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FORCE EFFECT OF POYNTING VECTOR

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Abstract: *This paper brings theory for derivation of force effect of Poynting vector. The theory of electromagnetic field is the base of presented approach together with close link to quantum mechanics, theory of relativity and vacuum physics. This paper serves as mathematical and physical appendix for preceding paper : Poyinting Vector - an Application for design of alternative engine unit. For the sake of truthfulness it is necessary to say that the work decribed here is a part of long term project with complex theoretical and practical tasks which are to be solved consequently.*

Key words: *Electromagnetic (EM) field, vacuum, Poynting vector, force, space technology*

INTRODUCTION

The paper contains additional mathematical and physical background completing preceding paper [2] titled “Poynting Vector - an Application for Design of Alternative Engine Unit”. That paper was dealing with a notion of vacuum, its possible physical structure, some of its important proerties, definition of the problem of new kind engine unit design and also did bring concrete simulation results.

This paper supports mentiond problem solution by wider description of mathematical tools used in this solution design journey. Paragraph by paragraph of [2] we will now make clear realized steps and obtained results.

1 VACUUM “FOAM” STRUCTURE

In [2] there was stated that vacuum has structure of the time-space foam. What can be possible obstacles while searching any exact mathematical description of such structure ? We can illustrate this situation on following example.

Let us assume any given physical system. We obtain some measured data and want to reveal a “law”, if there is any, generating this data.

The first natural step is to try to plot this data in a graph. This graph is in Fig.1. Here we see some development in time, also discrete character of measurement is obvious but nothing more. What is the core of this problem ? It is in used kind of “projection” of reality (our data) into a graph plane, or in mathematical words : mapping of set of data values into a two dimensional space (graph plane).

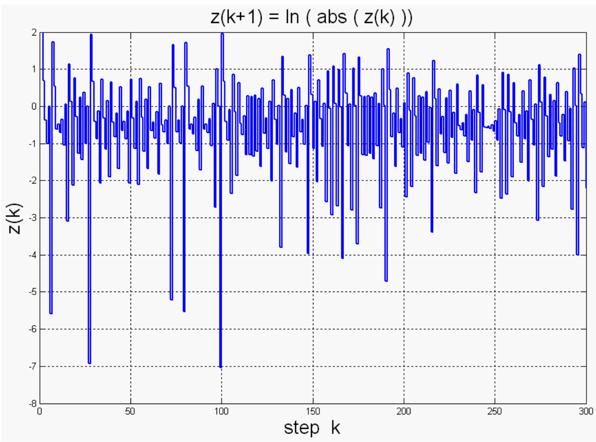

Fig.1: Measured data

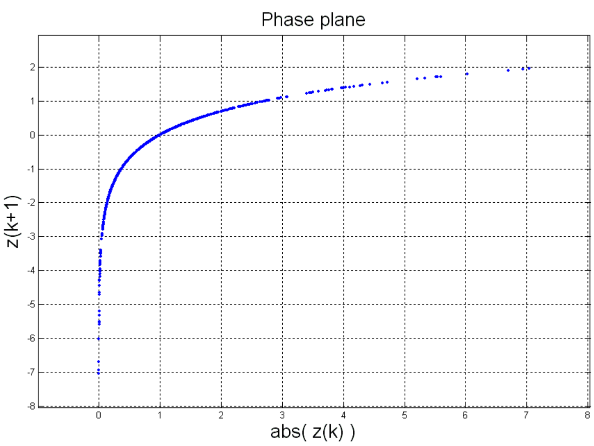

Fig.2: Phase plane graph of measured data

There are many other possible ways how to present this data. Let us plot this data into phase plane showing dependence of ordered pairs of data. It is shown in Fig.2.

Values on horizontal axis are drawn independently on their sign but it is not violation of data character because of mirroring of obtained graph branches.

From Fig.2 we can easily see logarithmical dependence between pairs of data values. It is true because this experimental example data was generated by discrete time system of the first order described by this equation :

$$z(k+1) = \log\left(abs\left(z(k)\right)\right)$$

with starting value

$$z(0) = 2.0$$

What is the result of this illustrative example ? Usage of unsuitable mapping can destroy our effort to discover the law of functionality of observed reality.

Of course it is possible to use stochastic or probabilistic approach which are offering any result in every case but the total cost for such result is fatal for our purpose - we would “pay” by lost of the only information hidden in obtained data. The situation can be compared with a butterfly under large hat - we have 100% sure that butterfly is under our probabilistic distribution “hat” but we do not know where it is, what is its behaviour etc. It is not general disadvantage of statistic, chaotic and other methods but they were designed for totally another kind of problems.

In this illustrative example another unsuitable approaches can be represented for example by using of FFT, or by replacement of obtained data series by mean value and deviation etc. They will tell us nothing about logarithm dependence.

Partial conclusion can be that statistic approach will not be used but deterministic approach is not known. Deterministic description must cover foam character of vacuum which results in infinite dimensional description of individual time-space “domains” and energetic couples among them. All of them are time varying of course.

A step forward can be done by using theory of EM field.

2 EM FIELD PROPERTIES OF VACUUM

Let us start with description of some basic terms and their meaning .

Mathematical definition of curl of vector is written very often in the following form.

$$\nabla \times \mathbf{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \qquad (1)$$

This relation is analogy of Cartesian product of two topological spaces descripted by their bases. Bases of these spaces are in this case following :

$$\begin{pmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{pmatrix} \qquad (2)$$

represents base of the first space,

$$\begin{pmatrix} A_x & A_y & A_z \end{pmatrix} \qquad (3)$$

represents base of the second space, and

$$\begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \end{pmatrix} \qquad (4)$$

represents base of resulting space.

This point of view brings a posibility of expansion of our problem description into a spaces with more than 3 dimensions.

Let us assume that the product $\nabla \times \mathbf{A}$ will be only one of components of cross product of vectors of spaces of dimensions greater than 3. Let us then define 4-dimensional space with base

$$\begin{pmatrix} \vec{i} & \vec{j} & \vec{k} & \vec{t} \end{pmatrix} \qquad (5)$$

This base can be seen as an idea motivated by Minkowski spaces koncept. Similar conception can be found in works of R. Hamilton, H. Lorenz and others.

Then the cross product would have the form (1) only if

$$\vec{t} = \vec{0} \qquad (6)$$

Now it is right place for some notes to this approach.

The fourth component of (5) represents time which is considered as vector. It is not very often in classical physics but is of big advantage in generalized theory of relativity. The cross product (1) is only a special case of curl of two vectors in case of fixed time. By admission of variable development of time vector we receive time-space deformations joind with vectors of mentioned cross product.

More detailed analysis is exceeding the scope of this paper and leads to a concept of description of deformations of time-space structures caused by electromagnetic field. In todays physical research these problems are in focus of studies and experiments concentrated on quark-gluon behaviour and properties of plasma which is little divergent from the target of this paper.

We herein conclude this part by following relations. Let us assume following cross product and look on its components after application of Cramer's rule :

$$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\partial\vec{i} & \partial\vec{j} & \partial\vec{k} & jc\partial\vec{t}\\ \vec{i} & \vec{j} & \vec{k} & \vec{t}\\ A_x & A_y & A_z & A_t\\ B_x & B_y & B_z & B_t\end{vmatrix} \qquad (7)$$

$$\mathbf{A}\times\mathbf{B}=\partial\vec{i}\begin{vmatrix}\vec{j} & \vec{k} & \vec{t}\\ A_y & A_z & A_t\\ B_y & B_z & B_t\end{vmatrix}-\partial\vec{j}\begin{vmatrix}\vec{i} & \vec{k} & \vec{t}\\ A_x & A_z & A_t\\ B_x & B_z & B_t\end{vmatrix}+$$

$$+\partial\vec{k}\begin{vmatrix}\vec{i} & \vec{j} & \vec{t}\\ A_x & A_y & A_t\\ B_x & B_y & B_t\end{vmatrix}-jc\partial\vec{t}\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix} \qquad (8)$$

Here in these relations $\vec{j}$ is base vector and j is imaginary unit, c is velocity of light.

Well known part described above is the last term of (8) for fixed time as was descussed. Minus sign has relation to fundamental basics of theory of electromagnetic field and Minkowski „four space".

Moreover this concept has close relation to generalized theory of algebraic structures and to so called „relation" understood as mapping from a set into itself and to „transformation" where used mapping is an element of another set.

More detailes can be found in rich literature, e.g. in [4]. Mathematical background can be found for example in [7] or [9].

For purposes of this paper we will now mention mainly the fact that cross product is very often used in description of EM laws, namely in Maxwell's equations or, as will be seen later in this paper, in definition of energy of EM field.

This approach is based on assignment of vectors in product the concrete meaning of components of EM field.

A notion of energy has important position in physics generally. In theory of EM field there are several definitions of energy. One of them is postulating Poynting vector by this known way :

$$\mathbf{S}(t)=\mathbf{E}(t)\times\mathbf{H}(t) \qquad (9)$$

By words : energy represented by Poynting vector $\mathbf{S}(t)$ is EM energy comming during unit time interval through unit surface the normal vector of which coincides with direction of vectors $\mathbf{E}(t)$ and $\mathbf{H}(t)$. By this way Poynting vector $\mathbf{S}(t)$ is defining direction in which energy of EM field is spreading in space. Details e.g. in [1], [5].

Remark : in some literature Poynting vector $\mathbf{S}(t)$ is being denoted $\mathbf{N}(t)$.

The relation (9) is based on unexpressed assumption of transformation from 4 dimension space into 3 dimension space. Without this transformation we would have to express Poynting vector using relation (7). This transformation is based on simple time-parametrization and on silent assumption of none time changes deformations, it means on assumtion of strict equality

$$\partial t=1 \qquad (10)$$

Then it is possible to use relation (9) for our further work.

Poynting vector has significant role for solution of engine problem solved in [2] because energy „flow" is basic condition for force occurence. Basic relation of EM field energy described by Poynting vector (see [3]) is :

$$\mathbf{F}(t)=\frac{1}{c^2}\frac{d\mathbf{S}(t)}{dt} \qquad (11)$$

From (11) can be easily seen reason why this force was not observed in electrotechnical practice. Poynting vector is being present in case of every EM wave spreading from its source. It means for example that all EM waves emited by antennas are pressing by this force as well as on emiting antenna system as on receiving antenna and other objects on which this transmitted signal is incidenting. However in (11) there is the term $\frac{1}{c^2}$ having approx. value $1.1\cdot10^{-17}$ ($m^{-2}s^2$) multiplying time derivative of Poynting vector. Even in the case of radio and TV transmitters working with output power ranging from many kW higher the time derivative of Poynting vector is decreased by this term to a value which can be neglected. One of reasons is usage of harmonic signal as we see later herein.

From mathematical point of view there is no problem to get from (9) this relation :

$$\frac{d\mathbf{S}(t)}{dt}=\left(\frac{d\mathbf{E}(t)}{dt}\times\mathbf{H}(t)\right)+\left(\mathbf{E}(t)+\frac{d\mathbf{H}(t)}{dt}\right) \qquad (12)$$

From solution demands perspective of problem defined in [2] there is inportant extremization of force defined by (11).

There are two basic possible approaches : either to use „classical" extremization calculus described e.g. in [9], or use heuristic or any other alternative approach together with knowledge of characteristics of electrotechnical signals.

The first approach leads to underdetermined systems of partial differential equations and because of it this way does not seem to be advantages one.

The second approach is based on the idea mentioned above - to use nonharmonic signal for feeding transmitting antenna system.

Befor next steps we will little more precise demands put on searched solution of the stated problem.

The force of EM field defined by (11) can not be monotonously growing. Monotonous character is meeting only mathematical demand of extremization but is in contradiction with axiom of limited total energy of closed physical system.

The force must be generated by signal periodical in time.

In accordance with [2] these conditions can be written in form of the following criteria :

$$\mathbf{G}(u) = \int_0^u \mathbf{F}(t)dt \qquad (13)$$

where $0 < u < +\infty$ has meaning of time period of feeding signal.

Obviously we want :

$$\mathbf{G}(u) > 0 \qquad (14)$$

which leads to demands :

$$\frac{\partial \mathbf{G}(u)}{\partial u} = 0 \qquad (15)$$

and

$$\frac{\partial^2 \mathbf{G}(u)}{\partial u^2} < 0 \qquad (16)$$

Because we search feeding signal - to find such definition of Poynting vector to be able to solve the engin problem - we are facing a task to find u, $\mathbf{E}(t)$ and $\mathbf{H}(t)$ such that (14), (15) and (16) will be fulfiled for (11), (12) and (13).

Of course there are many other phenomena having any influence on the solution of the given problem. Every „normal“ antenna has emitting characteristics described schematically in a polar graph of amplitude of emitted signal against horizontal angle. In this characteristics there are ususally back or side lobes representing EM energy which is lost from the point of view of our problem. It means additional extremization of direction restriction of emitted EM wave. Detailes e.g. in [6] or [8].

3 REALISATION NOTES

Summarising the relations and ideas mentioned above we can try to find solution of a time shape of feeding signal in a form of periodic nonharmonic signals. Because of the term $\frac{1}{c^2}$ in relation (11) it is desired to feed the engine by signals with strong peeks having high values of time derivative. Ideal approximation is Dirac impuls which is however only theoretical abstraction without practical realisation possibility. From electrotechnical experience is clear that possible realisable shapes of signals can be for example rectangle or triangle ones.

But using „sharp shaped waves“ is not itself sufficient to ensure desired results. The possible problems hidden in simplification can be illustrated on this example. Let us choose feeding vectors in the following form :

$$\mathbf{E}(t) = \left[0 \quad 0 \quad sign\left(\sin\left(\omega t + \frac{\pi}{4} \right) \right) \right] \qquad (17)$$

$$\mathbf{H}(t) = \left[\cos(\omega t) \quad \sin(\omega t) \quad 0 \right] \qquad (18)$$

Resulting time development of Poynting vector looks to be very promising, see Fig.3.

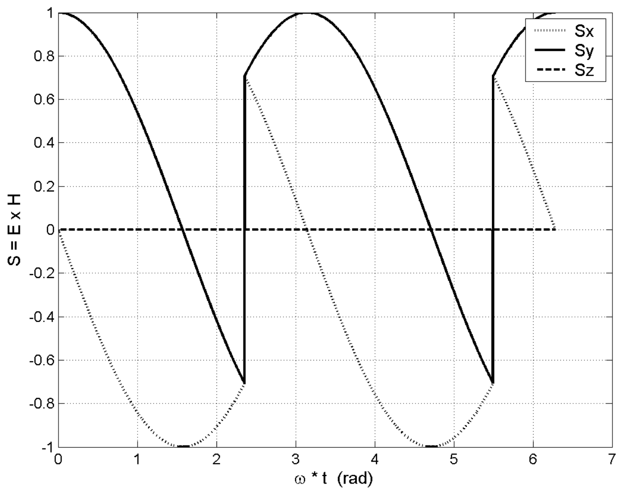

Fig.3:Components of Poynting vector

In Fig.3 it seems that Poynting vector has necessary „needles“ which can ensure enough high values of time derivatives to make resulting force effective.

However in Fig.4 there is polar graph of horizontal distribution of resulting Poynting vector’s time derivative in space - and from this graph it is clear that the resulting force will be uneffective. The resulting force which will act in coincidence with time derivative of Poynting vector is alternatively oriented in contradictory directions. In case of very little mass of emitting antenna system this system will be shaking on its initial position without effective move in any direction.

Moreover every similar solution has a small “surprise”. It can be seen after zooming of the center part of this polar graph. Here are two side lobes of circle shape opsite one another and their axis has different direction than “main” force. See Fig.5. Although diameter of this “circle lobes” is considerably less than 0.1 and so can be neglected in comparison to length of main branches in Fig.4, this effect could increase its influence in case of different signal type selection.

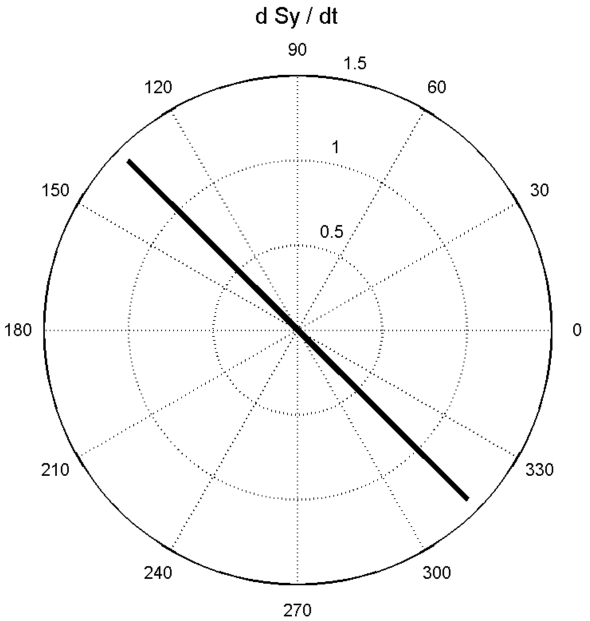

Fig.4: Polar graph of Poynting vector

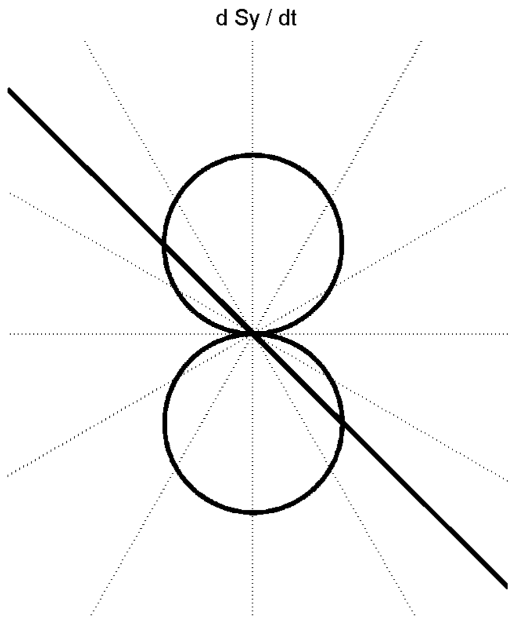

Fig.5: Polar graph of Poynting vector - detail of central part of graph in Fig.4

Another, really more effective solution with acceptable direction characteristic is described in [2] together with notes of concrete possible design arrangements of resulting antenna system elements.

4 Conclusion

Many other consequences of described theory represent a core of future development of this project.

Basic outlines of physical, electrotechnical and mathematical theories were described and used to formulate relations enabling to solve the stated problem of finding of new kind of engine.

Many theoretical aspects can be found in literature and hence they were not been rewriten here, namely tensor character of permitivity and permeability (see [5]), analogy between solution of electrical engineering problems and their mechanical equivalents (see this phenomenon on practical example of solved problem in [2]), quantum mechanics or theory of relativity (see [4]), theory of antenna systems (see [6], [8]) and many others.

Realisation note : all computer simulations and graphs were performed in environment of MATLAB.

5 Acknowledgments

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