Generalised transmission model of first order parametric section

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Abstract **This paper presents a model of the first order first order parametric section with non-periodically variable parameter. Those systems are used as parts of more complex parametric filters. A formula, describing filter response to any signal with finite average power, has been determined. Obtained results have been illustrated by an example.**

Keywords **LTV, parametric section**

I. Introduction

The transmission models of SISO systems (fig. 1) are described [1] by $\mathcal{P}$-operation which is the relation between input $x(t)$ and output $y(t)$ signals of the system.

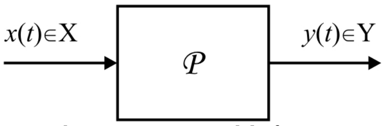

Fig. 1. The transmission model of SISO system

Those signals are part of various signals spaces. For parametric systems the $\mathcal{P}$-operation in the time domain can be expressed by:

- state equations
- parametric convolution, defined as:

$$y(t)=\int_0^t h(t,\tau)x(\tau)d\tau\ , \qquad (1)$$

where

$h(t,\tau)$ – the impulse response of a system.

The transmission description of the system using the convolution (1) allow to determine the system response $y(t)$ to any excitation $x(t)$, if the impulse response $h(t,\tau)$ is known. The impulse response determination of the system described by equation:

$$y'(t)+\omega(t)y(t)=x(t)\ , \qquad (2)$$

where:

$\omega(t)$ – parametric function,

further referred as a first order section. The determination of impulse response of higher order parametric system have been included in work [3]. In this work the parametric function variability has been assumed as:

$$\omega(t)=\omega_{\mathrm{g}}+C\mathrm{e}^{-\gamma t},\quad \omega_{\mathrm{g}},\gamma\in \mathrm{R}^{+}, C\in \mathrm{R}\ , \qquad (3)$$

$$\omega(t)=\omega_{\mathrm{g}}+\sum_{k=1}^{n}C_k\mathrm{e}^{-\gamma_k t},\quad \omega_{\mathrm{g}}\ \gamma_k\in \mathrm{R}^{+}, C_k\in \mathrm{R}\ . \qquad (4)$$

The parametric functions (3), (4) are only the subset of $\mathrm{L}^2\langle 0,\infty)$ space (with the omission of ω_0 component), so these function are not a representation of any function with limited energy. It can be proven [4], that any function $f(t)\in \mathrm{L}^2\langle 0,\infty)$ can be approximate by series:

$$f(t)=\omega_{\mathrm{g}}+\sum_{\substack{k=-N_1\\ n\neq 0}}^{k=N_1}C_k\mathrm{e}^{-p_k t},\quad C_k\ , p_k\in \mathrm{C}\ , \qquad (5)$$

where:

$$C_k=|C_k|e^{j\psi_k}\ ,\qquad C_k=C_{-k}^{*}\ , \qquad (6)$$

$$p_k=\gamma_k+\mathrm{j}\omega_k\ ,\qquad p_k=p_{-k}^{*}\ . \qquad (7)$$

The proper approximation conditions for limited number of terms (5) has been expressed by formula:

$$\bigwedge_{k\in N}\ \mathrm{Re}\{p_k\}<0\ . \qquad (8)$$

In this reason the numbers p_k have to be set in the left half plane of Gauss space, fig. 2.

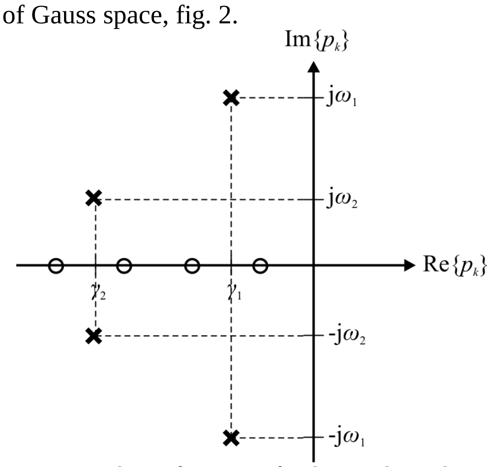

Fig. 2. The configuration of real „o" and complex „x" of coefficients p_k.

In this paper the generalised model (in comparison to earliest models considered in work [3]) of first order parametric section, described by eq. (2) with parametric function expressed as:

$$\omega(t)=\omega_g+e^{-\gamma t}(A\cos(\omega_1 t)+B\sin(\omega_1 t))\ , \qquad (9)$$

where:

$$\begin{aligned} &p=\gamma\pm \mathrm{j}\omega_1=|C|\,e^{j\psi},\\ &A=2\,|C|\cos\psi,\\ &B=2\,|C|\sin\psi. \end{aligned} \qquad (10)$$

The function $\omega(t)$ is the particular case of series (5) for N=1.

II. THE MODEL OF THE SECTION

The solutions to the equation (2) in closed form are known [5]. For zero initial conditions those solutions are determined by equation:

$$y(t) = \int_0^t e^{-(\alpha(t)-\alpha(\tau))} x(\tau) d\tau \,, \qquad (11)$$

where:

$$\alpha(t) = \int_0^t \omega(t) dt \,. \qquad (12)$$

After using the equations (9), (12) and comprising formulae (1) and (11) one can obtain the equation of the section model in convolution form:

$$y(t) = \int_0^t \exp(-\omega_g (t-\tau))$$

$$\exp[\exp(-\gamma t) \frac{(A\omega - B\gamma)\sin(\omega_1 t) - (A\gamma - B\omega)\cos(\omega_1 t)}{\gamma^2 + \omega^2} +$$

$$-\exp(-\gamma\tau) \frac{(A\omega - B\gamma)\sin(\omega_1 t) - (A\gamma - B\omega)\cos(\omega_1 t)}{\gamma^2 + \omega^2}])$$

$$x(\tau) d\tau, \qquad (13)$$

whereas the impulse response can be expressed as:

$$h(t,\tau) = \exp(-\omega_g (t-\tau))$$

$$\exp[\exp(-\gamma t) \frac{(A\omega - B\gamma)\sin(\omega_1 t) - (A\gamma - B\omega)\cos(\omega_1 t)}{\gamma^2 + \omega^2} +$$

$$-\exp(-\gamma\tau) \frac{(A\omega - B\gamma)\sin(\omega_1 t) - (A\gamma - B\omega)\cos(\omega_1 t)}{\gamma^2 + \omega^2}]). \qquad (14)$$

The equations (13) and (14) are describing a complete transmission model of generalised first order parametric section, with variable coefficient described by formula (9).

III. EXAMPLE

For section described by equation (2) the parametric function is expressed by equation (7). The waveforms of parametric functions are shown in fig. 3.

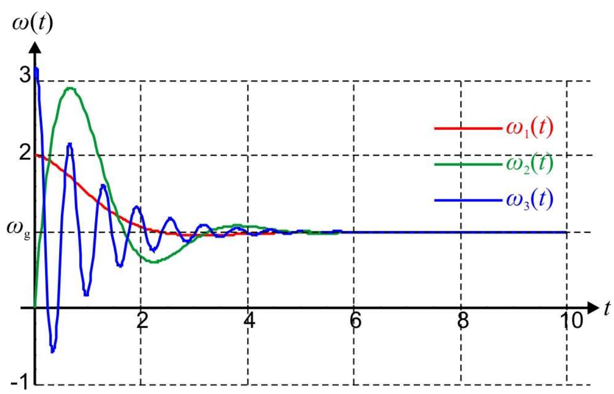

Fig. 3. Examples of waveforms of parametric functions $\omega(t)$.

The impulse response (for system with parametric function $\omega_3(t)$) has been plotted in fig. 4. The response of the system to unit step excitation has been presented in fig. 5. For comparison, in fig.5 a response of stationary section (where $\omega(t)=\omega_g$) has been also shown.

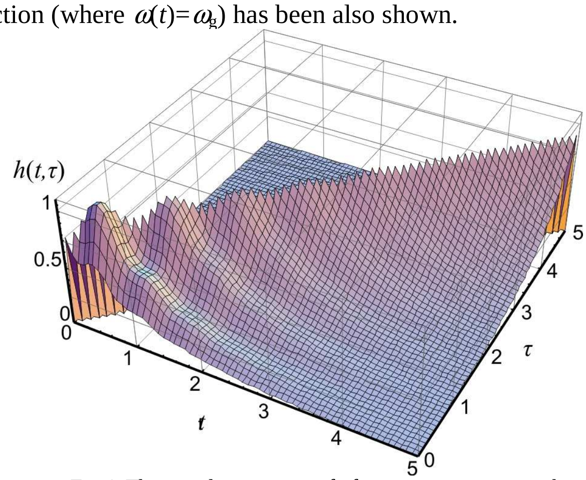

Fig.4. The impulse response of of parametric system with variable parameter ω_3(t)

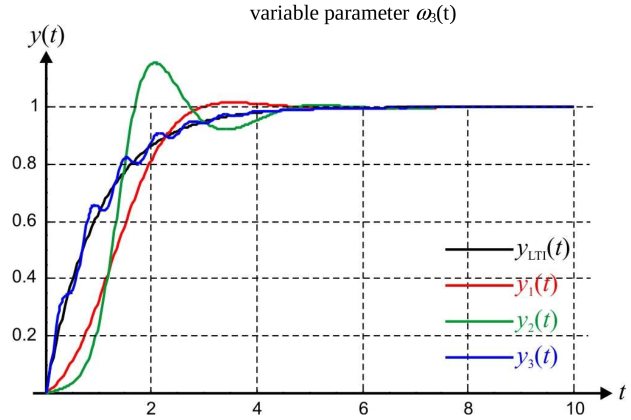

Fig. 5. The step responses of first order parametric sections

IV. CONCLUSIONS

The proposed analytical model of the first order generalised parametric low-pass section with non periodically variable parameter allows determination of the system response to any excitation with finite average power and limited energy.

In opposite to classical stationary sections the impulse responses of parametric section are functions of two variables – time, and moment of application of the excitation to the input of the system. Variability of parameters have an impact to the shape and parameters of the impulse response. In the stationary state the sections with variable parameters are equivalent to stationary sections and the advantages of proposed approach consist in improvement of dynamic properties by a proper choice of parametric function.

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