal{P}}_{22}$	-0.36	0.09	0.23	0.68	1	1	-0.10	0.09	0.23	0.42	0.9	0.9
$\tilde{\mathcal{P}}_{32}$	-0.16	0.25	0.36	0.73	1	1	0.08	0.25	0.36	0.52	0.9	0.9
$\tilde{\mathcal{P}}_{42}$	0.04	0.40	0.49	0.78	1	1	0.26	0.40	0.49	0.62	0.9	0.9
$\tilde{\mathcal{P}}_{52}$	0	0	0	0	1	1	0	0	0	0	1	1
$\tilde{\mathcal{P}}_{62}$	-0.16	0.25	0.39	0.78	1	1	0.10	0.25	0.39	0.57	0.9	0.9
$\tilde{\mathcal{P}}_{72}$	0	0	0	0	1	1	0	0	0	0	1	1

Tab. 7: The positive distances from the average solution (PDA matrix) – Part 2

	\tilde{p}_{ij}^U						\tilde{p}_{ij}^L					
	p_1^U	p_2^U	p_3^U	p_4^U	H_1^U	H_2^U	p_1^L	p_2^L	p_3^L	p_4^L	H_1^L	H_2^L
\tilde{p}_{82}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{p}_{13}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{p}_{23}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{p}_{33}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{p}_{43}	-0.06	0.31	0.42	0.76	1	1	0.17	0.31	0.42	0.58	0.9	0.9
\tilde{p}_{53}	0.04	0.38	0.47	0.76	1	1	0.24	0.38	0.47	0.61	0.9	0.9
\tilde{p}_{63}	-0.26	0.16	0.29	0.71	1	1	-0.01	0.16	0.29	0.48	0.9	0.9
\tilde{p}_{73}	-0.16	0.23	0.34	0.71	1	1	0.07	0.23	0.34	0.51	0.9	0.9
\tilde{p}_{83}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{p}_{14}	-0.75	-0.10	0.10	0.83	1	1	-0.37	-0.10	0.10	0.43	0.9	0.9
\tilde{p}_{24}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{p}_{34}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{p}_{44}	-0.02	0.63	0.83	1.35	1	1	0.37	0.63	0.83	1.05	0.9	0.9
\tilde{p}_{54}	0.27	0.85	0.98	1.35	1	1	0.59	0.85	0.98	1.12	0.9	0.9
\tilde{p}_{64}	-0.02	0.59	0.76	1.27	1	1	0.33	0.59	0.76	0.97	0.9	0.9
\tilde{p}_{74}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{p}_{84}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{p}_{15}	-0.38	0.03	0.15	0.55	1	1	-0.15	0.03	0.15	0.32	0.9	0.9
\tilde{p}_{25}	0	0.31	0.39	0.65	1	1	0.18	0.31	0.39	0.51	0.9	0.9
\tilde{p}_{35}	-0.38	0.03	0.15	0.55	1	1	-0.15	0.03	0.15	0.32	0.9	0.9
\tilde{p}_{45}	-0.19	0.17	0.29	0.65	1	1	0.04	0.17	0.29	0.46	0.9	0.9
\tilde{p}_{55}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{p}_{65}	0.09	0.38	0.43	0.65	1	1	0.25	0.38	0.43	0.53	0.9	0.9
\tilde{p}_{75}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{p}_{85}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{p}_{16}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{p}_{26}	-0.29	0.15	0.29	0.73	1	1	-0.02	0.15	0.29	0.49	0.9	0.9

Tab. 7: The positive distances from the average solution (PDA matrix) – Part 3

	\tilde{p}_{ij}^U						\tilde{p}_{ij}^L					
	p_1^U	p_2^U	p_3^U	p_4^U	H_1^U	H_2^U	p_1^L	p_2^L	p_3^L	p_4^L	H_1^L	H_2^L
\tilde{p}_{36}	-0.39	0.07	0.22	0.68	1	1	-0.12	0.07	0.22	0.41	0.9	0.9
\tilde{p}_{46}	-0.49	-0.02	0.12	0.58	1	1	-0.22	-0.02	0.12	0.31	0.9	0.9
\tilde{p}_{56}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{p}_{66}	0.01	0.37	0.47	0.78	1	1	0.23	0.37	0.47	0.61	0.9	0.9
\tilde{p}_{76}	-0.49	-0.02	0.12	0.58	1	1	-0.22	-0.02	0.12	0.31	0.9	0.9
\tilde{p}_{86}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{p}_{17}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{p}_{27}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{p}_{37}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{p}_{47}	0.08	0.64	0.78	1.22	1	1	0.40	0.64	0.78	0.96	0.9	0.9
\tilde{p}_{57}	-0.32	0.27	0.44	1.01	1	1	0.03	0.27	0.44	0.70	0.9	0.9
\tilde{p}_{67}	0.49	0.97	1.04	1.28	1	1	0.73	0.97	1.04	1.13	0.9	0.9
\tilde{p}_{77}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{p}_{87}	0	0	0	0	1	1	0	0	0	0	1	1

Source: own

Tab. 8: The negative distances from the average solution (NDA matrix) – Part 1

	\tilde{n}_{ij}^U						\tilde{n}_{ij}^L					
	n_1^U	n_2^U	n_3^U	n_4^U	H_1^U	H_2^U	n_1^L	n_2^L	n_3^L	n_4^L	H_1^L	H_2^L
\tilde{n}_{11}	-0.50	-0.01	0.13	0.63	1	1	-0.21	-0.01	0.13	0.36	0.9	0.9
\tilde{n}_{21}	0.03	0.52	0.67	1.12	1	1	0.33	0.52	0.67	0.87	0.9	0.9
\tilde{n}_{31}	0.25	0.71	0.83	1.17	1	1	0.51	0.71	0.83	0.98	0.9	0.9
\tilde{n}_{41}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{51}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{61}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{71}	-0.50	-0.01	0.13	0.63	1	1	-0.21	-0.01	0.13	0.36	0.9	0.9
\tilde{n}_{81}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{12}	-0.13	0.35	0.49	0.97	1	1	0.16	0.35	0.49	0.70	0.9	0.9

Tab. 8: The negative distances from the average solution (NDA matrix) – Part 2

	\tilde{n}_{ij}^U						\tilde{n}_{ij}^L					
	n_1^U	n_2^U	n_3^U	n_4^U	H_1^U	H_2^U	n_1^L	n_2^L	n_3^L	n_4^L	H_1^L	H_2^L
\tilde{n}_{22}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{32}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{42}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{52}	-0.53	-0.06	0.08	0.56	1	1	-0.25	-0.06	0.08	0.30	0.9	0.9
\tilde{n}_{62}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{72}	-0.53	-0.06	0.08	0.56	1	1	-0.25	-0.06	0.08	0.30	0.9	0.9
\tilde{n}_{82}	0.28	0.73	0.84	1.17	1	1	0.54	0.73	0.84	0.98	0.9	0.9
\tilde{n}_{13}	-0.11	0.36	0.49	0.96	1	1	0.17	0.36	0.49	0.70	0.9	0.9
\tilde{n}_{23}	-0.51	-0.04	0.09	0.56	1	1	-0.23	-0.04	0.09	0.31	0.9	0.9
\tilde{n}_{33}	-0.41	0.06	0.19	0.66	1	1	-0.13	0.06	0.19	0.41	0.9	0.9
\tilde{n}_{43}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{53}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{63}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{73}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{83}	0.19	0.65	0.79	1.16	1	1	0.47	0.65	0.79	0.95	0.9	0.9
\tilde{n}_{14}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{24}	0.19	0.81	0.98	1.48	1	1	0.56	0.81	0.98	1.21	0.9	0.9
\tilde{n}_{34}	-0.39	0.34	0.54	1.19	1	1	0.01	0.34	0.54	0.81	0.9	0.9
\tilde{n}_{44}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{54}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{64}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{74}	-0.25	0.45	0.65	1.26	1	1	0.16	0.45	0.65	0.92	0.9	0.9
\tilde{n}_{84}	-0.39	0.34	0.54	1.19	1	1	0.01	0.34	0.54	0.81	0.9	0.9
\tilde{n}_{15}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{25}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{35}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{45}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{55}	-0.46	-0.06	0.07	0.47	1	1	-0.22	-0.06	0.07	0.24	0.9	0.9
\tilde{n}_{65}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{75}	-0.13	0.30	0.42	0.85	1	1	0.13	0.30	0.42	0.62	0.9	0.9

Tab. 8: The negative distances from the average solution (NDA matrix) – Part 3

	\tilde{n}_{ij}^U						\tilde{n}_{ij}^L					
	n_1^U	n_2^U	n_3^U	n_4^U	H_1^U	H_2^U	n_1^L	n_2^L	n_3^L	n_4^L	H_1^L	H_2^L
\tilde{n}_{85}	0.34	0.75	0.85	1.13	1	1	0.58	0.75	0.85	0.97	0.9	0.9
\tilde{n}_{16}	-0.03	0.45	0.60	1.03	1	1	0.26	0.45	0.60	0.79	0.9	0.9
\tilde{n}_{26}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{36}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{46}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{56}	-0.43	0.06	0.20	0.68	1	1	-0.14	0.06	0.20	0.42	0.9	0.9
\tilde{n}_{66}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{76}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{86}	-0.33	0.16	0.30	0.78	1	1	-0.04	0.16	0.30	0.52	0.9	0.9
\tilde{n}_{17}	0.26	0.77	0.91	1.32	1	1	0.58	0.77	0.91	1.11	0.9	0.9
\tilde{n}_{27}	-0.68	-0.04	0.13	0.72	1	1	-0.33	-0.04	0.13	0.37	0.9	0.9
\tilde{n}_{37}	-0.68	-0.04	0.13	0.72	1	1	-0.33	-0.04	0.13	0.37	0.9	0.9
\tilde{n}_{47}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{57}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{67}	0	0	0	0	1	1	0	0	0	0	1	1
\tilde{n}_{77}	-0.41	0.23	0.40	0.99	1	1	-0.06	0.23	0.40	0.64	0.9	0.9
\tilde{n}_{87}	0.26	0.77	0.87	1.26	1	1	0.54	0.77	0.87	1.04	0.9	0.9

Source: own

A reciprocal relation between corresponding elements of PDA and NDA matrices can be seen in Tab. 7 and Tab. 8. If an element of one of these matrices is equal to $\tilde{0}$, the corresponding element of the other matrix is a positive IT2FS, and vice versa.

Steps 5 to 7. Based on the Tab. 7 and Tab. 8 and Equations (21) to (25), the weighted sum of positive and negative distances ($\tilde{s}p_i$ and $\tilde{s}n_i$), the normalized values of them ($\tilde{n}p_i$ and $\tilde{n}n_i$) and the appraisal scores are calculated for all alternatives. The results of these steps are shown in Tab. 9.

Step 8. According to Tab. 9, the ranking values of appraisal scores (\mathcal{RV}) can be calculated. The results are represented in Tab. 10. It can be seen that the ranking order of alternatives (subcontractors) is $\mathcal{A}_6 > \mathcal{A}_4 > \mathcal{A}_5 > \mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_7 > \mathcal{A}_1 > \mathcal{A}_8$. Therefore, \mathcal{A}_6

is the best subcontractor in terms of the seven criteria.

4. Comparison and Sensitivity Analysis

A comparison and a sensitivity analysis are performed in this section to validate the results of the proposed method. In recent years, many multi-criteria decision making methods have been introduced and developed in the interval type-2 fuzzy environment. Some methods which have good efficiency and could be implemented in the considered MCDM problem have been selected for the comparison. The selected methods for the comparative analysis are the methods proposed by Chen et al. (2012), Wang et al. (2012), Baležentis and Zeng (2013), Hu et al. (2013), Keshavarz Ghorabae et al. (2014)

Tab. 9: The weighted sum of distances, their normalized values and the appraisal scores – Part 1

\tilde{K}		\tilde{K}_{ij}^U						\tilde{K}_{ij}^L					
		k_1^U	k_2^U	k_3^U	k_4^U	H_1^U	H_2^U	k_1^L	k_2^L	k_3^L	k_4^L	H_1^L	H_2^L
$\tilde{s}p$	$\tilde{s}p_1$	-1.12	-0.08	0.23	1.37	1	1	-0.49	-0.08	0.23	0.71	0.9	0.9
	$\tilde{s}p_2$	-0.56	0.40	0.70	1.82	1	1	0.04	0.40	0.70	1.14	0.9	0.9
	$\tilde{s}p_3$	-0.81	0.25	0.55	1.74	1	1	-0.16	0.25	0.55	1.01	0.9	0.9
	$\tilde{s}p_4$	-0.76	1.78	2.62	5.46	1	1	0.71	1.78	2.62	3.77	0.9	0.9
	$\tilde{s}p_5$	-0.02	1.52	1.97	3.55	1	1	0.85	1.52	1.97	2.61	0.9	0.9
	$\tilde{s}p_6$	0.13	2.48	3.16	5.62	1	1	1.42	2.48	3.16	4.15	0.9	0.9
	$\tilde{s}p_7$	-0.47	0.08	0.24	0.89	1	1	-0.12	0.08	0.24	0.48	0.9	0.9
	$\tilde{s}p_8$	-0.42	0.05	0.19	0.66	1	1	-0.14	0.05	0.19	0.39	0.9	0.9
$\tilde{s}n$	$\tilde{s}n_1$	-0.60	1.15	1.74	4.04	1	1	0.39	1.15	1.74	2.67	0.9	0.9
	$\tilde{s}n_2$	-0.70	1.22	1.73	3.55	1	1	0.44	1.22	1.73	2.48	0.9	0.9
	$\tilde{s}n_3$	-1.01	1.00	1.51	3.38	1	1	0.17	1.00	1.51	2.24	0.9	0.9
	$\tilde{s}n_4$	0.00	0.00	0.00	0.00	1	1	0.00	0.00	0.00	0.00	0.9	0.9
	$\tilde{s}n_5$	-1.27	-0.06	0.25	1.51	1	1	-0.50	-0.06	0.25	0.75	0.9	0.9
	$\tilde{s}n_6$	0.00	0.00	0.00	0.00	1	1	0.00	0.00	0.00	0.00	0.9	0.9
	$\tilde{s}n_7$	-1.71	0.76	1.46	4.07	1	1	-0.22	0.76	1.46	2.54	0.9	0.9
	$\tilde{s}n_8$	-0.12	2.32	3.08	5.76	1	1	1.21	2.32	3.08	4.12	0.9	0.9
$\tilde{n}p$	$\tilde{n}p_1$	-0.40	-0.03	0.08	0.49	1	1	-0.17	-0.03	0.08	0.25	0.9	0.9
	$\tilde{n}p_2$	-0.20	0.14	0.25	0.65	1	1	0.01	0.14	0.25	0.40	0.9	0.9
	$\tilde{n}p_3$	-0.29	0.09	0.19	0.62	1	1	-0.06	0.09	0.19	0.36	0.9	0.9
	$\tilde{n}p_4$	-0.27	0.63	0.93	1.94	1	1	0.25	0.63	0.93	1.34	0.9	0.9
	$\tilde{n}p_5$	-0.01	0.54	0.70	1.26	1	1	0.30	0.54	0.70	0.93	0.9	0.9
	$\tilde{n}p_6$	0.05	0.88	1.12	1.99	1	1	0.50	0.88	1.12	1.47	0.9	0.9
	$\tilde{n}p_7$	-0.17	0.03	0.08	0.32	1	1	-0.04	0.03	0.08	0.17	0.9	0.9
	$\tilde{n}p_8$	-0.15	0.02	0.07	0.23	1	1	-0.05	0.02	0.07	0.14	0.9	0.9

Tab. 9: The weighted sum of distances, their normalized values and the appraisal scores – Part 2

\tilde{K}		\tilde{K}_{ij}^U						\tilde{K}_{ij}^L					
		k_1^U	k_2^U	k_3^U	k_4^U	H_1^U	H_2^U	k_1^L	k_2^L	k_3^L	k_4^L	H_1^L	H_2^L
\tilde{n}	\tilde{n}_1	-0.49	0.36	0.58	1.22	1	1	0.02	0.36	0.58	0.86	0.9	0.9
	\tilde{n}_2	-0.31	0.36	0.55	1.26	1	1	0.09	0.36	0.55	0.84	0.9	0.9
	\tilde{n}_3	-0.24	0.44	0.63	1.37	1	1	0.17	0.44	0.63	0.94	0.9	0.9
	\tilde{n}_4	1.00	1.00	1.00	1.00	1	1	1.00	1.00	1.00	1.00	0.9	0.9
	\tilde{n}_5	0.45	0.91	1.02	1.47	1	1	0.72	0.91	1.02	1.19	0.9	0.9
	\tilde{n}_6	1.00	1.00	1.00	1.00	1	1	1.00	1.00	1.00	1.00	0.9	0.9
	\tilde{n}_7	-0.50	0.46	0.72	1.63	1	1	0.06	0.46	0.72	1.08	0.9	0.9
	\tilde{n}_8	-1.12	-0.13	0.14	1.04	1	1	-0.52	-0.13	0.14	0.55	0.9	0.9
\tilde{h}	\tilde{h}_1	-0.44	0.17	0.33	0.85	1	1	-0.08	0.17	0.33	0.55	0.9	0.9
	\tilde{h}_2	-0.25	0.25	0.40	0.95	1	1	0.05	0.25	0.40	0.62	0.9	0.9
	\tilde{h}_3	-0.27	0.27	0.41	0.99	1	1	0.06	0.27	0.41	0.65	0.9	0.9
	\tilde{h}_4	0.37	0.82	0.96	1.47	1	1	0.63	0.82	0.96	1.17	0.9	0.9
	\tilde{h}_5	0.22	0.72	0.86	1.36	1	1	0.51	0.72	0.86	1.06	0.9	0.9
	\tilde{h}_6	0.52	0.94	1.06	1.50	1	1	0.75	0.94	1.06	1.23	0.9	0.9
	\tilde{h}_7	-0.33	0.25	0.40	0.97	1	1	0.01	0.25	0.40	0.62	0.9	0.9
	\tilde{h}_8	-0.64	-0.06	0.11	0.64	1	1	-0.28	-0.06	0.11	0.34	0.9	0.9

Source: own

Tab. 10: The ranking values of appraisal scores

Alternatives	(RV)
A_1	0.1633
A_2	0.1767
A_3	0.1778
A_4	0.1957
A_5	0.1938
A_6	0.1964
A_7	0.1753
A_8	0.1049

Source: own

and Keshavarz Ghorabae et al. (2015a). The symbols M-1 to M-6 are utilized for representing these selected methods, respectively.

For comparing the extended EDAS method with the other methods, the numerical example is solved using the selected methods separately, and the Spearman's rank correlation coefficient (r_s) is used to analyze the ranking results. We can say that there is a significant statistical correlation between results if the values of r_s are greater than 0.6 (Keshavarz Ghorabae et

al., 2016a). Tab. 11 represents the results of the comparison between the proposed method and the other methods. All correlation coefficients are greater than 0.6 according to this table. This fact demonstrates a strong or very strong relationship between the ranking results of the extended EDAS method and the other methods. Therefore, it can be said that the result of the proposed method is consistent with the results of other methods.

Tab. 11: Comparing different methods' results and the corresponding correlation (r_s)

Alternatives	Methods						
	M-1	M-2	M-3	M-4	M-5	M-6	EDAS-IT2FSs
A_1	7	7	7	7	7	4	7
A_2	5	4	5	5	4	6	5
A_3	4	5	6	4	5	7	4
A_4	2	2	2	2	2	2	2
A_5	3	3	3	3	3	3	3
A_6	1	1	1	1	1	1	1
A_7	6	6	4	6	6	5	6
A_8	8	8	8	8	8	8	8
r_s	1	0.98	0.9	1	0.98	0.76	—

Source: own

Tab. 12: The generated weights for sensitivity analysis

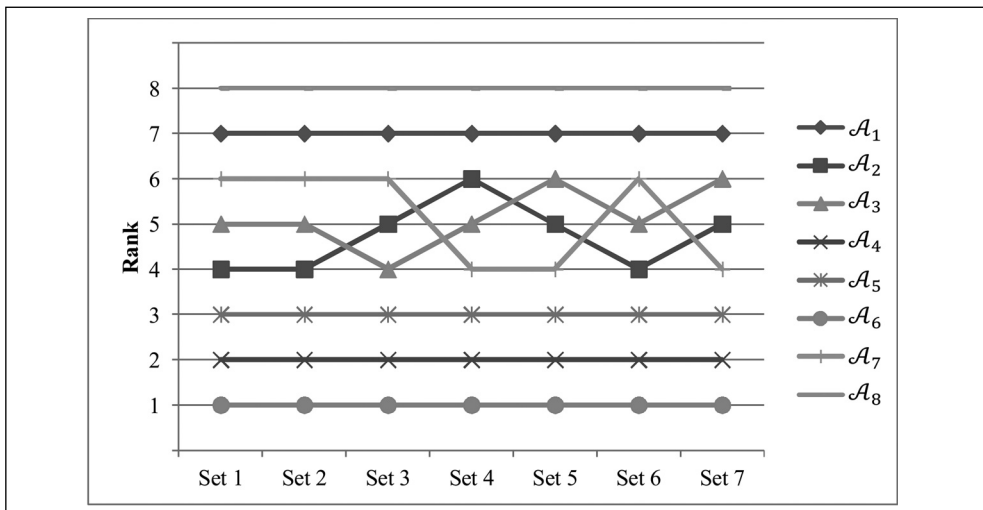
Sets	Criteria						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
1	0.0357	0.0714	0.1071	0.1429	0.1786	0.2143	0.2500
2	0.0714	0.1071	0.1429	0.1786	0.2143	0.2500	0.0357
3	0.1071	0.1429	0.1786	0.2143	0.2500	0.0357	0.0714
4	0.1429	0.1786	0.2143	0.2500	0.0357	0.0714	0.1071
5	0.1786	0.2143	0.2500	0.0357	0.0714	0.1071	0.1429
6	0.2143	0.2500	0.0357	0.0714	0.1071	0.1429	0.1786
7	0.2500	0.0357	0.0714	0.1071	0.1429	0.1786	0.2143

Source: own

In addition to this comparison, a sensitivity analysis based on varying the weights of criteria is performed. For this aim, according to the number of criteria in this problem, we generate seven sets of criteria weights with a simple pattern. In the pattern which is used for sensitivity analysis, one criterion has the highest weight, and one criterion has the lowest weight, and the other criteria have a weight between them. Using this pattern helps us to investigate the impact of changing the weights of criteria in a more effective way. The weights of criteria in each set are represented in Tab. 12.

As can be seen, a crisp weight is assigned to each criterion in these sets. The numerical example is solved with each set of generated weights separately. The ranking results with different sets are shown in Fig. 2. This figure represents less change in the rank of alternatives within different sets of generated criteria weights. Therefore, we can say that the proposed method has good stability when the weights of criteria are varied. These analyses demonstrate the validity and stability of the results of the extended EDAS method in the decision-making problem with multiple criteria and uncertain data.

Fig. 2: The ranking results of the sensitivity analysis



Source: own

Conclusions

The uncertainty is usually inevitable in the multi-criteria decision-making process. The fuzzy set theory is one of the efficient tools to deal with the MCDM problems in an uncertain environment. IT2FSs are flexible and give more degrees of freedom to decision-makers for modeling the decision-making problems. In this study, we have proposed an extended EDAS method with interval type-2 fuzzy sets. The concepts and arithmetic operations of IT2FSs have been utilized for extending this method. A numerical example of multi-criteria

subcontractor evaluation problem has been used to illustrate the process of the proposed method. Moreover, to validate the results, this example has been solved by some existing methods. Also, we have performed a sensitivity analysis with seven sets of criteria weights. These sets have been designed according to the number of criteria in the problem, and a pattern has been used to generate the weights of criteria in each set. These analyses show that the results of the extended EDAS method are relatively consistent with the other methods of the comparison and have good stability

in different sets of criteria weights. Future research can apply the proposed method to the MCDM problems such as supplier selection, robot selection, personnel selection, material selection and project selection.

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Mehdi Keshavarz Ghorabae, Ph.D. Candidate

Allameh Tabataba'i University
Faculty of Management and Accounting
Department of Industrial Management
m.keshavarz_gh@yahoo.com

Prof. Maghsoud Amiri, Ph.D.

Allameh Tabataba'i University
Faculty of Management and Accounting
Department of Industrial Management
m.amiri.atu@gmail.com

Prof. Edmundas Kazimieras Zavadskas, Ph.D.

Vilnius Gediminas Technical University
Research Institute of Smart Building
Technologies
edmundas.zavadskas@vgtu.lt

Prof. Zenonas Turskis, Ph.D.

Vilnius Gediminas Technical University
Research Institute of Smart Building
Technologies
zenonas.turskis@vgtu.lt

Abstract

MULTI-CRITERIA GROUP DECISION-MAKING USING AN EXTENDED EDAS METHOD WITH INTERVAL TYPE-2 FUZZY SETS**Mehdi Keshavarz Ghorabae, Maghsoud Amiri, Edmundas Kazimieras Zavadskas, Zenonas Turskis**

Multi-criteria decision-making (MCDM) methods are very useful in the real-world decision-making problems. We are usually confronted with the decision-making process in an uncertain environment, and the fuzzy set theory is an efficient tool to handle this uncertainty. Interval type-2 fuzzy sets are one of the extensions of the fuzzy sets which are very flexible to model an uncertain environment. This study is related to MCDM problems within the context of interval type-2 fuzzy sets (IT2FSs). The evaluation based on distance from average solution (EDAS) method is a new and efficient MCDM method, and assessment of alternatives in this method is based on the distance of them from average solution with respect to all criteria. In the EDAS method, each alternative has positive and negative distances which are used to determine the appraisal score of it. In this research, we present an extended EDAS method, which is named EDAS-IT2FSs, for dealing with multi-criteria group decision-making problems with interval type-2 fuzzy sets. Basic concepts of interval type-2 fuzzy sets and the arithmetic operations of trapezoidal IT2FSs are used to develop the extended EDAS method. A numerical example of multi-criteria subcontractor evaluation problem is used to illustrate the process of using the extended EDAS method. The example involves eight subcontractors that need to be evaluated with respect to seven criteria. A comparison and a sensitivity analysis based on different sets of criteria weights are also performed to show the validity of the proposed method. The results of these analyses show the efficiency and stability of the extended EDAS method.

Key Words: Multi-criteria decision-making, interval type-2 fuzzy sets, fuzzy MCDM, EDAS method.

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