Mathematical model of flux-switching permanent magnet machine

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Abstract: **The mathematical model of flux-switching permanent magnet machine based on Cauchy boundary problem is described. This model takes into account machine geometry and design in detail, enables to simulate starting, slow down and other transient modes, allows for iron and steel losses in wide range of speed.**

Keywords: **stator interior permanent magnet machine, flux-switching permanent magnet machine, mathematical model.**

I. INTRODUCTION

Nowadays a growing interest is shown in electrical machine with permanent magnets in stator or so called stator-PM machine [1, 2]. Advantages of this kind of machine are simplicity of rotor and protection magnets from centrifugal force. Moreover rotor temperature rise due to low heat sink from rotor may be a significant problem for low speed machine with permanent magnets on rotor, which cause demagnetization of magnets.

Different kinds of stator-PM machine are shown in [1,2]. Each of them has doubly-salient structure enabling change or periodically interruption of flux closure way. Alternating flux in core induces electromotive force in winding which enables electromechanical energy conversion.

Most explicitly flux closure way changing principal is used in so called flux-switching permanent magnet (FSPM) machine. Some kinds of their design is shown on fig. 1.

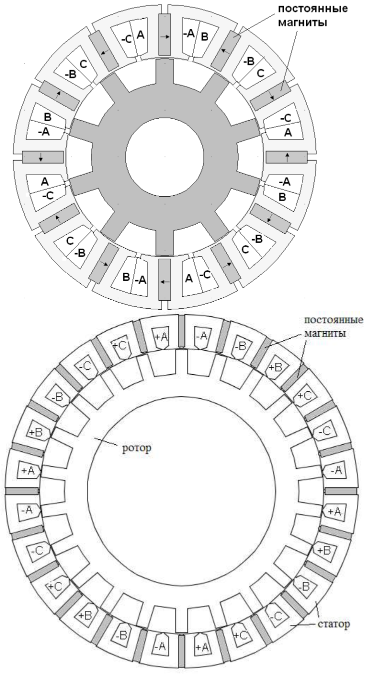

Fig. 1. Two designs of FSPM machine

Static mathematical models based upon boundary problem or equivalent circuit can be developed. However in this case some difficulties appear in estimation of the iron and magnet losses especially in high speed machines. So nowadays there is a lack of methods allowing to simulate and design the FSPM machine which has required outer dimensions torque, speed of the motor shaft and other characteristics.

In this article the FSPM machine model based on boundary initial value problem is discribed. The advantages of the model are:

- Model takes into account machine geometry and design in detail;
- Model enables to simulate starting, slow down and other transient modes.
- Model allows for iron and steel losses in wide range of speed.

II. MATHEMATICAL MODEL

It is very convenient to set constitutive equitation in common shape for all subdomain of the model

$$J_z = \sigma E_z + J_z^e, \qquad (1)$$

$$\mathbf{H} = \frac{\mathbf{B} - \mathbf{B}^r}{\mu\mu_0}.$$

where the conductivity σ is not zero only in permanent magnet subdomains; external current density J_z^e is not zero only in winding subdomains; relative magnetic permeability μ is not unity and highly nonliner only in magnetic core subdomains; μ_0 is space permeability; $\mathbf{B}^r$ is a vector, which sense is different on various subdomain. In permanent magnet subdomain $\mathbf{B}^r$ is as usual remanent magnetic flux density. But in magnetic core subdomain $\mathbf{B}^r$ is a special vector accounting for iron losses.

Let assume that iron losses are approximated by

$$P_i = \alpha \left(\frac{\partial \mathbf{B}}{\partial t} \right)^2, \qquad (2)$$

where α is a coefficient, may be dependant on frequency. The volume power of changing magnetic flux density is $\mathbf{H}\frac{\partial \mathbf{B}}{\partial t} = \frac{\mathbf{B}}{\mu\mu_0}\frac{\partial \mathbf{B}}{\partial t} - \frac{\mathbf{B}^r}{\mu\mu_0}\frac{\partial \mathbf{B}}{\partial t}$. Requiring second summand to be equal P_i we have

$$\mathbf{B}^r = -\mu\mu_0 \alpha \frac{\partial \mathbf{B}}{\partial t}. \qquad (3)$$

For each magnet is electrically open in transversal direction total current through each magnet is zero so we need to include in our model potential difference along machine length given in each permanent magnet domains V_i to make the total current through each magnet equal to zero.

Accepting widely spread assumption that magnetic field is in cross-section plane and currents are perpendicular this plane we have an equation in A_z

$$-\frac{\partial}{\partial x}\left(\frac{1}{\mu\mu_0}\frac{\partial A_z}{\partial x}\right)-\frac{\partial}{\partial y}\left(\frac{1}{\mu\mu_0}\frac{\partial A_z}{\partial y}\right)-\frac{\partial}{\partial x}\frac{B_y^r}{\mu\mu_0}+\frac{\partial}{\partial y}\frac{B_x^r}{\mu\mu_0}=$$

$$=-\sigma\left(\frac{\partial A_z}{\partial t}+\frac{V_i}{L}\right)+J_z^e=J_z\,. \qquad (4)$$

where L and J_z are the machine length and total current density; V_i is potential difference along the machine length. V_i is given for each permanent magnet domain, independant on *x* and *y* inside each magnet domain, but can differ in different magnet domains. So for FSPM with N_m magnets we have N_m additional degrees of freedom V_i which are potential difference along magnets.

Requiring total current through each magnet to be zero we have

$$\iint\limits_{\Omega_i^{mag}} J_z = 0\,, \qquad (5)$$

where Ω_i^{mag} is *i*-th magnet subdomain.

As usual the equations set is complemented with the electrical circuit and mechanical equations which can have some additional degrees of freedom.

For solving the equations set numerically we must add the initial and boundary conditions. As a boundary condition it is convenient to choose absence of a field on sufficient removal from the engine $A_z = 0$. In most engines the yoke of the stator closes magnetic circuit along the entire length of the stator so the magnetic field outside the motor is very small. The computational domain boundary for models of such engines may be the outer diameter. But FSPM machine has complex stator and magnetic flux can close outside the machine. That is why we must add an air subdomain outside the machine.

Initial conditions are the value of the field and of the additional degrees of freedom in the initial time. For the correct formulation of the Cauchy problem we must choose as the initial time such a state in which the degrees of freedom does not depend on the history of the system and can be calculated directly by virtue of the steady-state problem. As the rotor rotates even when the winding is open dissipative processes take place in the steel and in the magnets and state description contains time derivatives. Therefore, as the initial time we must choose a state with stationary rotor and with steady-state or zero currents in the coils.

In the steady-state initial moment there is no electromotive force induced in the magnets, and consequently the initial potential difference along magnet is zero $V_i = 0$.

The initial state of the magnetic field is described by the equation

$$J_z^e=-\frac{\partial}{\partial x}\left(\frac{1}{\mu\mu_0}\frac{\partial A_z}{\partial x}\right)-\frac{\partial}{\partial y}\left(\frac{1}{\mu\mu_0}\frac{\partial A_z}{\partial y}\right)-\frac{\partial}{\partial x}\frac{B_y^r}{\mu\mu_0}+\frac{\partial}{\partial y}\frac{B_x^r}{\mu\mu_0}\,. \qquad (6)$$

To simulate the rotation the computational region is divided with the circle into two areas: area A includes the area of the stator, and half of the air gap, area B includes the area of the rotor and the area of the other half of the air gap.

Components of the magnetic vector potential are joined on the common boundary of areas A and B with time dependant boundary condition.

A_z at the point (x, y) on the boundary of area A sets A_z at the point on the boundary of area B with the coordinates

$$\begin{aligned} x' &= x\cos\theta + y\sin\theta,\\ y' &= -x\sin\theta + y\cos\theta, \end{aligned} \qquad (7)$$

where θ is the angle of rotor position.

To demonstrate the capabilities of the model a trial calculation of the generating mode was carried out in which the power supply terminals is connected to the resistors in a star schema with a common wire, and uniformly accelerated motion of the rotor was assumed.

Calculation result sample is shown in fig. 2. It is seen that the power increases with time, which is due to the increase in angular velocity.

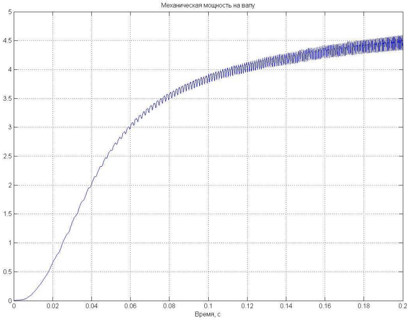

Fig. 2. Calculated power of FSPM machine

III. CONCLUSION

The mathematical model of flux-switching permanent magnet machine based on Cauchy boundary problem is described. It has been shown that dissipative processes in FSPM machine including magnet and iron losses can be allowed for in model based on Cauchy boundary problem.

Besides The described model takes into account machine geometry and design in detail, andenables to simulate starting, slow down and other transient modes.

IV. REFERENCES

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