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MATHEMATICAL MODELING OF ELECTRICAL TRANSMISSION SYSTEMS WITH SYNCHRONOUS NON-SALIENT POLE MACHINES USING MATLAB/SIMULINK

PROF. PETRO STAKHIV, D.SC.
OKSANA HOHOLYUK, PH.D.

Abstract: *In the paper problem of electromagnetic transient processes simulation for power transmission research is considered. Mathematical model of AC power transmission block "power source-power transformer-transmission-line-power autotransformer-load" is presented. The problem of mathematical modeling of electric power systems with synchronous non-salient poles generators is considered. Expediency of electric power system elements creation and their adaptation to MATLAB/Simulink environment on the basis of mathematical models and macromodels is shown.*

Key words: *Mathematical modeling, electric power systems, MATLAB/Simulink, macromodel*

INTRODUCTION

Research and analysis of electric systems transients are important stages during their development and exploitation. Electric systems belong to complex dynamic systems which contain great amount of elements with stiff interconnections. Investigation of electric system in complex demands usage of adequate mathematical models of their elements, development methods and computer tools for practical implementation – programs for mathematical modeling and simulation [1, 3, 6].

Among programs intended for electric systems transients simulation the following ones can be mentioned: EMTP-type programs (ATP, NETOMAC, PSCAD), and MATLAB /Simulink [1, 2]. During electric power systems modeling when high adequacy is required usage of most of programs is complicated because models built into their libraries frequently are simplified and do not take into account configuration, parameters and nonlinear characteristics of system elements. Due to expanded toolboxes with libraries of matrix and special mathematic functions MATLAB/Simulink environment became a standard for technical systems modeling. Besides it MATLAB/Simulink can be effectively used for electric systems transients simulation owing to SimPowerSystems Blockset models [2, 6].

In literature various mathematical models of electric system elements can be found [1, 3]:

- power sources (synchronous and asynchronous generators of electric stations);
- power transformers and autotranformers (suitable for high- and low-frequency transients analysis) when eddy currents, hysteresis, winding mutual inductancs, leakage inductances and etc are considered;
- transmission lines (with lumped and distributed parameters and frequency-dependent parameters);
- elements of load.

In spite of considerable amount of publications an absence of researches on complex electric systems analysis, their generalized mathematic models and development methods is felt [1, 4 – 6]. Therefore the goal of presented paper is development of model for AC power transmission transients simulation in the form "power source–power transformer– transmission line–power autotransformer–load" and its implementation in MATLAB/Simulink on the basis of improved models of their elements.

1 MODELING OF ELECTRIC POWER SYSTEMS WITH NON-SALIENT POLE SYNCHRONOUS GENERATORS

Researched AC power transmission system is shown in Fig. 1. Electric transmission mathematical model is formed in coordinate basis of currents and voltages of electric circuits branches, flux linkages and magnetic branches voltages based on principles presented in [6]

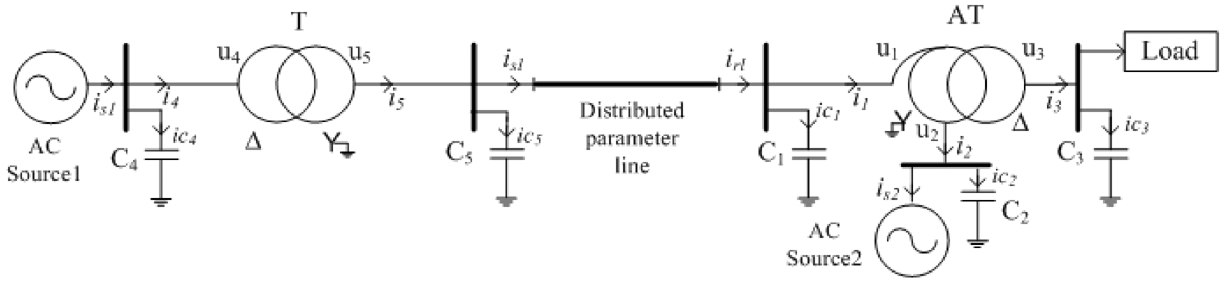

Fig.1: AC power transmission scheme

and procedure from [7] using separate models of elements.

In order to develop effective digital and mathematical models of electric power systems with high level of adequacy it is necessary to take into account parameters and nonlinear characteristics of electromechanical converters, considerable part of which consists of non-salient poles synchronous machines (turbogenerators).

Electric power system sources consist of set of synchronous and asynchronous generators of electric power stations and connections between them. During electric transmission transients analysis power sources frequently are represented in simplified way. The system which contains group of power stations connected by high and ultra-high voltage lines can be replaced by equivalent electromotive force applied before equivalent impedance. From mathematical point of view such simplification leads to reducing of differential and integral state equations order and to, as a consequence, distortion of frequency characteristics of investigated circuit. If an equivalent system has a transformer connection then simplifying of power sources does not have great influence on accuracy of transients calculation in switched line. Hence power source can be replaced by equivalent electromotive force and equivalent impedance. In other cases it is necessary to develop detailed mathematical models.

Mathematical model of synchronous generator can be written down using Park's variables. Generator state equations in *dq0* coordinates take the following form:

$$u_{sd} = R_s i_d + L_{\sigma s}\, di_d/dt + d\Psi_{\delta d}/dt - \\ - \omega_r (L_{\sigma s} i_s + \Psi_{\delta q}) \tag{1}$$

$$u_{sq} = R_s i_q + L_{\sigma s}\, di_q/dt + d\Psi_{\delta q}/dt + \\ + \omega_r (L_{\sigma s} i_s + \Psi_{\delta d}) \tag{2}$$

$$u_f = R_f + L_{\sigma f}\, di_f/dt + d\Psi_\delta/dt \tag{3}$$

$$0 = R_{\partial d} i_{\partial d} + L_{\sigma\partial d}\, di_{\partial d}/dt + d\Psi_{\delta d}/dt \tag{4}$$

$$0 = R_{\partial q} i_{\partial q} + L_{\sigma\partial q}\, di_{\partial q}/dt + d\Psi_{\delta q}/dt \tag{5}$$

$$i_d + i_{fd} + i_{\partial d} - i_m(\Psi_\delta)\,\Psi_{\delta d}/\Psi_\delta = 0 \tag{6}$$

$$i_q + i_{\partial q} - i_m(\Psi_\delta)\,\Psi_{\delta q}/\Psi_\delta = 0 \tag{7}$$

$$\Psi_\delta = \sqrt{\Psi_{\delta d}^2 + \Psi_{\delta q}^2} \tag{8}$$

$$\frac{J}{p_0} d\omega_r/dt = -\frac{3}{2} p_0 (\Psi_{\delta d} i_q - \Psi_{\delta q} i_d) + M(\omega_r) \tag{9}$$

where u_{sd}, u_{sq}, i_d, i_q are projections of image vectors of stator voltage and currents on orthogonal *dq* coordinate axes; $\Psi_{\delta d}, \Psi_{\delta q}$ are projections of image vectors of flux linkages caused by main magnetic flux on these axes; $i_{\partial d}, i_{\partial q}$ are currents of equivalent longitudinal and lateral damping circuits reduced to stator winding; u_f, i_f are voltage and current excitation winding; i_m, Ψ_δ are magnetization current and main flux linkage; $R_s, R_f, R_{\partial d}, R_{\partial q}$, $L_{\sigma s}, L_{\sigma f}, L_{\sigma\partial d}, L_{\sigma\partial d}$ are resistances and leakage inductances of stator and rotor circuits; p_0, J is a number of poles and rotor inertia constant; M is turbine moment; ω_r is angular speed of rotor rotation reduced to pole region.

Mathematical models of electric power systems should be formed in phase physical coordinates on the basis of algebraic and topological methods that make it impossible to use described model of synchronous generator in orthogonal *dq0* coordinates. Let us convert in equations (1), (2), (6), (7) stator voltages and currents from *dq0* coordinates to phase coordinates using Park transformation matrices. For synchronous generator direct and inverse Park transformation matrices can be written down in the following form:

$$P_{dq}(x) = \frac{2}{3}\begin{vmatrix} x_2 & (\frac{\sqrt{3}}{2}x_1 - \frac{x_2}{2}) & -(\frac{\sqrt{3}}{2}x_1 + \frac{x_2}{2}) \\ -x_1 & (\frac{x_1}{2} + \frac{\sqrt{3}}{2}x_2) & (\frac{x_1}{2} - \frac{\sqrt{3}}{2}x_2) \end{vmatrix} \tag{10}$$

$$P_{dq}^{-1}(x) = \begin{vmatrix} x_2 & -x_1 \\ (\frac{\sqrt{3}}{2}x_1 - \frac{x_2}{2}) & (\frac{x_1}{2} + \frac{\sqrt{3}}{2}x_2) \\ -(\frac{\sqrt{3}}{2}x_1 + \frac{x_2}{2}) & (\frac{x_1}{2} - \frac{\sqrt{3}}{2}x_2) \end{vmatrix}, \tag{11}$$

where $x_1 = \sin(\theta)$, $x_2 = \cos(\theta)$, θ is angle between *d* axis of rotor and *a* phase axis of stator, $x = (x_1, x_2)_t$ is a column vector of trigonometric functions.

In order to obtain closed system of state equations of synchronous generator in phase and *dq* coordinates obtained equations should be supplemented by coupling equations of angular speed ω_r with trigonometric functions of Park's transformation matrices that look like as:

$$dx_1/dt = \omega_r x_2 \,;\; dx_2/dt = -\omega_r x_1 \,. \qquad (12)$$

Mathematical model of synchronous generator can be introduced into MATLAB/Simulink environment as Simulink Submodel. In it's structure generator model consists of submodel in *dq0* coordinates and direct and inverse transformation blocks from phase coordinates into dq0 coordinates.

Voltage of electric power sources for electric transmission system is formed on capacitive cross-sections.

Electrical power systems may be classified as complex dynamic systems that contain a plenty of elements with rigid constraint. In addition to traditional elements for electric circuits and networks, they may contain elements which are rather complex objects itself, for example, ordinary electrical complexes or devices, electrical power apparatus or the whole power plants, transformer substations, automatics systems and etc. Such aggregation is possible and expedient in many cases, and essentially simplifies the further analysis of electrical power systems processes also. High order of mathematical models of electric power systems causes a necessity of essential simplification without adequacy losses. One of possible approaches to solve this task is creation of mathematical macromodels. Absence of parameters necessary for creation of detailed mathematical models of electric power system elements causes expediency of macromodels creation for separate elements (for example, turbogenerators) can be considered as ponderable factor for special macromodels creration.

Creation of mathematical macromodels can be carried out using discrete state equations in the following form:

$$\begin{cases} \mathbf{x}^{(k+1)} = \mathbf{F}\mathbf{x}^{(k)} + \mathbf{G}\mathbf{v}^{(k)} + \mathbf{\Phi}\left(\mathbf{x}^{(k)}, \mathbf{v}^{(k)}\right) \\ \mathbf{y}^{(k+1)} = \mathbf{C}\mathbf{x}^{(k+1)} + \mathbf{D}\mathbf{v}^{(k+1)} \end{cases} \qquad (13)$$

where $\mathbf{x}$ is a state variable vector, $\mathbf{v}$ is input variables vector, $\mathbf{y}$ is output variables vector, $\mathbf{F}$, $\mathbf{G}$, $\mathbf{C}$, $\mathbf{D}$ are matrices of corresponding sizes, $\mathbf{\Phi}$ is nonlinear function of ($\mathbf{x}$, $\mathbf{y}$) vectors, k is a discrete order number.

During mathematical modeling of high dimension systems continuous and discrete macromodels can be used. Electrical power system elements have a number of specific properties that have an essential influence on implementation of macromodel creation procedure. In our opinion the basic specificity consists in the following:

1. It is very difficult to conduct measurements on physical objects to obtain input information to create mathematical macromodel.

2. The significant amount of electrical power systems elements is electromechanical and that fact predetermines necessity of their state description by different type variables (electric and mechanical). That problems lead to essential time constant scattering of their mathematical models.

3. External factors (atmospheric, topographical, etc.) which influence is difficult for mathematical simulation may have significant influence on parameters of electrical power system elements. Due to this fact an absolute repeatability of results is impossible.

The mentioned above features of electrical power system elements predetermine some singularity of the macromodel construction procedure. First of all it concerns the choice of external (input or output) variables of the element to be modeled. Not only instant values of currents and voltage, but also speed of rotation, the mechanical moments, frequency, peak values of quasi-periodical currents and voltages can be used as state variables. For example, for the synchronous machine it is possible to choose instantaneous values of currents and voltages, the mechanical moment on a shaft and a angle speed of rotor rotation as state variables. In the next step it is necessary to evaluate approaches suitable for obtaining needed *a priori* information for the macromodel construction. As it is known, for this purpose there are two main approaches: natural experiment and computer modeling on the basis of full mathematical model of the investigated object. The first way for electric power systems research has the essential restrictions caused by fact that it is necessary to obtain information about input and output signals of macromodel to be developed. Another difficult problem is that it is impossible to consider influence of external factors.

The most difficult task is to develop method of construction for concrete object maromodel. In particular, it is not always possible to divide process of parameters identification of linear and nonlinear macromodel part and to find out mathematical form of their description. Often it is expedient to divide the identification procedure of linear and nonlinear parts of macromodel on two different procedures of identification that essentially complicates this process. Nevertheless, due to the diacoptic approach and optimization procedures implementation identification of electrical power system macromodel parameters became possible.

As to the second method to obtain a priori information by mathematical modeling, restrictions are caused by complexity of full electrical power system elements models first of all. Modern capability of computer engineering, calculus mathematics and information technologies makes it possible to develop adequate mathematical macromodels practically for all elements of electric power systems, namely electromechanical converters, electromagnetic apparatus, transmission lines etc. Therefore it is expedient to develop approaches for macromodels design especially for electric power systems.

As far as available systems of transients modeling and simulation do not operate with discrete electric signals, it is necessary to replace discrete macromodel by continuous one. MATLAB/Simulink environment makes it possible to use mathematical macromodels in discrete and continuous form. Composition of macromodels of separate elements macromodels with traditional mathematical model of electric power system can be

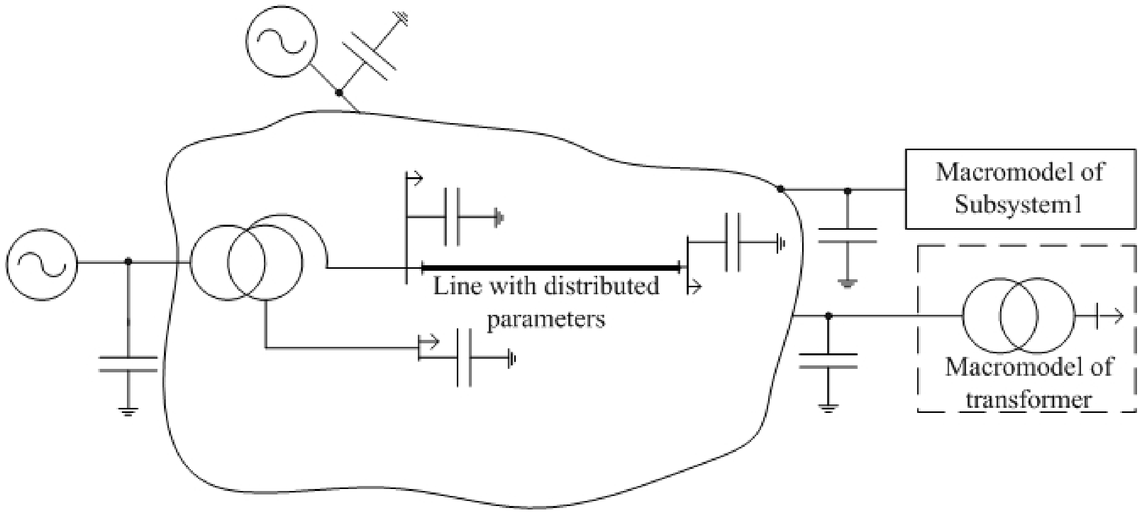

Fig. 2. Generalized scheme of electric system for transient research on the basis of mathematical models and macromodels

carried out using equations of capacitive cross-sections. For this purpose equation of cross-section in the following form can be used:

$$\mathbf{C}_{cs}\, d\mathbf{U}_{cs}/dt = \mathbf{\Pi}_{cs}(\mathbf{i}_{cs1}, \mathbf{i}_{cs2})_t \qquad (14)$$

where $\mathbf{i}_{cs_1}$ is a column vector of mathematical models currents connected with capacitive cross-sections; $\mathbf{i}_{cs_2}$ is a column-vector of macromodels currents connected with capacitive cross-sections, $\mathbf{C}_{cs}$ is matrix of cross-section phase and phase-to-phase capacitances.

Proposed composition approach defines mathematical macromodel form, so input signals during macromodels creation should be phase voltages of capacitive cross-sections and currents of elements for which macromodel is under construction will be output values. Generalized scheme of electric system for transient research on the basis of mathematical models and macromodels is shown in the Fig. 2.

In Simulink environment discrete macromodel can be directly formed using Simulink Subsystems with the help of programming of mathematical expressions corresponding to structure of nonlinear macromodel where phase voltages will be input signals and maacromodel currents will be output signals.

2 CONCLUSIONS

Proposed mathematical model of electrical transmission is suitable for analysis of transient processes in MATLAB/Simulink environment and can contain in its structure mathematical models and macromodels of separate elements.

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Prof. Petro Stakhiv, D.Sc, Head of Department of Theoretical Electrical Engineering, Institute of Electric Power Engineering and Control Systems, Lviv Polytechnic National University, 12 S. Bandera str, Lviv, 79013, Ukraine
spg@polynet.lviv.ua

Oksana Hoholyuk, Ph.D., Department of Theoretical Electrical Engineering, Institute of Electric Power Engineering and Control Systems, Lviv Polytechnic National University, 12 S. Bandera str, Lviv, 79013, Ukraine
oph@astra.lviv.ua