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MAGNETIC CIRCUITS WITH PERMANENT MAGNETS, GENERATING STRONG MAGNETIC FIELDS[1]

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Abstract. The paper is devoted to the problems of magnetic circuits with permanent magnets for generating strong homogeneous magnetic fields. An appropriate mathematical model is expressed by a system of nonlinear partial differential equations of the second order for vector potential $A(x, y)$. The model is solved by the finite element method using the professional Quick Field program. An illustrative example allows demonstrating basic properties of topological structures of magnetic circuits with permanent magnets, including the possibilities for the next improvement.

Key words. Strong magnetic field, magnetic circuits with permanent magnets, numerical solution, finite elements method.

1. Introduction

Some branches of natural sciences, engineering and medicine make use of apparatuses and appliances whose functioning in a certain space region—operational space—necessitates a strong, steady-state, homogeneous magnetic field. For instance, this is the case of

- physics of moving electric particles, where the deviation of the particle trajectory allows determining the momentum of the particle [1],
- magnetic cooling devices [2],
- genetic engineering, where biological specimens are subjected to strong, long-lasting magnetic fields [3],

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- nanotechnologies, where some new materials are made under the influence of strong homogeneous magnetic fields [4],
- levitation systems [5],
- magnetic separators [6],
- medical diagnostic systems using magnetic resonance [7], and still other regions.

The required magnetic field is usually generated either by classical electromagnets or by supraconvective coils. At present, it is permanent magnets that are widely used thanks to modern materials with high remanent magnetic induction. An important factor in implementing the new technology rests with optimizing the *geometrical configuration* of permanent magnets in the magnetic circuit.

In the year 1985, American physicist Klaus Halbach designed an accelerator of particles with special configuration of permanent magnets, with the aim to obtain an exceptionally strong magnetic field. The permanent magnets were made by powder metallurgy of rare earth with special composition of neodymium-iron-boron (NdFeB). A cuboid form of the magnets is shown in Fig. 1, where the arrow-heads indicate the direction of magnetization. The magnetic fields of particular magnets are superimposed on each other, obtaining a surprising disposition: in the bottom part of the disposition there appears a strong magnetic field, whereas in the upper part the field is very weak (Fig. 2). This adjustment is nowadays denoted *the Halbach Array* or shortly *the H system.* The magnetic field of the H system possesses a wide variability as against a homogeneous field. However, in some applications this feature may turn out to be a disadvantage. In the half of the nineties, the H system of magnets was used by Richard F. Post to design a high-pitched generator, and at the same time the H system was applied to designing the first passive magnetic bearings. R. F. Post [9], [10] applied for a patent for a magnetically levitated transport system using the H system of magnets under the denotation *Inductrack*. In comparison with the hitherto transport systems maglev, the Inductrack is simple, inexpensive and embodies expedient properties [11], [12]. This project is supported by NASA for catapulting cosmic rockets and space shuttles. The presented applications indicate that the problem of *magnetic circuits synthesis* becomes topical for obtaining magnetic fields of required properties in chosen operational regions.

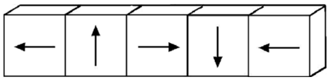

Fig. 1. Halbach system of permanent magnets—basic arrangement.

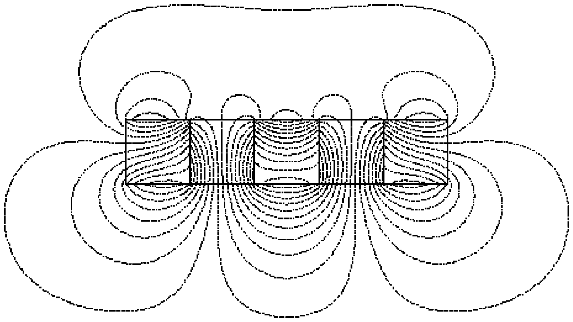

Fig. 2. Halbach system of permanent magnets–courses of lines of force.

The aim of the paper is to show that there exist still other topological configurations that, despite being different from the H system, can yield an operation space with a strong homogeneous magnetic fields. Such configurations are composed of:

- permanent magnets with a demanded direction of magnetization,
- bodies made from soft iron–the *shielding* and/or the *focusers* for concentrating magnetic field into a chosen operational space usually of a nonferromagnetic nature, where a strong homogeneous field is required.

2. Mathematical model of magnetic circuit and its computation

A mathematical model of the magnetic circuit shown in Fig. 3a is put down in a planar and orthogonal $(x,\, y)$ coordinate system. The model consists of

- definition region,
- differential equations relevant to the region considered,
- boundary conditions satisfying definiteness of the solution.

2.1. Definition regions. *The topological arrangement* of the magnetic circuit is composed of three subregions Ω_{1-3} that are confined by a fictive boundary Γ_∞, for modelling the conditions in the infinite. The position of the fictitious boundary Γ_∞ was found expediently by iteration, to enable defining the mean value of magnetic induction in the air gap, preserving three digits data validity. The subregions are as follows:

Ω_1 ... permanent magnets **M1**, **M8**, with marked orientation of the magnetization towards the coordinate system,

Ω_2 ... focusers **F1**, **F2** and ferromagnetic shielding **FS**,

Ω_3 ... operational space **C** and surrounding air **A**.

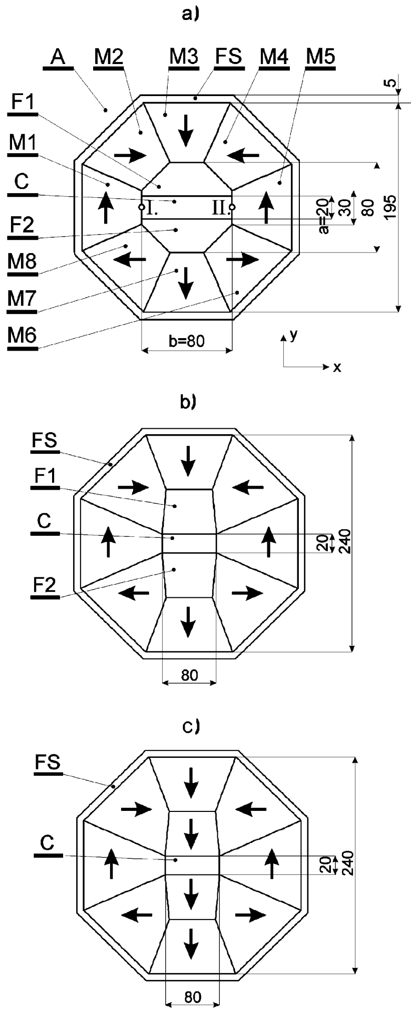

Fig. 3. Arrangement of magnetic circuits: a) alternative A, b) alternative B, c) alternative C, **M1**, **M8** permanent magnets, **F1**, **F2** ferromagnetic focusers of magnetic field, **C** operational space, **FS** ferromagnetic screening, **A** surrounding air.

2.2. Differential equation. The magnetostatic field is characterized for instance by [13]

$$\Omega_1 : \qquad \nabla \times \left(\frac{1}{\mu} \nabla \times \boldsymbol{A} - \boldsymbol{H}c \right) = 0 \tag{1}$$

$$\Omega_2 : \qquad \nabla \times \left(\frac{1}{\mu} \nabla \times \boldsymbol{A} \right) = 0 \tag{2}$$

$$\Omega_3 : \qquad \nabla \times \nabla \times \boldsymbol{A} = 0 \tag{3}$$

Vector potential $\boldsymbol{A}$ possess a nonzero component only in the direction $\boldsymbol{z}_0$, so that equations (1) to (3) may be put down in the form

$$\boldsymbol{A}(x,y) = \boldsymbol{x}_0 0 + \boldsymbol{y}_0 0 + \boldsymbol{z}_0 Az(x,y) \ .$$

The term vector $\boldsymbol{B} = \operatorname{rot} \boldsymbol{A}$ can be expressed in the form

$$\boldsymbol{B}(x,y) = \boldsymbol{x}_0 B_x(x,y) + \boldsymbol{y}_0 B_y(x,y) + \boldsymbol{z}_0 0 \,.$$

2.3. Boundary conditions. The fictive boundary Γ_∞ is characterized by

$$\boldsymbol{A}(x,y) = 0 \,. \tag{4}$$

2.4. Computer model and its accuracy of solution. The mathematical model was solved by the finite element method, using the professional MKP program *Quick Field* [14]. Observing the convergence of solution, as well as a desirable position of the fictive boundary Γ_∞, it was found out that the accuracy of three digits data validity by computing the mean value of magnetic induction in the operational space **C** necessitates using the net with 64 up to 80 nodal points respectively, depending on whether magnetic screening **FS** has been omitted or taken into consideration.

3. Illustrative example

3.1. Setting of the problem. Three alternative magnetic circuits with permanent magnets shown in Fig. 3 are considered. Each of the alternatives is considered

- without a screening casing—modification *a*,
- with a screening casing—modification *b*.

The masses of permanent magnets of all the above arrangements are identical. The physical properties of particular alternatives are shown in Tab. 1.

Table 1. Physical parameters of components in the magnetic circuit.

component	material	parameter	rate	dimension
permanents magnets	RECOMA 28 [15]	coercitive force $\boldsymbol{H}c$ remanence $\boldsymbol{B}_r$ permeability μ_r	720 1070 1.05	kA/m mT –
focusators of shielding	steel 12 040	characteristic $B(H)$	Fig. 4	

The aims of the study are

- to determine which of the magnetic circuits can in its operational space **C** induce the most effective and homogeneous induction,

- to compare this induction with the induction between two permanent magnets of the same mass and in the same operational space.

The physical parameters of all the constituents of the magnetic circuits are presented in Tab. 1., including i) the magnetization characteristics $B(H)$ of focusers **F1**, **F2** and ii) relevant screening **FS** (Fig. 4.)

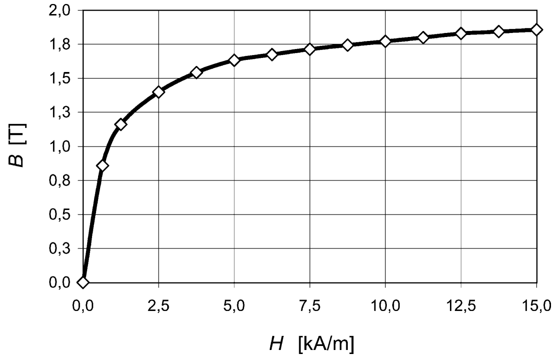

Fig. 4. Magnetic characteristic $B(H)$ of steel 12 040.

3.2. Results and their discussion. The obtained results allow to draw the following conclusions.

- Without applying the shielding **FS** (see e.g. circuit A, modification *a* in Fig. 5a), the magnetic circuit develops a certain dispersion of magnetic field into the surroundings. The dispersion can be eliminated by a shielding cover (Fig. 5b, circuit A, modification *b*).
- Shielding **FS** and focusators **F1**, **F2** have the following influence on the magnetic field:
- Shielding **FS** intensifies the magnetic induction in operational space **C** in all cases (compare Figs. 6a, 6b), however, its influence on the homogeneity of the magnetic field is insignificant (Figs. 7a, 7b).
- The presence of focusators **F1**, **F2** intensify favourably the magnitude of magnetic field in the operational space in both the two modifications (compare circuits B, C, in Figs. 6a, 6b). It is evident that:
- The comparison of the suggested magnetic circuits with a simple arrangement of a pair of permanent magnets **M1**, **M2** of identical mass (see Fig. 8) indicates that the simple arrangement exhibits an impressive decrease of both the magnetic field intensity and homogeneity in the operation space **C** (Fig. 9, 10 and 11) in comparison with the magnetic circuits shown in Fig. 3 a,b,c.

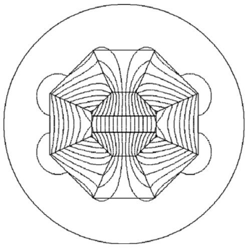

Fig. 5a. Lines of force of magnetic field, (magnetic circuit of type A—without screening cover).

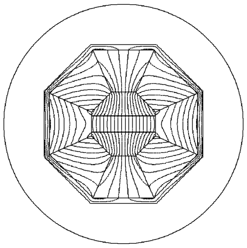

Fig. 5b. Lines of force of magnetic field, (magnetic circuit of type A—with screening cover).

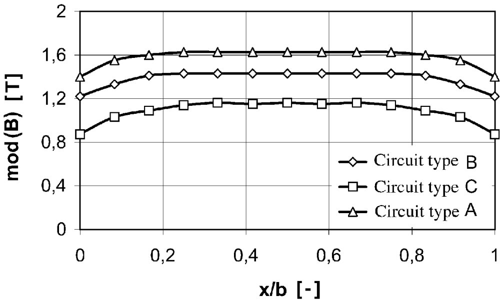

Fig. 6a. Homogeneity of magnetic field in working chamber, (magnetic circuit of type A—without screening cover).

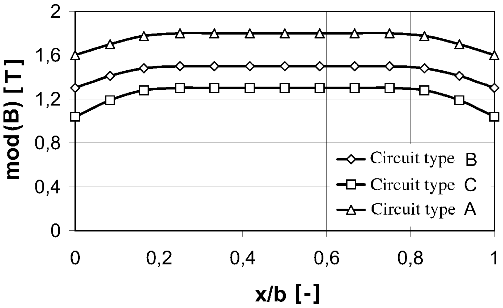

Fig. 6b. Homogeneity of magnetic field in working chamber, (magnetic circuit of type A—with screening cover).

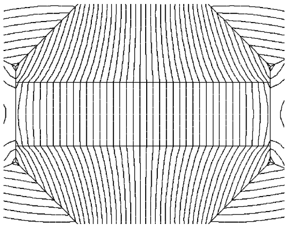

Fig. 7a. Course of magnetic induction mod($\boldsymbol{B}$) in working chamber along trajectory $\overline{\text{I. II.}}$ in Fig. 3A, (magnetic circuit of type A—without screening cover).

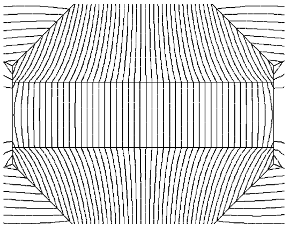

Fig. 7b. Course of magnetic induction mod($\boldsymbol{B}$) in working chamber along trajectory $\overline{\text{I. II.}}$ in Fig. 3A, (magnetic circuit of type A—with screening cover).

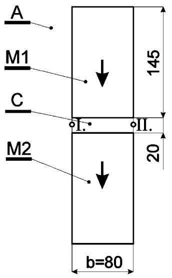

Fig. 8. Elementary control arrangement of a pair of permanent magnets, **M1**, **M2** equivalent permanent magnets, **C** operational space, **A** surrounding air.

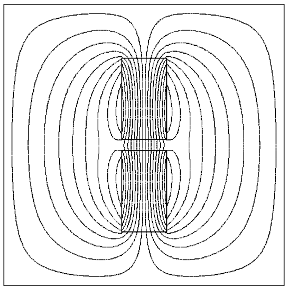

Fig. 9. Lines of force of magnetic field of control arrangement of magnets (in Fig. 8).

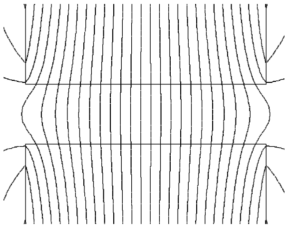

Fig. 10. Homogeneity of magnetic field in working chamber of control arrangement of magnets (see Fig. 8).

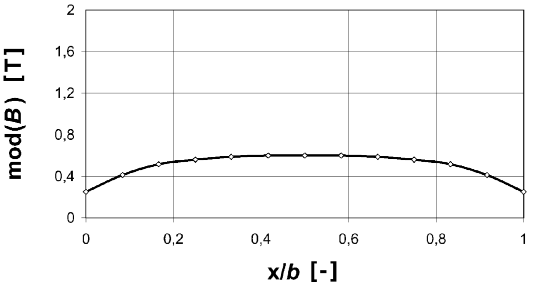

Fig. 11. Course of magnetic induction mod($\boldsymbol{B}$) in working chamber of control arrangement of magnets along trajectory I. II (see Fig. 8).

4. Conclusion

The paper is an introductory study to the problems of synthesis of magnetic circuits with permanent magnets. From a general point of view it is possible to design a magnetic circuit expediently, to endow the operational space with a strong and homogeneous magnetic field exceeding the capacity of a simple magnetic pair of the same mass. From the concepts under study, it is the magnetic circuit with the shielding shown in Fig. 3c that proved to have optimal properties.

Prospective progress in this field is expected in developing the theory of topological structures with permanent magnets. By analogy to the theory of electric circuits, magnetic circuits develop i) intrinsic *topological structure* characterizing mutual adjustment ('linkage') of the magnetic circuit elements, and ii) *physical structure* defining the physical properties of the elements. However, the situation is here much more complicated: While the topological structure of electric circuits is one-dimensional, which means that it can be completely described by an orientated graph (i.e. by 1D simplex that can be displayed algebraically by an incidental matrix), the topological structure of magnetic circuits is two- or three-dimensional, i.e. it depends on the geometrical design of the matrix elements. The dimension of the structure is here also relevant for setting the orientation of the matrix elements, expressing the direction of magnetization of permanent magnets. Hence, the topology of magnetic circuits is represented by much more complicated structure demanding a mathematical treatment. The model developed in this way is supposed to be applied to solving concrete problems, for instance for the analysis or synthesis of a magnetic circuit, including its possible optimalization.

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