KTE
September 7. - 9.9.2009
Cheb, Czech Republic

WAVELET ANALYSIS OF STRONGLY NONLINEAR SYSTEMS

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Abstract: *The paper deals with a particular method for analysis strongly nonlinear systems. The main emphasis is placed on differentiation of chaotic behavior of systems from other types of behavior like periodical, quasi-periodical or stochastic behavior.*

Key words: *wavelets, wavelet analysis, strongly nonlinear system, chaotic system, chaotic behavior*

INTRODUCTION

The chaotic systems investigation was developed mainly in the last two decades. At the present time there is a broad theoretical background developed and chaos theory is used in many practical applications. However it is impossible to say that all related problems have been already solved. One of the problems which investigation is still in progress is a differentiation between main types of behavior of dynamical systems. Nowadays many of created methods are based on reconstruction of the trajectory of dynamical system in the state space. Usually some variations of the time delay methods are used. However reconstruction of trajectory doesnÕt give any additional information and in fact it is only the way how to see data from different point of view.

1 CHAOTIC BEHAVIOR

Chaotic behavior can be, without an effort of mathematical accuracy, described like a no periodical, irregular, unflagging switching of intervals stability and instability of dynamical systems.

Consider autonomous dynamical system described by system representation in the form

$\mathcal{R}\{S\}$:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}), \quad y = h(\boldsymbol{x}). \tag{1.1}$$

Assume that it is not possible to observe (or measure) the state vector $\boldsymbol{x}$ directly. The only information we have about the system behavior is the vector or scalar signal y. In fact the problem of reconstruction of the trajectory in the state space could be seen as a state reconstruction Ð common problem in control systems theory.

2 CHAOTIC SYSTEM STATE RECONSTRUCTION

In the most general (and the most complicated) case we are only able to observe the output signal y. The goal of state reconstruction is to estimate order, structure and parameterization of the system which generated observed (or measured) signal.

As a simple example could be mentioned Fourier decomposition of signal generated by autonomous linear dynamical system. If this system is conservative and has a finite order, it is obvious fact that the signal y has to be periodical and therefore it is possible to express it as a sum of harmonic component. Each harmonic component can be generated by a second-order linear dynamical system. So the Fourier decomposition of signal can be understood as a simple example of signal generating system estimation. It should be said that, Fourier decomposition itself does not give any information about structure of system representation but it enables, for chosen system representation, find an appropriate parameterization.

Similarly it is possible reconstruct dissipative linear dynamical system using any time-frequency analysis method like Short-time Fourier transform, Floating Fourier transform or Wavelet analysis.

In the most general case, if it is necessary to reconstruct order, structure and parameterization of system representation described by equation (1.1) from output signal only, the situation is much more complicated. At the present time are used methods focused mainly on reconstruction of trajectory of the system and then on calculation of fractal dimension of the obtained set [2,3].

It is important to say that on the basis of fractal dimension it is possible to estimate complexity of system and consequently the order of the given system [4].

3 CONTINUOUS WAVELET TRANSFORM

Continuous wavelet transformation is an integral transformation where the kernel of transformation is a general base function ψ [1] defined by equation

$$F(s,t) = \int_{-\infty}^{\infty} f(\tau)\psi_{s,t}(\tau)\,\mathrm{d}\tau, \quad s,t \in \mathbb{R}, \qquad (1.2)$$

where $F(s,t)$ is an image of function $f(t)$ in wavelet transformation, $\psi_{s,t}(\tau)$ is a particular wave with scale s and time-shift τ.

The inverse wavelet transformation is defined by the equation

$$f(\tau) = \frac{1}{C}\iint_{\mathbb{R}^2} \frac{1}{|s|^2} F(s,t)\psi_{s,t}(\tau)\,\mathrm{d}s\mathrm{d}t, \qquad C = \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{\omega}\,\mathrm{d}\omega, \qquad (1.3)$$

where function $\hat{\psi}(\omega)$ is a Fourier transform of the base function.

4 WAVELET TRANSFORMATION OF CHAOTIC SYSTEMS

Wavelets derived from base wavelet have self-similarity property. This is an important liaison between wavelet transformation and the theory of fractals. By a proper choice of base wavelet it is possible to decompose chaotic signals. On the base of such decomposition we can calculate an estimate of signal generating system order.

Wavelet analysis of the signal generated by the well-known Lorenz system is depicted at the picture Fig. 1. For wavelet analysis the Morlet base wavelet was used. The shape of the Morlet base wavelet is shown in the picture Fig. 2.

It is important to say that results of continuous wavelet transformation strongly depend of used base wavelet.

The evolution of state variables of the Lorenz system is depicted in the picture Fig. 3.

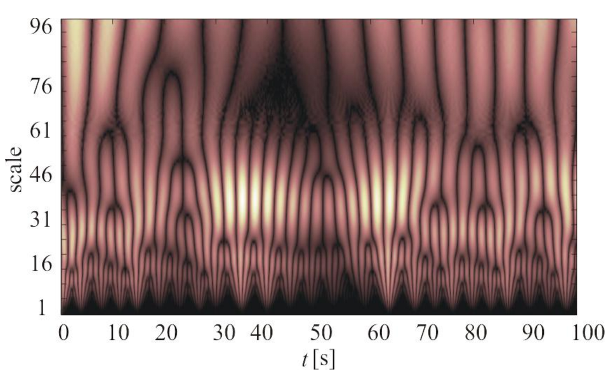

Fig. 1: Wavelet analysis of the signal generated by the Lorenz system (used base wavelet Morlet)

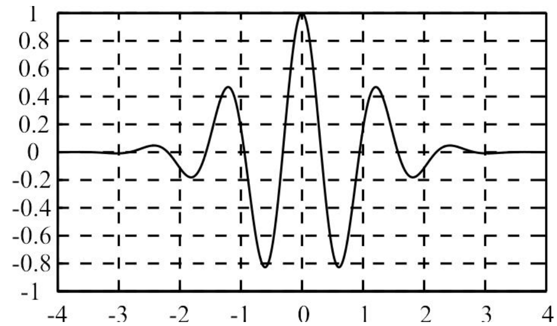

Fig. 2: Shape of the Morlet Wavelet

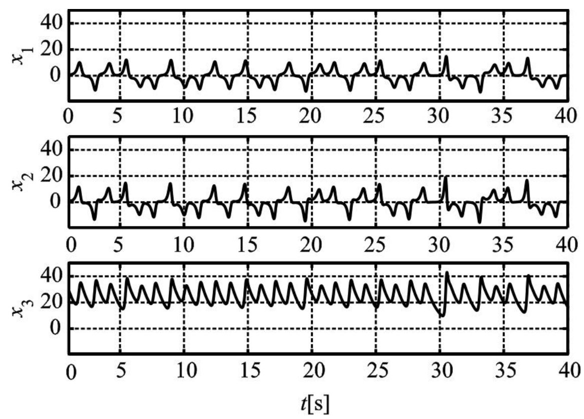

Fig. 3: Evolution of states variables of Lorenz system

5 CONCLUSIONS

The paper deals with possibilities of wavelet analysis of chaotic systems. There are present methods of chaotic system order and fractal dimension estimation discussed at the paper. It is shown that signal generated by chaotic system can be decomposed using wavelet transformation with properly chosen base wavelet. The choice of base wavelet is a crucial point. The results obtained using different base wavelets can be very different.

6 REFERENCES

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