# Adaptive Error Estimators for Solving of Wave Equations

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***Abstract*** **Adaptive techniques based on different criteria are presented and compared. The criteria represented by the several error estimators are applied to *h*-adaptive, *p*-adaptive and *hp*-adaptive approaches to the numerical solution of a typical problem solved by own code Agros2D based on a fully adaptive higher-order element method. The results are compared and discussed.**

***Keywords*** **spatial-adaptive FEM, wave equation, error estimator**

## I. Introduction

The space adaptivity represents a promising technique for numerical modeling of many problems in physics [1] and other disciplines. When properly applied, it may significantly reduce both the number of degrees of freedom (DOF) and time of computation. For years, the authors have been developing their own application Agros2D [2] based on a fully adaptive higher-order finite element method that cooperates with the library deal.II [3]. In this frame they propose and different criteria of using techniques based on estimation of their errors.

## II. Illustrative Example

Consider electric field (TE mode) of a WR-90 X-band waveguide containing symmetrical inductive diaphragms (Fig. 1).

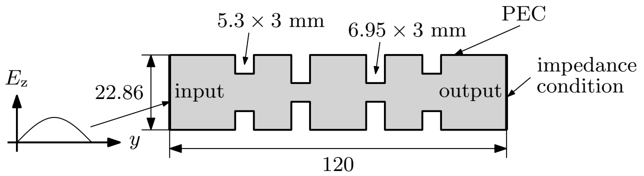


Fig. 1. Basic arrangement and dimensions

Harmonic electric field in waveguide is described by wave equation in the form

$$\mathrm{curl}\ \frac{1}{\mu}\ \mathrm{curl}\ \underline{\boldsymbol{E}} - \mathrm{j}\omega(\sigma + \mathrm{j}\omega\varepsilon)\underline{\boldsymbol{E}} = 0,$$

where $\mu$ stands for permeability, $\varepsilon$ permittivity, $\sigma$ electric conductivity, $\underline{\boldsymbol{E}}$ phasor of electric field intensity and finally $\omega$ denotes frequency of the wave. The waveguide and iris parts are modeled as perfect electric conductors. On the left side is harmonic source of electric field and on the opposite side impedance boundary condition.

Figure 2 compares resulting mesh after 8 iteration steps of adaptive process. It is obvious that the resulting mesh after *p*-adaptivity is significantly coarser and leads to considerably less degrees of freedom than the second mesh.

Figure 3 compares convergence curves of several types of investigated adaptivity.

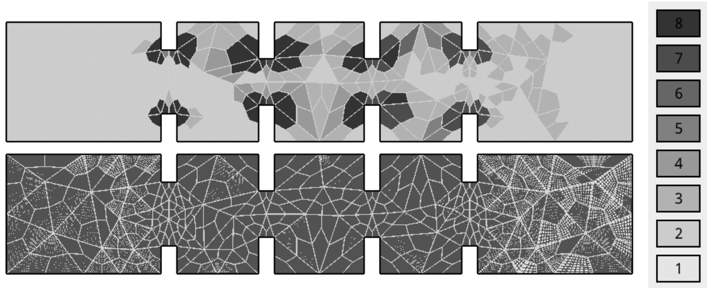


Fig. 2. Initial (light) and computational (dark) mesh with polynomial order after 8 iterations (*h* (bottom) and *p* (top) adaptivity)

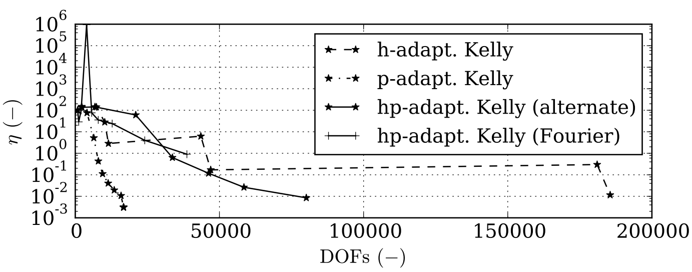


Fig. 3. Convergence of results for *h*-, *p*- and *hp*-adaptivity (adaptive process starting in all cases with $p = 1$)

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## References

[1] Dular, P., Zuqi, T., Le Menach, Y., Creuse, E., Piriou, F.: ”Comparison of Residual and Hierarchical Finite Element Error Estimators in Eddy Current Problems”, IEEE Transactions on Magnetics, Volume 50, Issue 2, 2014, DOI: 10.1109/TMAG.2013.2285254

[2] Karban, P., Mach, F., Kus, P., Panek, D., Dolezel, I.: ”Numerical Solution of Coupled Problems Using Code Agros2D”, in Computing, Volume 95, Issue 1 Supplement, 2013, pp. 381408.

[3] Bangerth, W., Heister, T., Heltai, L., Kanschat, G., Kronbichlerand, M., Maier, M., Turcksin, B., Young, T. D.: ”The deal.II Library, Version 8.1”, arXiv preprint, http://arxiv.org/abs/1312.2266v4, 2013.