Analysis of Transients in Transformer Winding Respecting Space-varying Inductance

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Abstract **The paper deals with the influence of the space-varying inductance on transients in the transformer winding. The method for the inductance evaluation respecting the manner of turns layout is presented. Obtained results have been implemented in our algorithm for the numerical analysis of transients in the circuit model with distributed parameters which has been solved in the time domain. The comparison of the time-space voltage distribution for the case with the homogenous inductance and the non-homogenous inductance has been carried-out.**

Keywords **Inductance, transformer winding, distributed parameters, non-homogenous isolation.**

I. INTRODUCTION

Very fast transient phenomena produced by switching-on and switching-off processes can cause the very high overvoltage in the transformer winding. It can result in damaging of this winding. The adequate model of the transformer winding is needed to determine the dangerous place at the winding and the dangerous voltage peak value. The algorithm published at [1] provides the time-space voltage and current distribution which allows receiving such a data. But, for obtaining useful and reliable results is necessary to determinate correct values of model parameters which respect not only the high frequency of surge phenomena but also the arrangement of turns. In [2], [3] was carried-out an evaluation of inductances and capacitances but authors used it for analysis in the frequency domain. They are focused on frequency dependences of these parameters. Our analysis is carried-out in the time domain [4] where the influence of the space-varying capacitance was studied. In this presented paper is introduced an algorithm for an evaluation of the space-varying inductance respecting an arrangement of turns and the inductive coupling between them.

II. TRANSFORMER WINDING MODEL

The algorithm for a numerical solution of very quick transient phenomena in a transformer winding was presented in our previous works. There some types of basic elements of the winding model were discussed. Now, the element depicted in fig. 1 is used. This circuit with space-varying parameters can be described with following partial differential equations of hyperbolic type

$$\begin{aligned}
-\frac{\partial i_{\mathrm{K}}(t,x)}{\partial x} &= G_{\mathrm{K}}(x)\frac{\partial u_K(t,x)}{\partial x} + \mathrm{C}(x)\frac{\partial u(t,x)}{\partial t} \\
&\quad + \frac{\partial i_{\mathrm{L}}(t,x)}{\partial x} + G_{\mathrm{C}}(x)u(t) \\
-\frac{\partial i_{\mathrm{L}}(t,x)}{\partial t} &= \frac{1}{L(x)}\frac{\partial u(t,x)}{\partial x} + \frac{R(x)}{L(x)} i_{\mathrm{L}}(t,x)\,. \qquad (1) \\
-\frac{\partial u_K(t,x)}{\partial t} &= -\frac{i_{\mathrm{K}}(t,x)}{K(x)} \\
-\frac{\partial u(t,x)}{\partial x} &= u_K(t,x)
\end{aligned}$$

This system of equations is solved numerically with the Wendroff´s difference formula. Boundary conditions express the relations between the voltage and the current at the input and the output of the winding.

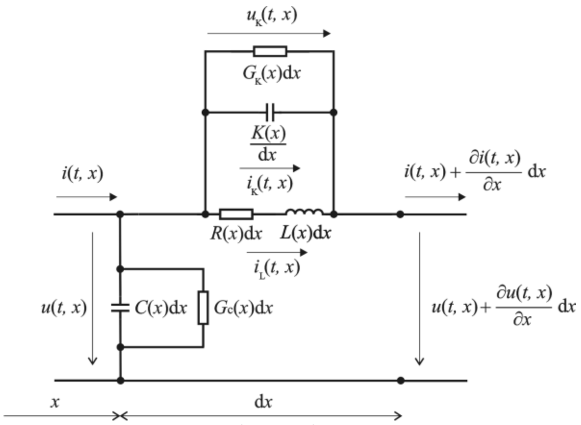

Fig. 1. Basic element for transformer winding model

III. WINDING PARAMETERS

Parameters should respect the arrangement of a given winding. At first, we supposed only a one-layer winding. In this case the turn-to-turn capacitance and the turn-to-earth capacitance can be considered as non-space-varying. We focus on evaluation of the turn inductance respecting the inductive coupling between turns in various distances. The configuration is depicted in fig. 2. The inductive coupling between two turns i and j is given by the Neumann´s formula which can be evaluated analytically or numerically. We used both ways.

$$L_{ij} = \frac{\mu}{4\pi} \oint_{Ci}\oint_{Cj} \frac{\overrightarrow{dl_i}\,\overrightarrow{dl_j}}{r_{ij}} \qquad (2)$$

The analytical solution of integral (2) needs to evaluate the elliptic integrals $E(k)$ and $K(k)$. The self-inductance L_{ii} was calculated numerically via magnetic flux coupled with i-th turn.

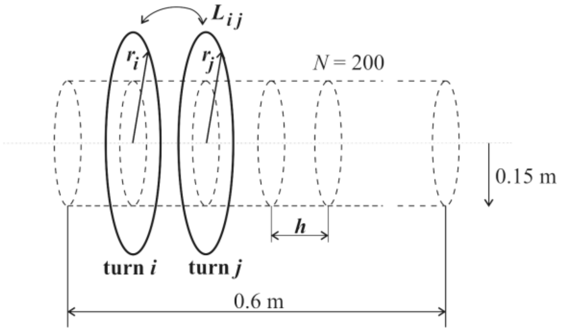

Fig. 2. Inductive coupling between turns *i* and *j*

Having both inductances L_{ii} and L_{ij}, i, j = 1, 2...N for each turn the inductance of i-th turn is given by the following formula

$$L_i = L_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{N} L_{ij} \,. \tag{3}$$

The inductance distribution function $L(x)$ is depicted in fig. 3. With regard to the number of turns N and real length of the winding it is possible to norm this dependence on the norm unit length.

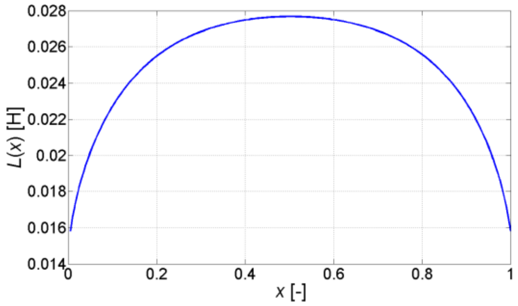

Fig. 3. Space-varying inductance $L(x)$ for norm unit length

IV. ILLUSTRATIVE EXAMPLE

The suggested method was applied on one illustrative example. The transformer winding model with parameters R = 20.7 mΩ/m, C = 20.7 nF/m, K = 2.07 pF/m, G_C = 0.15 nS/m, G_K = 0.15 pS/m was considered. The evaluation was carried-out for the norm unit length. The calculated distribution of the inductance in the winding is depicted in the fig. 3. The numerical analysis of eq. (1) was made for following boundary conditions: the transformer input was connected to the source of surge voltage pulse (0.1 μs/1 μs, 200 V), the inner source resistance R_0 = 2 Ω. The winding output was grounded. Fig. 4 shows the voltage distribution along the whole transformer winding.

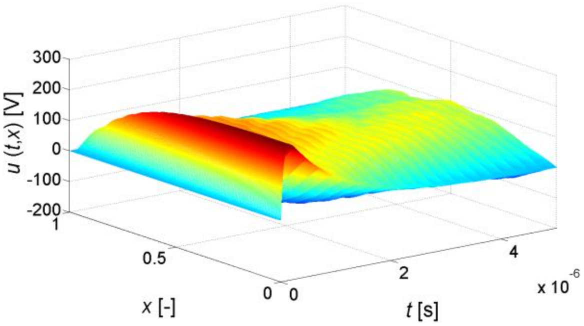

Fig. 4. Voltage distribution in transformer winding

The maximal voltage peak value about 25% higher then nominal one was detected at the inner point of the winding and reached U_{MAX} = 242 V. The evaluation was made for two cases, the first one for the winding with the varying inductance according to fig. 3 and the second one with the non-varying inductance. The average value of the inductance L = 0.027 H/m was used. The comparison is presented in the fig. 5. From curves of the time voltage distribution at the point x = 0.1 (cut of fig. 4) is seen that the influence of the varying inductive coupling causes higher values of the voltages. The different voltage frequency has influence to the time response.

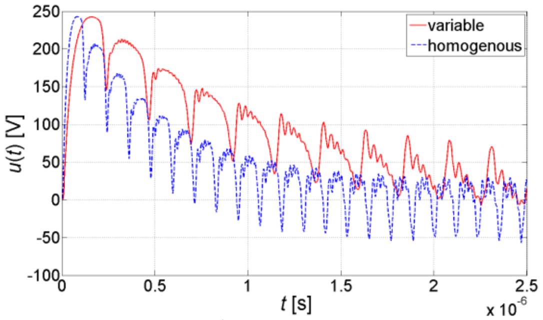

Fig. 5. Voltage distribution for varying and non-varying inductance

V. CONCLUSION

A computer model based on the transmission line approach and taking into account the space-varying inductance of the transformer coil has been presented. It allows respecting a different inductive coupling between turns along the winding. The suggested method for evaluation of the mutual turn-turn inductances is based on the Neumann´s formula. It can be solved analytically or numerically. We applied this algorithm on the one-layer coil but it is possible to extend the proposed method on more-layer coils according to an arrangement of a considered transformer. Obtained results were compared with analysis of the winding model with the non-varying inductances. It was proved that the influence of the varying inductance is not negligible.

VI. ACKNOWLEDGEMENTS

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