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SHORT-CIRCUIT STRESS IN LARGE POWER AIR CORE REACTORS

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Abstract: *This paper report on a modeling of three Phase AC Reactors by means of an Inductance-Substitution Circuit. This model is helpful in analyzing of the behavior of real reactors in distribution system and applications. Short-circuit current and short-circuit forces between the three adjacent coils of reactor in case of a line-to-ground and line-to-line fault was computed and presented. The model of the reactor is based on the system of differential equations and makes a computation of transient currents and forces possible. The method of virtual work to determine short-circuit forces was used.*

Key words: *Series Three Phase AC Reactor, Short-circuit Currents, Short-circuit Forces*

1 INTRODUCTION

Current limiting reactors are mainly used to limit short-circuit current on the load side of the reactor to prevent fault currents from exceeding to values dangerous for the equipment Reactors are connected in series with the line or power supply and consist of three phase coils generally placed on each other with support insulators between them.

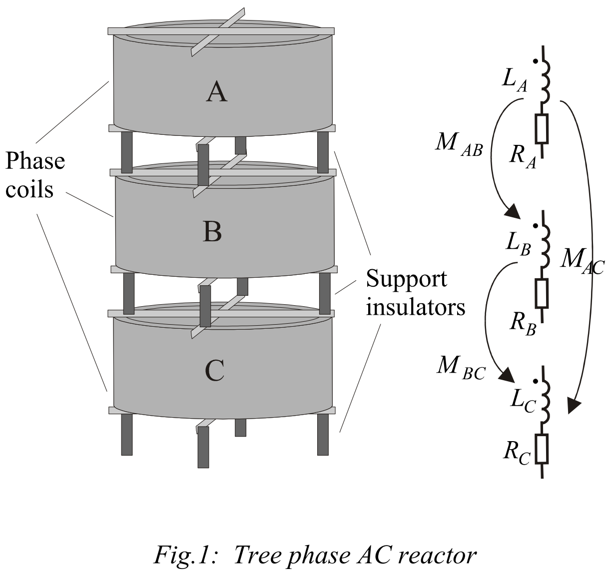


Fig.1: Tree phase AC reactor

The support insulators provide sufficient insulation clearance between the phases and they make enough space between the phases available to ensure, that the mutual inductance between the coils is negligible compared to the main inductance. Air core, dry type construction with fiberglass isolated and epoxy impregnated windings is frequently used. The winding consist often aluminum conductors with welded current carrying aluminum profile terminations.

2 SHORT-CIRCUIT CURRENT IN REACTOR

In a reactor, whose input side is fed from the supply network and output side is short-circuited, the short-circuit currents and electromagnetic forces occur. Short-circuit current may be considered as the sum of a steady-state and a transient component. Steady-state current is sinusoidal and it has constant amplitude. Transient component is aperiodic, depended on the instant of the fault and it relatively rapid decay.

Magnitude of short-circuit current are depended first on the type and duration of the short-circuit. In this work, two substitution circuits are presented. One is designed for solidly grounded neutral systems and the other for isolated neutral system. Both circuits are described with a system of linear ordinary differential equations and they contain the load resistances and inductances as well as self and mutual inductances and resistances of the reactor.

The substitution circuits makes the computing of a three-phase-, phase-to-phase-,phase-to-earth- and phase-to-phase-to-earth short- circuit possible. The substitution circuit suitable for use in system with solidly grounded neutral look like the figure below:

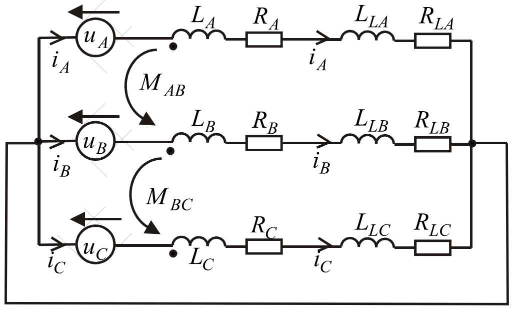


Fig.2: Substitution circuit for system with solidly grounded neutral

System of differential equations according to the Fig.2 is given as follows:

$$\left(R_A + R_{LA}\right) i_A + \left(L_A + L_{LA}\right)\frac{\mathrm{d}i_A}{\mathrm{d}t} + M_{AB}\frac{\mathrm{d}i_B}{\mathrm{d}t} + \\ + M_{AC}\frac{\mathrm{d}i_C}{\mathrm{d}t} = U_m \sin(\omega t - \varphi_0) \tag{1}$$

$$\left(R_B + R_{LB}\right) i_B + \left(L_B + L_{LB}\right)\frac{\mathrm{d}i_B}{\mathrm{d}t} + M_{AB}\frac{\mathrm{d}i_A}{\mathrm{d}t} + \\ + M_{AC}\frac{\mathrm{d}i_C}{\mathrm{d}t} = U_m \sin(\omega t - \varphi_0 - \frac{2}{3}\pi) \tag{2}$$

$$\left(R_C + R_{LC}\right) i_C + \left(L_C + L_{LC}\right)\frac{\mathrm{d}i_C}{\mathrm{d}t} + M_{AC}\frac{\mathrm{d}i_A}{\mathrm{d}t} + \\ + M_{BC}\frac{\mathrm{d}i_B}{\mathrm{d}t} = U_m \sin(\omega t - \varphi_0 - \frac{4}{3}\pi) \tag{3}$$

M_{AB}, M_{AC}, M_{BC}, L_A, L_B, L_C, R_A, R_B, R_C	Self and mutual inductances and resistances of winding of the reactor
I_A, I_B, I_C	Currents in the phase-coils of the reactor
R_{LA}, R_{LB}, R_{LC}, L_{LA}, L_{LB}, L_{LC}	Resistances and inductances of the load
φ_0	The angle defining instant of short-circuit

The substitution circuit suitable for use in system with isolated neutral look like this on the Fig.3:

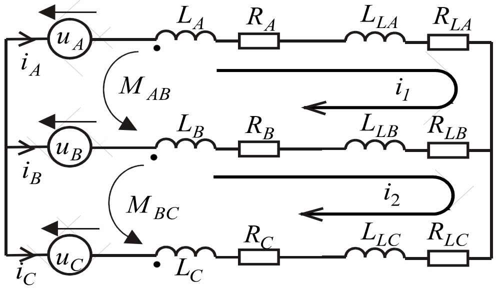


Fig.3: Substitution circuit for system with isolated neutral

Using the mesh current method, system of differential equations is obtained as follows:

$$\left(R_A + R_{LA}\right) i_1 + \left(L_A + L_{LA}\right)\frac{\mathrm{d}i_1}{\mathrm{d}t} - \\ - M_{AB}\left(\frac{\mathrm{d}i_1}{\mathrm{d}t} + \frac{\mathrm{d}i_2}{\mathrm{d}t}\right) + M_{AC}\frac{\mathrm{d}i_2}{\mathrm{d}t} + \\ + \left(R_B + R_{LB}\right)\left(i_1 + i_2\right) + \left(L_B + L_{LB}\right)\left(\frac{\mathrm{d}i_1}{\mathrm{d}t} + \frac{\mathrm{d}i_2}{\mathrm{d}t}\right) - \\ - M_{AB}\frac{\mathrm{d}i_1}{\mathrm{d}t} - M_{BC}\frac{\mathrm{d}i_2}{\mathrm{d}t} + \\ + U_m \sin(\omega t - \varphi_0 - \frac{2}{3}\pi) - U_m \sin(\omega t - \varphi_0) = 0 \tag{4}$$

$$\left(R_C + R_{LC}\right) i_2 + \left(L_C + L_{LC}\right)\frac{\mathrm{d}i_2}{\mathrm{d}t} + \\ - M_{BC}\left(\frac{\mathrm{d}i_1}{\mathrm{d}t} + \frac{\mathrm{d}i_2}{\mathrm{d}t}\right) + M_{AC}\frac{\mathrm{d}i_1}{\mathrm{d}t} + \\ + \left(R_B + R_{LB}\right)\left(i_1 + i_2\right) + \left(L_B + L_{LB}\right)\left(\frac{\mathrm{d}i_1}{\mathrm{d}t} + \frac{\mathrm{d}i_2}{\mathrm{d}t}\right) - \\ - M_{AB}\frac{\mathrm{d}i_1}{\mathrm{d}t} - M_{BC}\frac{\mathrm{d}i_2}{\mathrm{d}t} + \\ + U_m \sin(\omega t - \varphi_0 - \frac{2}{3}\pi) - U_m \sin(\omega t - \varphi_0 - \frac{4}{3}\pi) = 0 \tag{5}$$

$$i_A = i_1 \qquad i_B = i_2 - i_1 \qquad i_C = -i_2 \tag{6}$$

3 Electromagnetic forces acting on the windings of the reactor

The electromagnetic forces are proportional to the square of short-circuit current and they can be categorized as internal or external forces. Biggest short-circuit forces occur, when a current corresponding to the first peak of short circuit flows in the windings.

The forces can be evaluated by means of virtual work principle like a differentiation of energy with respect to the direction of acting force:

$$\left|F_x\right| = \frac{\partial W_m}{\partial x} \tag{7}$$

In this case, the total magnetic field energy of reactor is given by:

$$W_m = \frac{1}{2}L_A I_A{}^2 + \frac{1}{2}L_B I_B{}^2 + \frac{1}{2}L_C I_C{}^2 + M_{AB} I_A I_B + \\ + M_{AC} I_A I_C + M_{BC} I_B I_C \tag{8}$$

Internal forces (Fig.4) act in each one phase coil. The winding of the reactor can be regarded as a tube expanded from the inside by the radial force and compressed from the outside by the axial forces. Axial and radial forces should not cause dangerous deformation in the winding. The force acting on the observed part (e.g. on one turn of the middle phase-coil of the winding) is subject of computing in this case. According to virtual work principle, the force is given by the formula:

$$F_x = I_A I_B \frac{\partial M_{AX}}{\partial x} + I_B{}^2 \frac{\partial M_{BX}}{\partial x} + I_C I_B \frac{\partial M_{CX}}{\partial x} \qquad (9)$$

I_i Currents in the phase-coils of the reactor

M_{iX} Mutual inductances between the phase-coils and one turn in the observed part of the winding

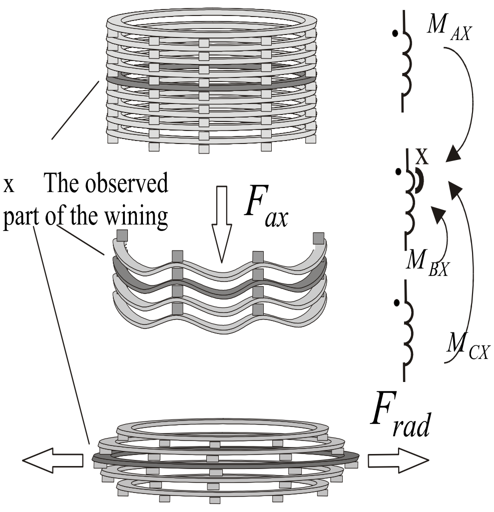


Fig.4: Axial and radial forces acting on the winding

External forces act between three adjacent phase-coils (Fig.5). In this case, the insulators and supporting structure of the reactor are subjected to tensile or compression stress. The sliding forces should not cause damage or destruction of the insulators. Subject of computing is the force acting on the whole phase coil of reactor. Total external force acting on the phase A by other phases is given by formula

$$F_A = I_A I_B \frac{\partial M_{AB}}{\partial x} + I_C I_C \frac{\partial M_{AC}}{\partial x} \qquad (10)$$

In the same way, total external force acting on phase B and C is given by formulas

$$F_B = I_A I_B \frac{\partial M_{AB}}{\partial x} + I_B I_C \frac{\partial M_{BC}}{\partial x} \qquad (11)$$

$$F_C = I_A I_C \frac{\partial M_{AC}}{\partial x} + I_B I_C \frac{\partial M_{BC}}{\partial x} \qquad (12)$$

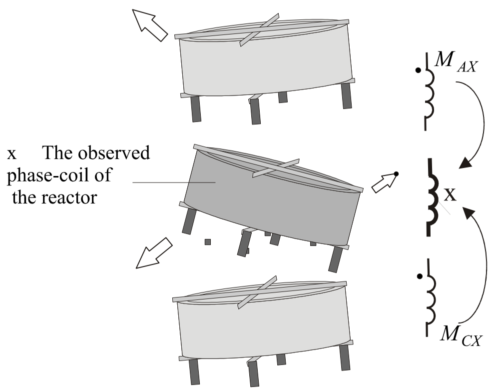


Fig.5: External forces between adjacent phase-coils

4 MUTUAL INDUCTANCES

The mutual inductances needed for evaluation of the forces can be simply obtained under assumption of the thin coaxial solenoid coils. In this case, mutual inductance of such two-coils according to Fig.6 is given by formula:

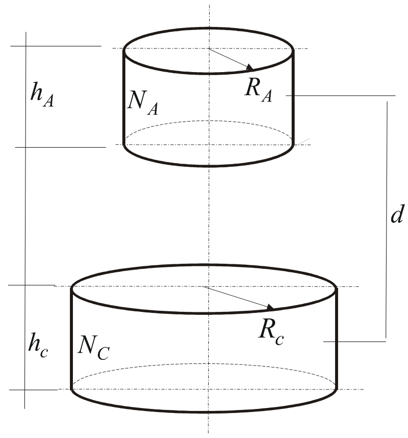


Fig.6: Mutual inductance of two cylindrical coils

$$M_{A,c} = \frac{4}{3}\mu_0 (R_A R_c)^{\frac{3}{2}} n_A n_c \sum_{i=1}^{4} (-1)^i Z(\beta, k_i) \qquad (13)$$

In this formula: n_A, n_C is number of turns of the coils per unit length:

$$n_A = \frac{N_A}{h_A} \quad n_c = \frac{N_C}{h_C} \qquad (14)$$

$Z(\beta, k)$ is special function given by formula:

$$Z(\beta,k) = \frac{1}{k}\left[\frac{1-k^2}{k^2}(K(k)-E(k)) + \frac{3\beta-1}{2}E(k) + \right.$$

$$\left. + \frac{3}{2}(1+\beta)(1-k^2)\Pi\left(\frac{2-(1+\beta)k^2}{\beta-1}, k\right)\right] \tag{15}$$

where β is :

$$\beta = \frac{R_A{}^2 + R_c{}^2}{2R_A R_c} \tag{16}$$

$E(k)$ in this formula is the complete elliptic integral of the first kind of the modulus k:

$$E(k) = \int_0^{\frac{\pi}{2}} \sqrt{1-k^2\sin^2\varphi}\,\mathrm{d}\varphi \tag{17}$$

$K(k)$ is the complete elliptic integral of the second kind of the modulus k:

$$K(k) = \int_0^{\frac{\pi}{2}} \frac{\mathrm{d}\varphi}{\sqrt{1-k^2\sin^2\varphi}} \tag{18}$$

$\Pi(\rho,k)$ is the complete elliptic integral of the third kind of the modulus k:

$$\Pi(\rho,k) = \int_0^{\frac{\pi}{2}} \frac{\mathrm{d}\varphi}{\left(1+\rho\sin^2\varphi\right)\sqrt{1-k^2\sin^2\varphi}} \tag{19}$$

The modulus k is given by formula

$$k_i = \frac{4R_A R_c}{\left(R_A + R_c\right)^2 + \alpha_i{}^2} \tag{20}$$

$$\alpha_1 = d + \frac{h_A}{2} + \frac{h_c}{2} \qquad \alpha_2 = d + \frac{h_A}{2} - \frac{h_c}{2}$$
$$\alpha_3 = d - \frac{h_A}{2} + \frac{h_c}{2} \qquad \alpha_4 = d - \frac{h_A}{2} - \frac{h_c}{2} \tag{21}$$

Under consideration of real coil thickness, more accurately value for mutual inductance is obtained by integration of formula (13).

5 NUMERICAL METODS FOR COMPUTING SOLUTION

For the evaluation of the elliptic integrals, a special functions icluded in MATLAB was used.

For the solution of ordinary differential equations system (initial value problem) the Matlab's built-in ODEs method was used. All in the MATLAB implemented solvers solve systems of equations in the form

$$y' = f(t,y)$$

. To the converting of differential equations system in this form, Symbolic Math Toolbox in MATLAB was used.

6 EXAMPLE OF EVALUATION

In the following text, an Example of computation of short-circuit current and the forces in the reactor is presented. Dimensions and technical parameters of the reactor are put on the Fig.6. The reactor is insulated for the nominal system voltage 6kV and designed to provide continuously output up to 250 kVA. Three-phase active load 250 kW is supposed in this example (phase-load-resistance 144Ω).

Computed mutual inductance between two phases is:

$$M_{AB} = M_{BC} = 13.4\ mH$$

$$M_{AC} = 3.8\ mH$$

Computed resistance of coils is:

$$R_A = R_B = R_C = 2\,\Omega$$

Computed differential coefficient of mutual inductance is:

$$\frac{\partial M_{AB}}{\partial x} = \frac{\partial M_{BC}}{\partial x} = 4.56\cdot 10^{-2}\ H/m$$

$$\frac{\partial M_{AC}}{\partial x} = 8.82\cdot 10^{-3}\ H/m$$

Differential coefficient of mutual inductance between one turn in the middle of phase-A and phase-A in radial direction, needed for radial force evaluation is:

$$\frac{\partial M_{AX}}{\partial r} = 4.66\cdot 10^{-5}\ H/m$$

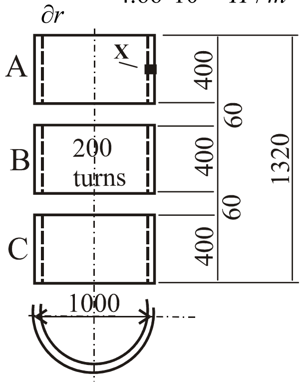


Fig.6: Three-phase air core reactor

Rated phase current at nominal load and voltage is given by (rms)

$$I_n = \frac{S_n}{\sqrt{3}U_n} = \frac{250\cdot 10^3}{\sqrt{3}\cdot 6000} = 24\ A$$

Peak value of the rated current is given:

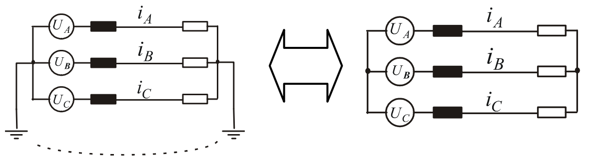
$$I_{n\_\max} = \sqrt{2}I_n = 34\ A$$

To the nominal magnitude of current correspond nominal forces. All the undermentioned magnitudes of the currents and forces are related to this current values.

By the standard fault-free operation, the system with solidly grounded neutral and system with isolated neutral is identical. Magnitudes of currents and forces are of nominal value, in this case. Force- and current-time response is demonstrated on the figure (7,8,9).

The instant of short-circuit is assumed at zero voltage of phase A.

6.1 Standard fault-free operation



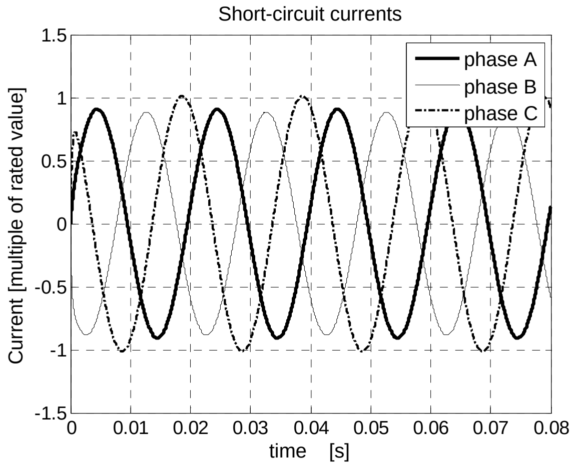


Fig.7 Currents (standard fault-free operation)

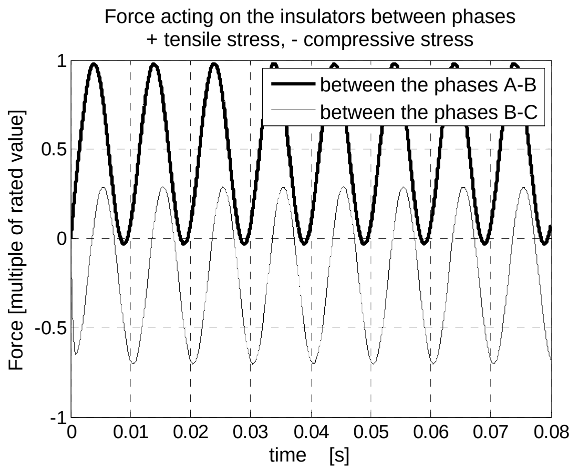


Fig.8 Forces (standard fault-free operation)

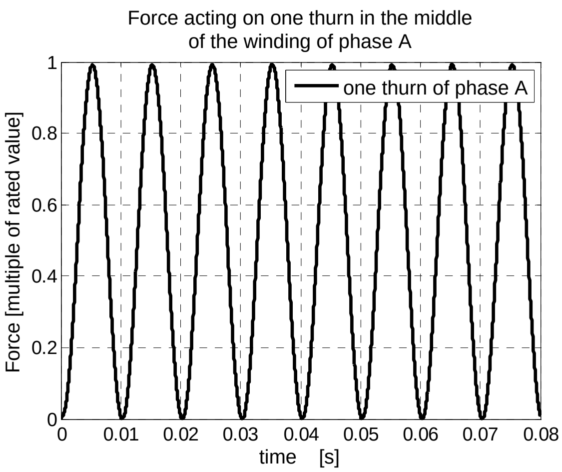
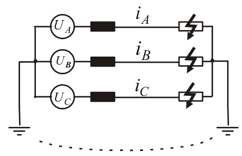


Fig.9 Force on one turn (standard fault-free operation)

6.2 Three-phase-to-earth-fault (solidly grounded neutral)



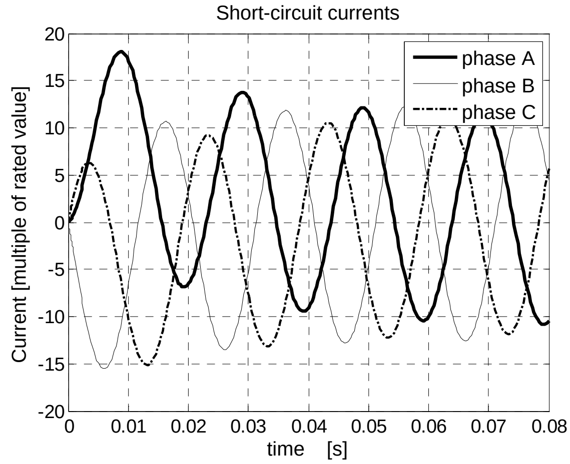


Fig.10 Currents (Three-phase-to-earth-fault)

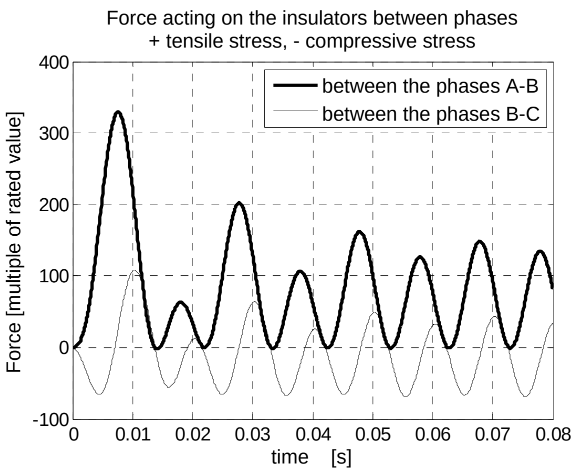


Fig.11 Forces (Three-phase-to-earth-fault)

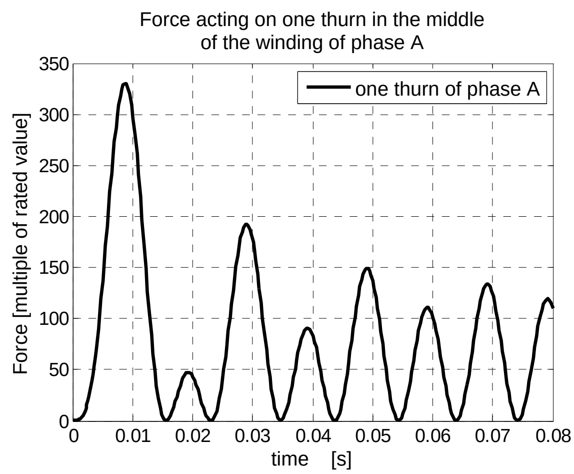
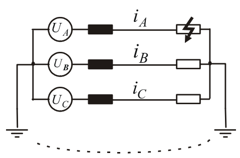


Fig.12 Force on one turn (Three-phase-to-earth-fault)

6.3 Single-phase-to-earth-fault (solidly grounded neutral)



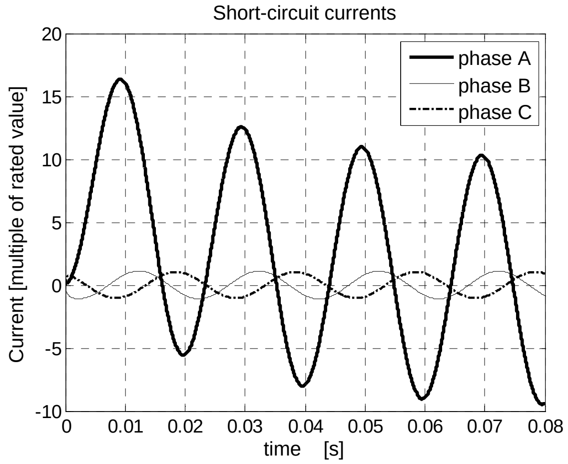


Fig.13 Currents (Single-phase-to-earth-fault)

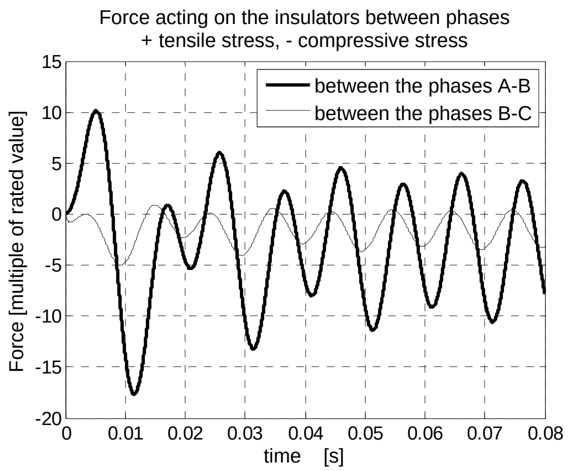


Fig.14 Forces (Single-phase-to-earth-fault)

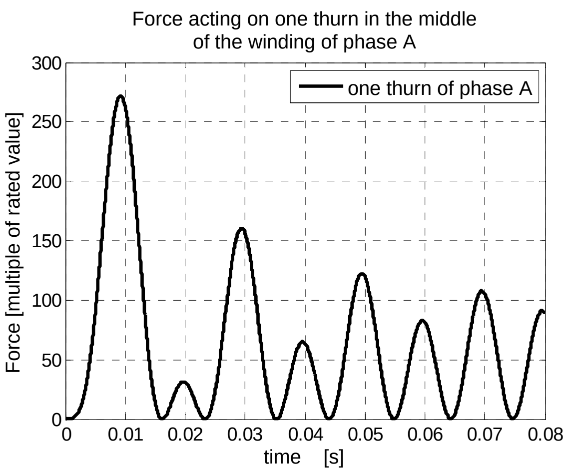
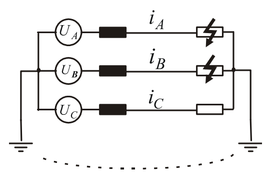


Fig.15 Force on one turn (Single-phase-to-earth-fault)

6.4 Two-phase-to-earth-fault (solidly grounded neutral)



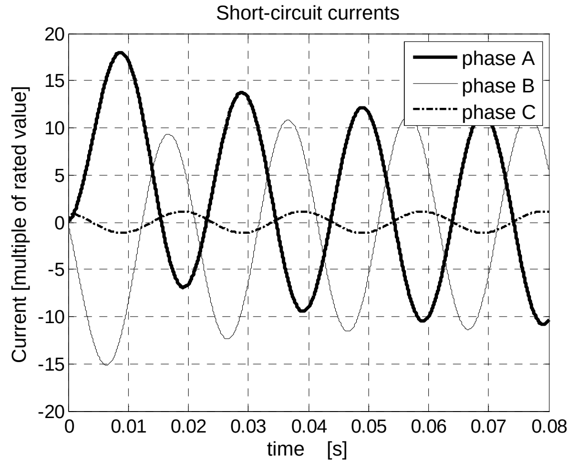


Fig.16 Currents (Two-phase-to-earth-fault)

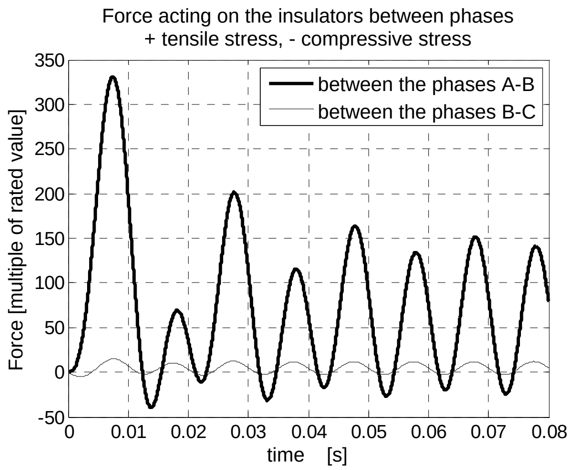


Fig.17 Forces (Two-phase-to-earth-fault)

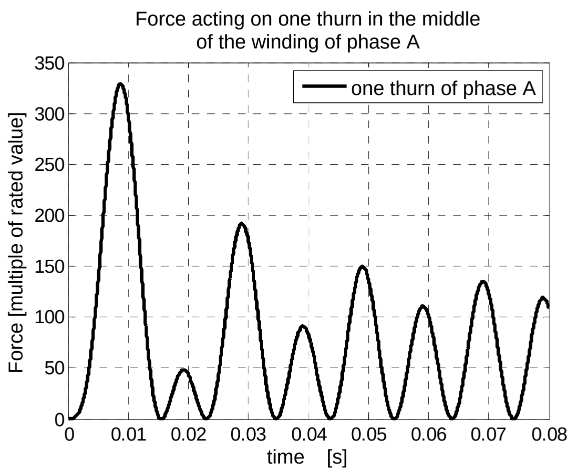
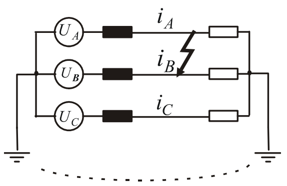


Fig.18 Force on one turn (Two-phase-to-earth-fault)

6.5 Phase-to-phase-fault (solidly grounded neutral)



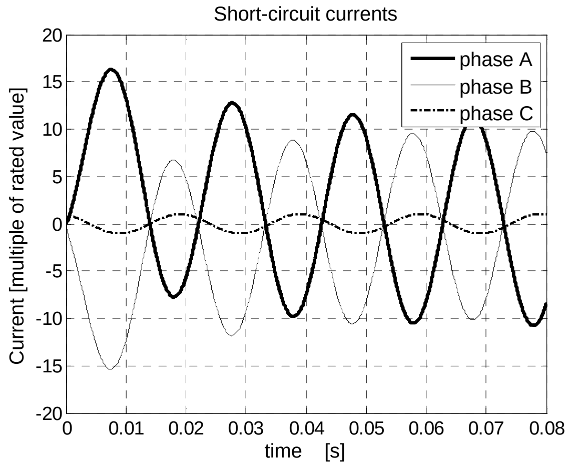


Fig.19 Currents (Phase-to-phase-fault)

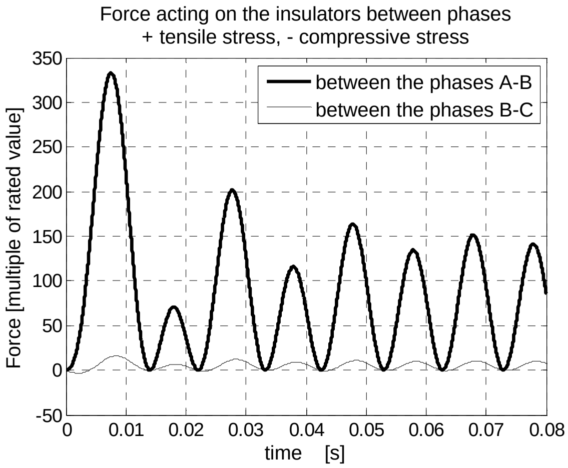


Fig.20 Forces (Phase-to-phase-fault)

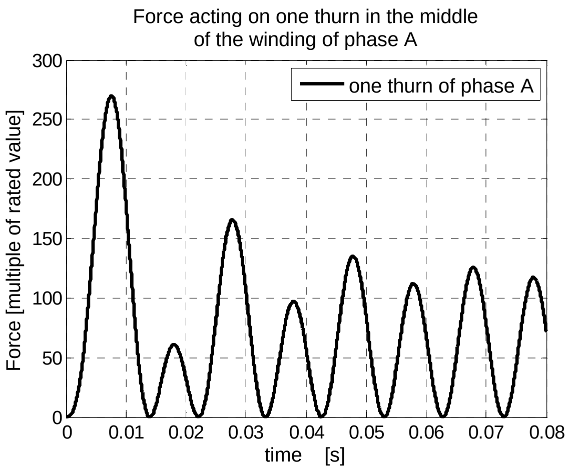
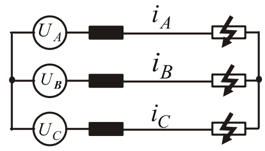


Fig.21 Force on one turn (Phase-to-phase-fault)

6.6 Three-phase-to-neutral fault (isolated neutral)



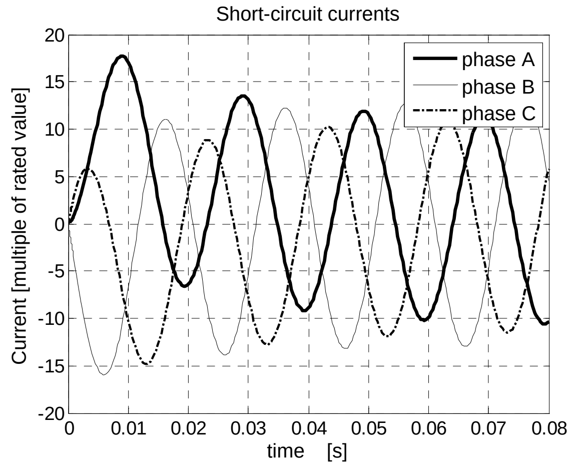


Fig.22 Currents (Three-phase-to-neutral fault)

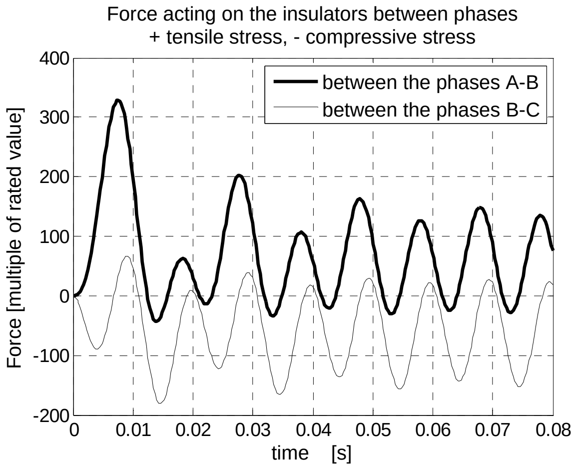


Fig.23 Forces (Three-phase-to-neutral fault)

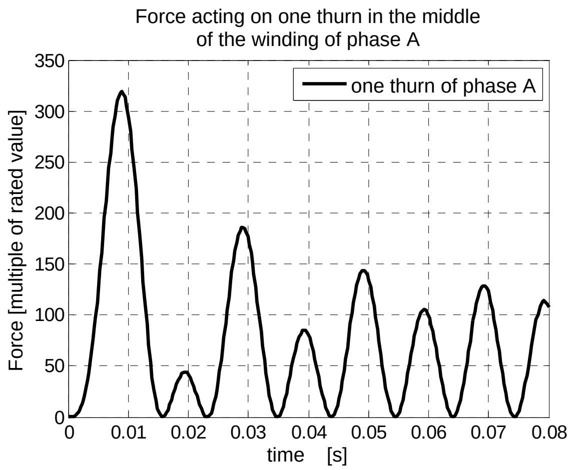
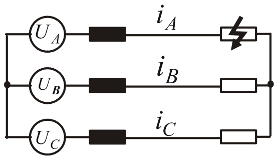


Fig.24 Force on one turn (Three-phase-to-neutral fault)

6.7 Single-phase-to-neutral fault (isolated neutral)



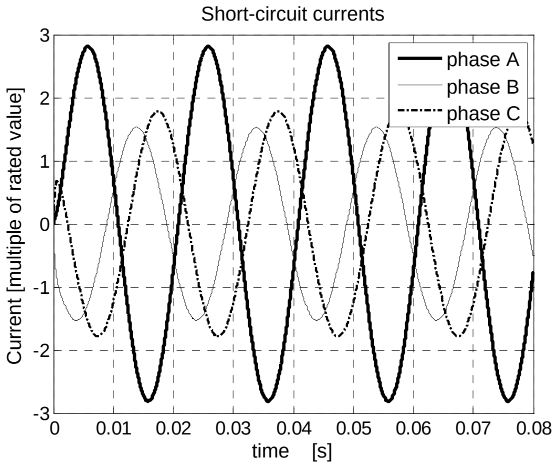


Fig.25 Currents (Single-phase-to-neutral fault)

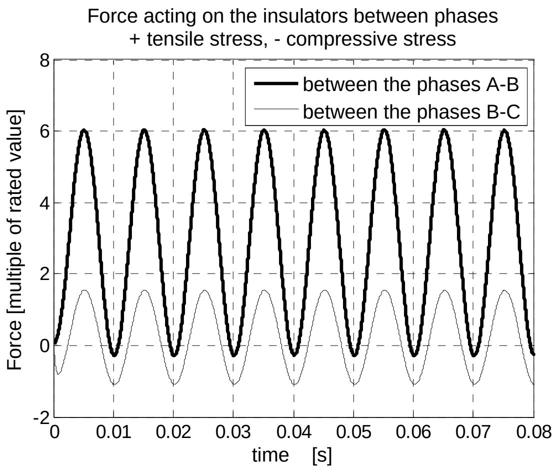


Fig.26 Forces (Single-phase-to-neutral fault)

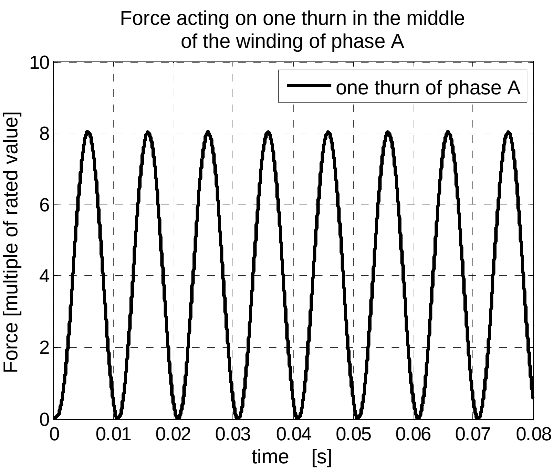
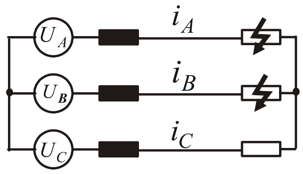


Fig.27 Force on one turn (Single-phase-to-neutral fault)

6.8 Two-phase-to-neutral fault (isolated neutral)



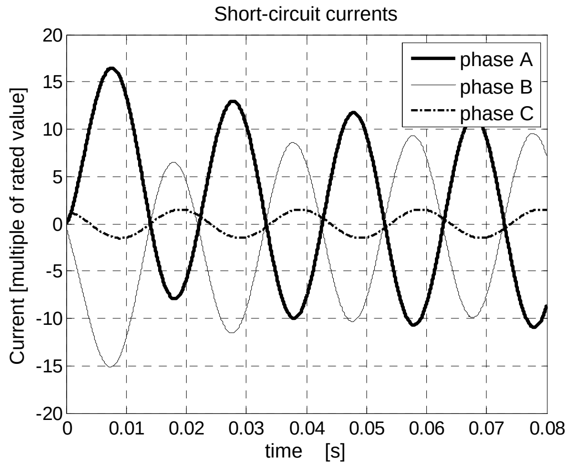


Fig.28 Currents (Two-phase-to-neutral fault)

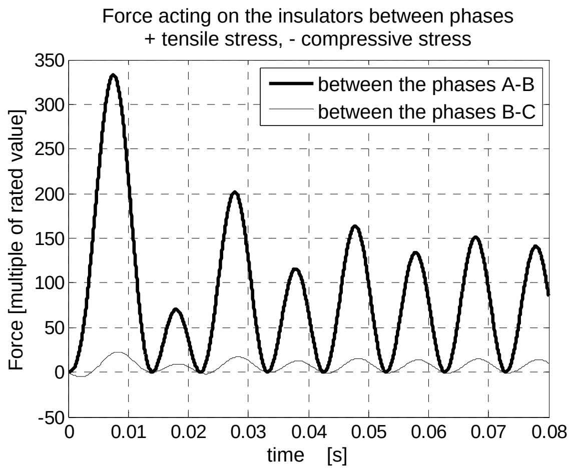


Fig.29 Currents (Two-phase-to-neutral fault)

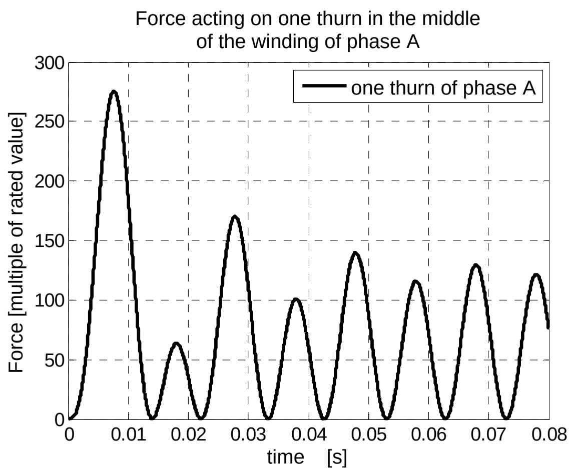
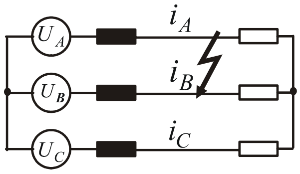


Fig.30 Force on one turn (Two-phase-to-neutral fault)

6.9 PHASE-TO-PHASE FAULT (ISOLATED NEUTRAL)



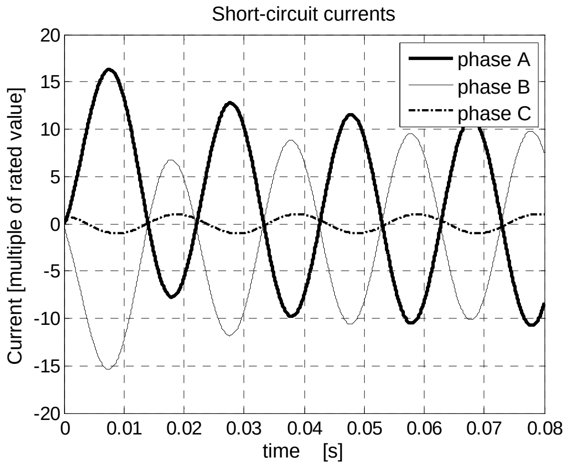


Fig.31 Currents (Phase-to-phase fault)

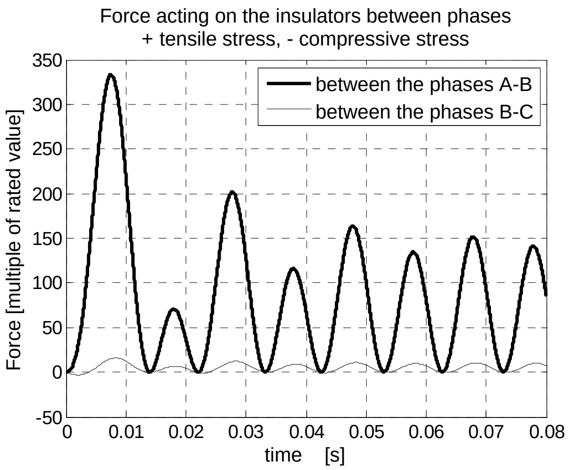


Fig.32 Forces (Phase-to-phase fault)

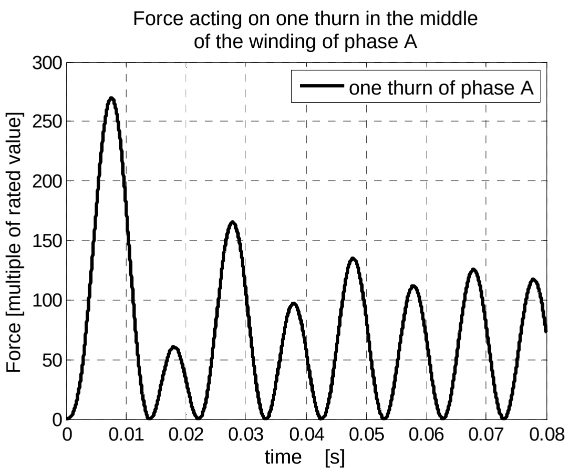


Fig.33 Force on one turn (Phase-to-phase fault)

Short-circuit currents and Forces System with isolated / grounded neutral				
Service condi-tion	Peak value of current	steady -state current	Peak value of Forces A-B	Peak value of Forces B-C
Rated Load	1 1	1 1	1 1	1 1
Three-phase-to-neutral fault	18 17.7	12 11.9	327 329	179 181
Single-phase-to-neutral fault	16.4 2.8	10.3 2.8	17.7 6.1	5 1.6
Two-phase-to-neutral fault	18 16.5	11.5 11.3	332 334	15 22.7
Phase-to-phase fault	16.3 16.3	11.1 11.1	333 333	16 15.8

Tab.1 Short-circuit currents and Forces (aggregate table)

7 REFERENCES

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