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# „CURVILINEAR SQUARES “ IN THE TEACHING OF ELECTRIC FIELDS

RNDR. CTIBOR HENZL, PH.D.
JAN ROZHON

**Abstract**: *The method of “the curvilinear squares” was first described by Dr. Lehmann in year 1909 [1]. It allows a graphical solution of partial differential equations of the elliptic type. It was used for solving technical problems in the first half of last century. More effective numerical methods have been developed since this time. We think, that the method of the „curvilinear squares“has a strong appeal as a teaching method. It cultivates in the students a sense for better understanding of the physical reality. It facilitates the active work in field mapping. The field quantities as E and j can be readily computed. The same can be said about the estimation of sources of the field Q and parameter C. The drawing of the equipotential lines and the field lines can be using certain programs, e.g. Microsoft Word.*

**Key words**: *Curvilinear squares, equilines, lines of force, visualization of electric field*

## INTRODUCTION

The first article concerning the method of “curvilinear squares” appeared in “Elektrotechnische Zeitschrift” in 1909. The title of the article is “Graphische Methode zur Bestimmung des Kraftlinienverlaufes in der Luft”, by Dr. Lehmann from Belfort, France. Lehmann’s Method or Graphic Relaxation are other names for method of “curvilinear squares“.

This method makes possible visualization of the fields and also provides numerical results. It was used for the solution to practical problems in the first half of last century. Later this method could not compete with modern sophisticated numerical methods implemented on modern computers. Nevertheless many Czech and foreign textbooks make reference to it, see [2], [3].

Method of “curvilinear squares“ requires very modest equipment. In past it was only a pencil, eraser and a sheet of paper. Today we are attempting to replace these methods by using Word.

## 1 A BRIEF DESCRIPTON OF THE METHOD

### 1.1 Description of the method

Electric field intensity is expressed using equations

$$E = \frac{\Delta\varphi}{\Delta n} \text{ resp. } E = \frac{\Delta\psi_E}{\Delta S}$$

By comparing left members we get $\frac{\Delta\varphi}{\Delta n} = \frac{\Delta\psi_E}{\Delta S}$.

For $\Delta n = \Delta S$ we have $\Delta\varphi = \Delta\psi_E$

*Equilines and lines of force create a net of squares.*

### 1.2 Drawing of the map of field

The map of the field is drawn using Drawing module in Word, using Autoshapes.

Three rules are:

1. A surface of conductors is equipotential
2. Lines of force and equilines are perpendicular
3. The square is “curvilinear” if it is possible to put a circle into, see Fig.1.

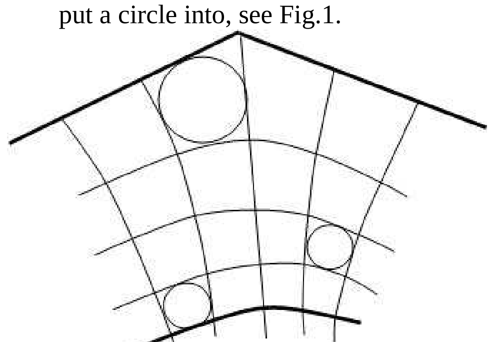

*Fig.1: Curvilinear squares*

## 2 CURVILINEAR SQUARES IN THE TEACHING

The method of “curvilinear squares” is incorporated in the teaching of the subject “Theoretical electrical engineering 2”, see [2]

### 2.1 The defining of the task

Draw the map of an electric field using method of graphic relaxation. Choose one of the configurations of electrodes from Fig. 2. Determine a size of the charges on electrodes and total capacitance of the system. Choose any point between electrodes and determine module of electric intensity *E*.

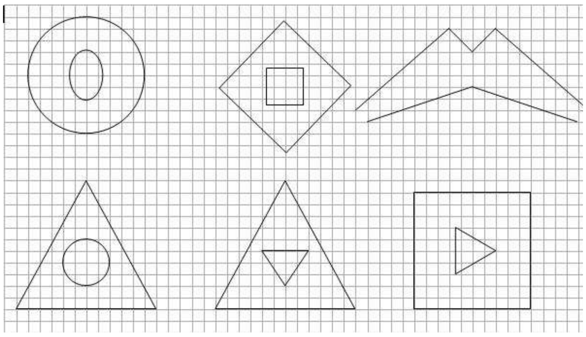

*Fig.2: Configuration of electrodes*

### 2.2 Processing in WORD

The third configuration was chosen. Electrodes are open. This configuration is considered as difficult and is not popular with students. Potential between electrodes is 40 V, air is dielectric. Dimensions are in centimeters. The task is solved in half-symmetry, see Fig. 3.

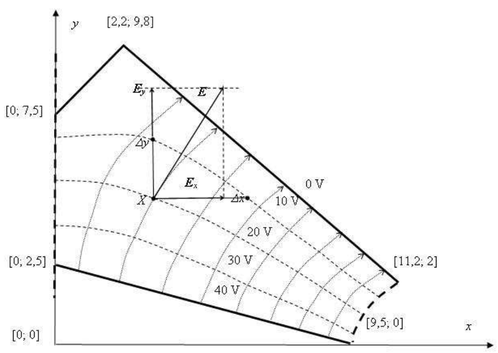


*Fig.3: Graphic solution*

For calculation of electric field intensity the point $X$ = [3,2; 4,7] was chosen. Fig. 3 has $\Delta x$ = 0,031 m, $\Delta y$ = 0,019 m.

$$E_x = \frac{\Delta\varphi}{\Delta x} = \frac{10}{0{,}031} = 323\,\text{V/m}$$

$$E_y = \frac{\Delta\varphi}{\Delta y} = \frac{10}{0{,}019} = 526\,\text{V/m}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{323^2 + 526^2} = 617\,\text{V/m}$$

$$\Delta Q = \Delta\psi_D = \varepsilon_0 E \Delta s = \varepsilon_0 \frac{\Delta\varphi}{\Delta n}\Delta s = \varepsilon_0 \Delta\varphi = 88{,}5\,\text{pC}$$

$$Q = \sum \Delta Q = 9\cdot\Delta Q = 8\cdot 10^{-10}\,\text{C}$$

$$C = \frac{Q}{U} = \frac{8\cdot 10^{-10}}{40} = 0{,}2\cdot 10^{-10}\,\text{F} = 20\,\text{pF}$$

## 3 COMPARISON WITH QUICKFIELD

For comparison of results see Tab. 1. Relative variation δ is computed. Numerical values are regarded as correct. Variation of $E_x$ and $E_y$ is greater then variation of *E*. For the course of equilines, see Fig. 4. Dot lines are derived from graphical solutions, full lines are from QUICKFIELD.

| | Method | | δ (%) |
|---|---|---|---|
| | Graphic | Numeric (FEM) | % |
| $E_x$ (V/m) | 323 | 194 | 68 |
| $E_y$ (V/m) | 526 | 635 | 17,3 |
| $E$ (V/m) | 617 | 664 | 7,2 |
| $Q$ (nC) | 0,8 | 0,85 | 7 |
| $C$ (pF) | 20 | 22 | 9,1 |

*Tab. 1: Comparison of the results*

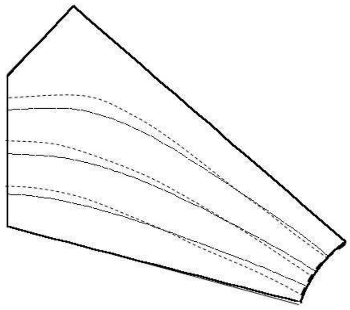

*Fig.4: Comparison of equilines*

## 4 MORE ABOUT OF SOLUTION OF REFERRED EXAMPLE

### 4.1 Steps of the solution of above referred example

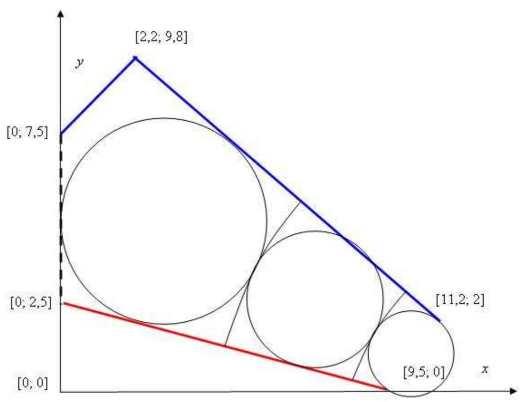


*Fig.5: First step of solution*

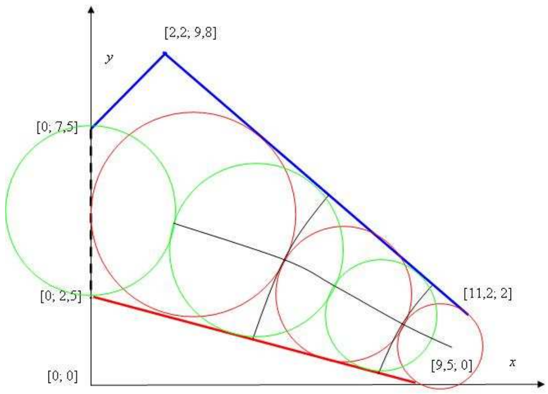


*Fig.6: Second step of solution*

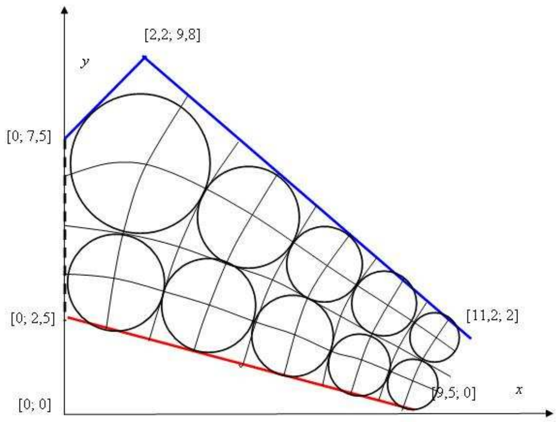


*Fig.7: Final step of solution*

The comparison with QUICKFIELD is made, see Fig 8. Dot lines are derived from graphical solution, full lines are from QUICKFIELD

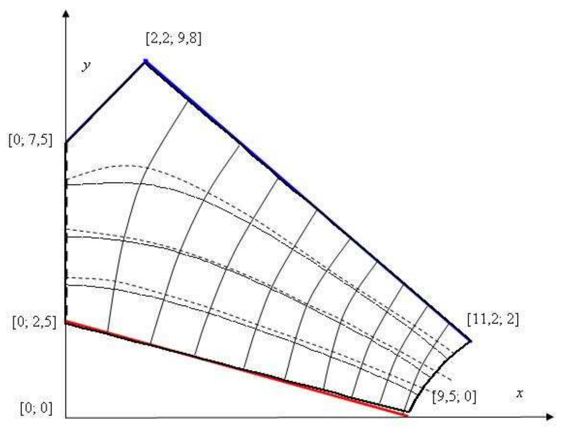


*Fig.8: Comparison of Equilines*

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## 5 PROBLEMS

### 5.1 Problem 1

Sketch the map of the field of following configurations of the electrodes:

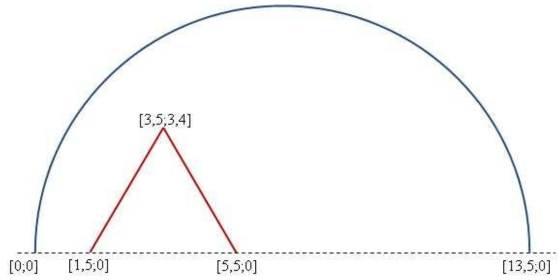


*Fig.9: Configuration of electrodes*

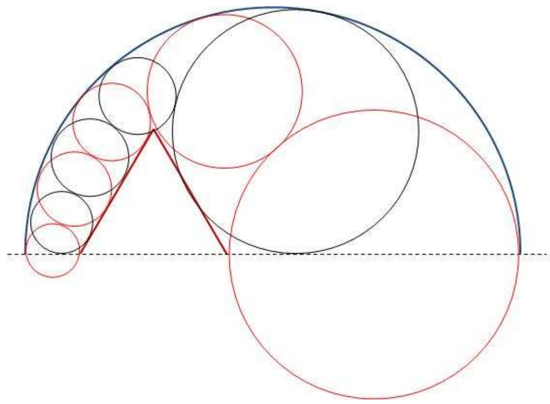

*Fig.10: First step of solution*

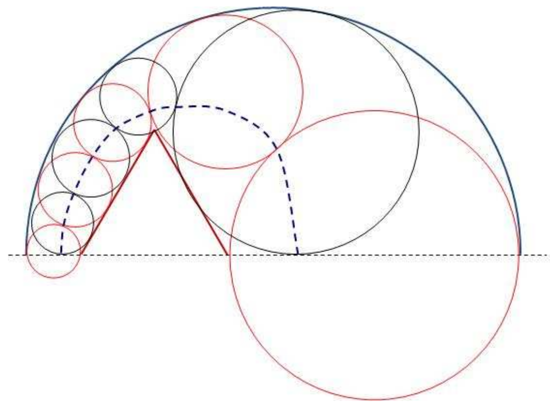

*Fig.11: Second step of solution (with central equiline)*

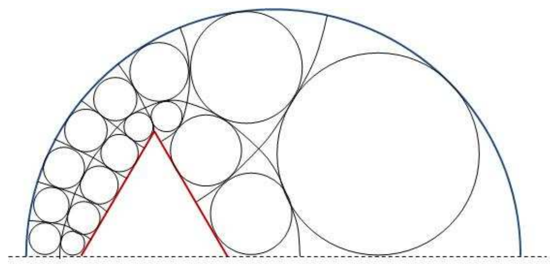

*Fig.12: Final step of solution*

The comparison with QUICKFIELD is made, see Fig 13. Dot line is derived from graphical solution, full line is from QUICKFIELD

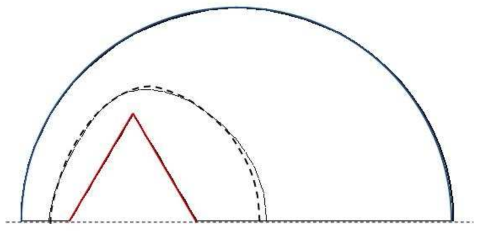

*Fig.13: Comparison of Equiline*

## 5.2 Problem 2

Sketch the map of the field of following configurations of the electrodes:

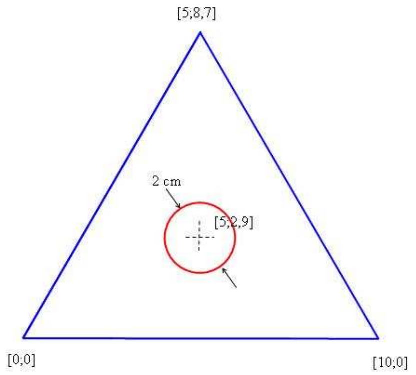


*Fig.14: Configuration of electrodes*

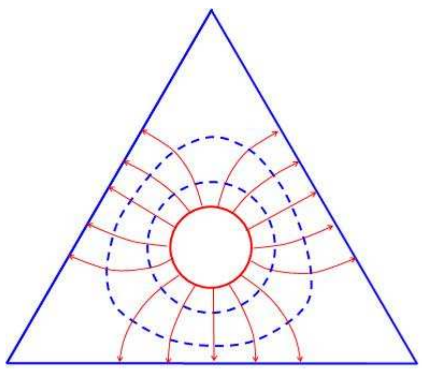

*Fig.15: Final solution*

The comparison with QUICKFIELD is made, see Fig 16. Dot lines are derived from graphical solution, full lines are from QUICKFIELD

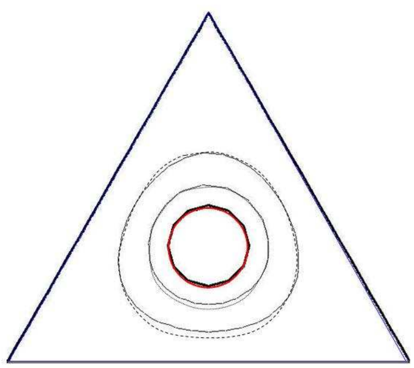

*Fig.16: Comparison of Equilines*

## 5.3 Problem 3

Construct a curvilinear square map for a coaxial capacitor of 3 cm inner radius and 8 cm outer radius. Use your sketch to calculate the capacitance per meter length, $\varepsilon_r = 1$. Calculate an exact value for the capacitance per unit length. [4]

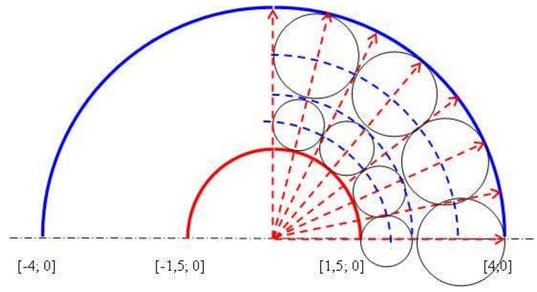


*Fig.17: Field map for coaxial capacitor*

Exact value of capacitance is

$$C = \frac{2\pi\varepsilon_0}{\ln\frac{r_2}{r_1}} = 5{,}7 \cdot 10^{-11} F$$

Using QUICKFIELD we have

$C = 5{,}8 \cdot 10^{-11} F$

Using graphical method we have

$$C = \frac{4 \cdot 7\varepsilon_0}{4} = 7 \cdot 8{,}85 \cdot 10^{-12} = 6{,}2 \cdot 10^{-11} F$$

# 6 Conclusions

- Method of “curvilinear squares” has its own place in the teaching of electromagnetic field.
- It provides a graphical solution, from which numerical results can be computed.
- Manual sketching of the field map is a more active approach than simple use of sophisticated programs.
- There is less accuracy of results than when using a numerical approach.

# 7 References

[1] Lehmann, Th.: Graphische Methode zur Bestimmung des Kraftlinienverlaufes in der Luft, ETZ 1909
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Ctibor Henzl, RNDr.,Ph.D,
VŠB - Technická univerzita Ostrava, 17. listopadu 15
708 33Ostrava 8, Ctibor.Henzl@vsb.cz